

# Performance Analysis of MUSIC Algorithm for DOA Estimation

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**Abstract** - Direction of Arrival plays an important role in increasing the throughput and capacity of network by focusing the reception and transmission of signal in the desired direction. In practical scenario multiple signals arrive from multiple sources towards the antenna. The capacity of the system can be increased if multiple signals are focused in desired direction. This paper shows the performance analysis of MUSIC algorithm using uniform linear array. The MUSIC algorithm performs well if the signals are incoherent but if the signal has coherent nature the performance of MUSIC algorithm degrades.

**Key Words:** Antenna array; Direction of Arrival Estimation; Array signal processing; MUSIC Algorithm

## 1. INTRODUCTION

DOA estimation is an important technique in the field of signal processing in which the exact location of the source is find form where the signals are arriving. This technique is used in the array signal processing for the purpose of wireless communication and audio/speech processing systems.

One technique used for DOA estimation is using fixed antenna in which there is one disadvantage of limiting the antennas main lobe beam width which reduces the efficiency and capacity of the antenna.

There is an inverse relationship between antennas main lobe and physical size. So by increasing the physical size for improving the accuracy of angle measurement is not a practical solution[1].

The smart antenna uses array of antenna instead of using a single antenna improves the efficiency and resolution of the DOA estimation. Many methods are proposed for estimating the direction and number of signals. The DOA estimation algorithm proposed in past are Capon, Barlett, Esprit and MUSIC. The MUSIC algorithm is accurate method and has high resolution which is used in design of smart antenna. In this paper MUSIC DOA estimator is simulated in MATLAB 2008 for number of elements, elements spacing, effect of snr, number of snapshots, and its evaluation for coherent signals.

## 2. DATA MODEL

The general equation for received signal in uniform linear array can be written[3] as

$$y(n) = x(n) + v(n) = As(n) + v(n)$$

$$\sum_{i=1}^M R_i z_i^n + v(n) \quad (1)$$

Where,  $y(n)$  is the received signal in a uniform linear array of  $N$  elements.  $M$  is the number of signals with additive white Gaussian noise impinging on the array.  $n = 0, 1, \dots, N-1$  &  $s(n) = [s_1 s_2 \dots s_n]^T$  is a  $M \times 1$  vector of signal amplitudes,  $v(n)$  is a  $N \times 1$  vector of additive noise process at  $i^{th}$  element and  $A = [a(\theta_0), a(\theta_1), \dots, a(\theta_{N-1})]$  is a  $N \times M$  matrix of the steering vectors whose  $i^{th}$  column

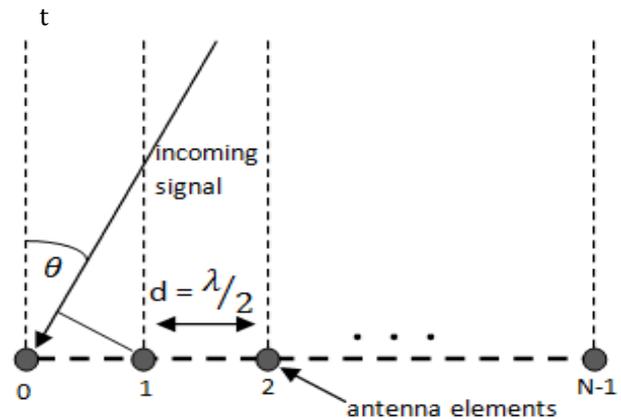


Fig -1: Uniform linear antenna array[3]

$a(\theta_i) = [1, e^{j\varphi_i}, \dots, e^{j(M-1)\varphi_i}]^T$  denotes the steering vector of source coming from antenna and  $\lambda$  is the wavelength of the carrier,  $\varphi_i = \frac{2\pi}{\lambda} d \sin(\theta_i)$ ,  $i=1,2,\dots,M$ .  $R_i$  is the complex amplitude at  $i^{th}$  element and  $z_i = e^{j2\pi d \sin(\theta_i)}$ . Here  $d = \frac{\lambda}{2}$  is the distance between two consecutive array elements. Our objective is to estimate the DOA of the  $M$  incident signals using MUSIC algorithm.

## 3. MUSIC ALGORITHM

MUSIC stands for Multiple Signal Classification which is the most classic DOA techniques based on eigen decomposition

[1]. This algorithm was proposed by Schmidt in 1979. In this algorithm first characteristics decomposition is performed for the covariance matrix for any output data of the array. The characteristics decomposition results in signal subspace which is orthogonal containing a noise subspace in the signal component. The two orthogonal subspaces are used to construct a spectrum function. In the spectrum function the peaks of the signals are searched and DOA signals are detected.

The implementation of the MUSIC algorithm is as follows

- In the first step the estimation of the input covariance matrix is based on N received signal vector or data estimation is done by collecting the data which is in below equation.

$$R_x = E\{x x^H\}$$

- The second important step is to find the eigenvalue which is find by decomposing the covariance matrix from the aforementioned Equation.

$$R_x = A R_s A^H + \sigma^2 I \tag{3}$$

- The eigenvalue is calculated from above equation 2 and then according to order of Eigenvalues take that eigenvector and eigenvalue which are equal to number of signal D as and take that as a part of space.
- The remaining M-D eigenvectors and eigenvalues are taken as a part of noise.
- After this the noise matrix is obtained is through following equation  $E_n = A^H v_i = 0$
- Compute the MUSIC spectrum According to formulae

$$P_{MUSIC}(\theta) = \frac{1}{a(\theta)^H E_n E_n^H a(\theta)} \tag{4}$$

Vary the value of  $\theta$ .

- In the last step spectrum function is calculated and the estimated value of DOA is find by searching the peak values.

#### 4. SIMULATION RESULTS

The simulations were performed in matlab 2007. The simulation parameters were defined as the incident angles are  $10^\circ, 30^\circ$  and  $50^\circ$  respectively [1]. The three signals are independent narrow band signals which are not correlated. The model of the noise is taken as additive white Gaussian noise (AWGN). The SNR is considered as 20 db. The number of element is 10. The spacing of the array element is 10. The spacing of the array element is half of the wavelength of input signal. The number of snapshot taken is 200 in this simulation result shows the relationship between the key factors effecting the DOA estimation and MUSIC algorithm [1][2].

4.1 Basic DOA Estimation for MUSIC Algorithm  
 Figure 2 shows the basic simulation of the MUSIC algorithm in which two independent signals are taken for the hypothetical situation. By using MUSIC algorithm a spectrum peak algorithm can easily be constructed. It can estimate the direction and number of incident signal accurately. If accurate model is used DOA estimation is very accurate in MUSIC algorithm and it performs very well with high resolution in the multiple signal environments.

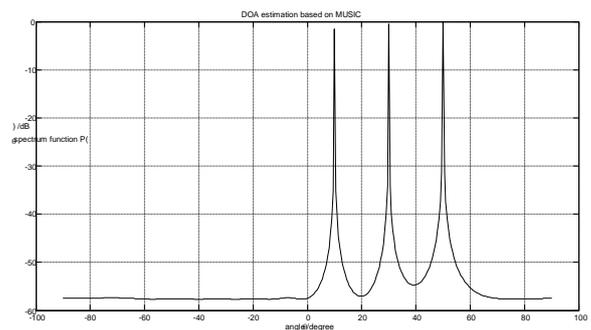
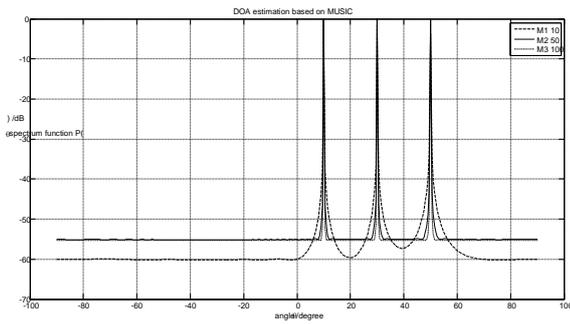


Fig.-2: Music Algorithm

#### 4.2 MUSIC Algorithm and Number of Array Elements

In the figure 3 the relationship between the number of array elements and DOA estimation is shown. With above simulation parameters the array element number is 10, 50 and 100 and the results are shown.



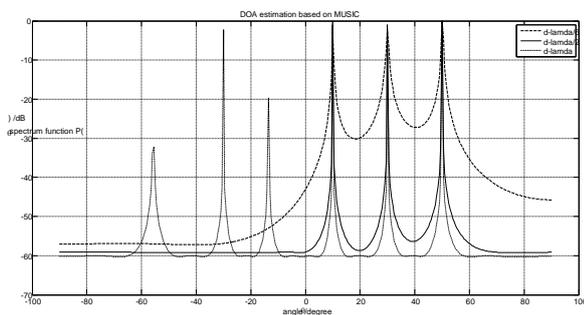
**Fig-3:** Relationship between number of elements and MUSIC algorithm

Keeping simulation parameters same and increasing the number of arrays elements is shown that spectral beam width becomes narrow, directivity increases which enhance the ability to distinguish spatial signals. By increasing the number of array elements the doa estimation is improved but the data processing speed decreases.

### 4.3 MUSIC Algorithm and the Element spacing

In figure 4 relationship between element spacing and DOA estimation is shown. The simulation parameters are same, number of array element is 10 and the array spacing is  $\lambda, \lambda/6, \lambda/2$ .

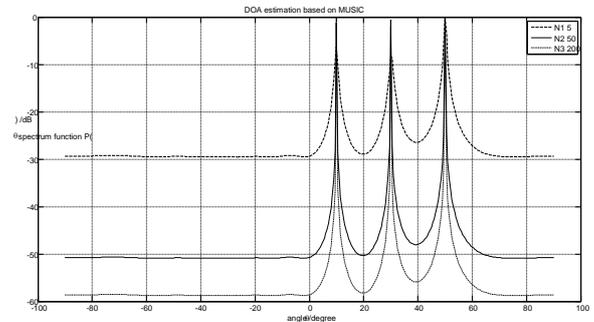
In the below figure it can be seen from the results that as the spacing between array elements is increasing beam width of DOA estimation is becoming narrower. The resolution of MUSIC algorithm increases with increase in spacing, however, when the spacing is greater than  $\lambda/2$ , except for the signal source direction, other directions appeared as false peaks.



**Fig.-4:** Effect of spacing of array of elements.

### 4.3 MUSIC Algorithm and Number of snapshot.

The relationship between snapshot and MUSIC algorithm is shown in figure 4. The parameters are the same just the snapshot is varied to 5, 50 and 200.

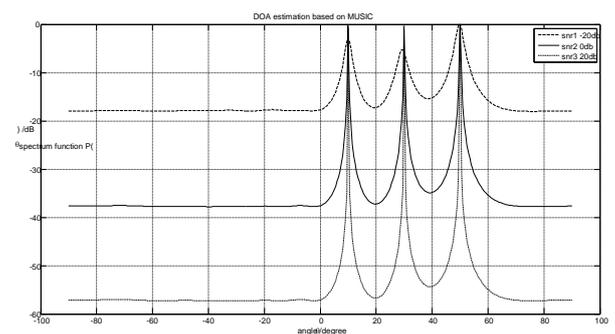


**Fig-5:** Relationship between snapshots on MUSIC algorithm

The result shows that as the number of snapshots is increased the spectrum of DOA estimation increase and accuracy of MUSIC algorithm is also increased but complexity of processing of data is in there.

### 4.4 MUSIC Algorithm and SNR

The figure 6 shows the relationship of SNR and DOA estimation. All other simulation parameters are kept constant, snapshot is kept 200 and SNR is varied -20dB, 0dB and 20dB.



**Fig-6:** Effect of SNR on MUSIC algorithm

All other simulation parameters are kept constant and if the SNR is increased the direction of signal becomes clear and accuracy of MUSIC algorithm improves. The spectrum of DOA estimation becomes narrow.

### 4.5 MUSIC Algorithm and Incident angle

Figure 7 shows the effect of signal incident angle difference. All simulation parameters are kept constant, SNR is 20dB and the incident angle is 25,30 and 60degree.

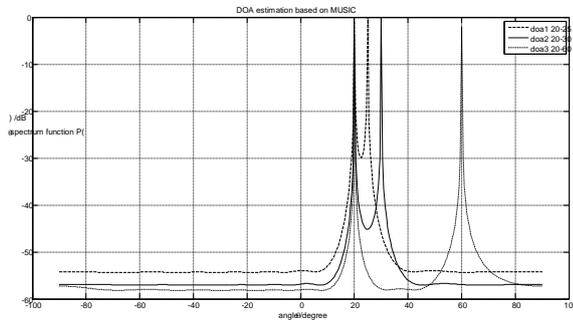


Fig.-7: Effect of incident angle on MUSIC algorithm

The incident angle shows how MUSIC algorithm recognizes three signals. The results shows that if incident angle in increased the resolution of signal is increased and direction of signal is improved.

### 4.6 MUSIC Algorithm for Coherent Signal

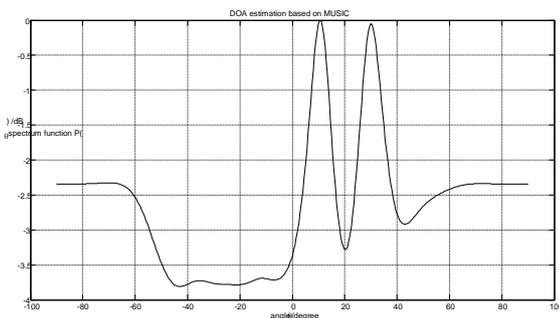


Fig-8: shows the effect of coherent signals on MUSIC algorithm.

## 5. CONCLUSION

In this paper DOA Estimation for uniform linear array using MUSIC algorithm investigated in detail. The simulation results shows that the MUSIC algorithm provide superior resolution due to its inherent Eigen-structure. It can be seen from the simulation that the more the number of element and snapshot, the larger the difference between the incident angles, the greater the array elements spacing, the MUSIC algorithm has a high resolution.

The MUSIC algorithm performed well in the case of incoherent signals but degrade its performance in the presence of coherent signals.

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