DIFFERENTIAL GAME APPROACH FOR THE ANALYSIS OF TWO AREA LOAD FREQUENCY CONTROL

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Abstract—In the traditional interconnected power systems, the LFC plays an important role, which constantly requires different Control Areas (CA) to share the regulation burden of the control area that lacks regulation capacity by providing the power supports via the tie-line. Such process results in extra regulation costs due to wear and tear of generating units which may result in unfairness and CA’s deviate from the LFC command. This situation becomes even more serious with the integration of intermittent renewable energy such as wind and solar power. The main objective of this paper is to control the load and frequency of the power system by using the Differential Game approach. In previous research, the PI controller with Genetic Algorithm (GA) was implemented. But the disadvantage associated with the GA is that it requires the repeated fitness function evolution for complex problem and also the stop criterion is not clear. In the proposed method, the Differential Game (DG) approach is implemented. Despite the challenge associated with the computational complexity, the DG theory is a powerful mathematical tool for solving the multiplayer multi objective optimization problems. With two-area LFC model, non-co-operative equilibrium solutions and two cooperative equilibrium solutions with different time consistency properties are derived. The proposed strategies are then applied to a two-area power system and compared with the traditional Proportional–Integral (PI) controller and optimal controller. The proposed method implemented in MATLAB/Simulink and maintains the stability of the system frequency by balancing the total generation and total loads.

Key Words: Differential Game (DG), Control Area (CA), Load Frequency Control (LFC), Cooperative Solution

1. INTRODUCTION

Electrical power systems are evolving toward the smart grids all over the world, which are generally regarded as the next-generation power grids. More and more advanced techniques of control, communications and computation will be employed. Electric power systems are fundamentally reliant on control, communications, and computation to ensure stable, reliable, and efficient operation. Generators rely on governor and automatic voltage regulator to counter the effects of disturbances that continually affect power systems.

At a higher level, energy management systems use supervisory control and data acquisition to collect data from expansive power systems and then use sophistication analysis tools to ensure secure and economic operational conditions. Load Frequency Control (LFC) is a distributed closed-loop control scheme that optimally reschedules generator power set points to maintain frequency and tie-line flows at their specified values. With the development of renewable energy such as wind and solar power, power system must overcome a number of technical difficulties to deliver renewable energy in significant quantities. Control is one of the key technologies for accommodation of renewable energy into power systems.

A game-theoretic approach to cooperative control by highlighting the connection between cooperative control problems and potential games is proposed. A new class of games and enhanced existing learning algorithms is introduced to broaden the applicability of game-theoretic methods to cooperative control setting. The cooperative control of multiple agents under different communication scenarios is discussed. Cooperative control laws are proposed with the aid of suitable transformations and results from graph theory. Stable control laws robust to communication delays are also developed and studied. The different control trajectories are compared for the two area system.

2. LOAD FREQUENCY CONTROL

The power systems is the interconnection of more than one control areas through tie lines. The generators in a control area always vary their speed together (speed up or slow down) for maintenance of frequency and the relative power angles to the predefined values in both static and dynamic conditions. If there is any sudden
load change occurs in a control area of an interconnected
power system then there will be frequency deviation as
well as tie line power deviation.

The two main objective of Load Frequency
Control (LFC) are

- To maintain the real frequency and the desired
  power output (megawatt) in the interconnected
  power system.
- To control the change in tie line power between
  control areas.

If there is a small change in load power in a
single area power system operating at set value of
frequency then it creates mismatch in power both for
generation and demand. This mismatch problem is
initially solved by kinetic energy extraction from the
system, as a result declining of system frequency occurs.
As the frequency gradually decreases, power consumed
by the old load also decreases. In case of large power
systems the equilibrium can be obtained by them at a
single point when the newly added load is distracted by
reducing the power consumed by the old load and power
related to kinetic energy removed from the system.
Definitely at a cost of frequency reduction we are getting
this equilibrium. The system creates some control action
to maintain this equilibrium and no governor action is
required for this. Each output frequency finds the
information about its own area and the tie line power
deviation finds the information about the other areas.
Thus the load frequency control of a multi area power
system generally incorporates proper control system, by
which the area frequencies could bring back to its
predefined value or very nearer to its predefined value
so as the tie line power, when the is sudden change in
load occurs.

2.1 INTERCONNECTED SYSTEM

According to practical point of view, the load frequency
control problem of interconnected power system is
much more important than the isolated (single area)
power systems. Whereas the theory and knowledge of an
isolated power system is equally important for
understanding the overall view of interconnected power
system.

Generally, now days all power systems are tied with
their neighboring areas and the Load Frequency Control
Problem become a joint undertaking. Some basic
operating principle of an interconnected power system
is written below:

- The loads should strive to be carried by their
  own control areas under normal operating
  conditions, except the scheduled portion of the
  loads of other members, as mutually agreed
  upon.
- Each area must have to agree upon adopting,
  regulating, control strategies and equipment
  which are beneficial for both normal and
  abnormal conditions.

2.2 Two Area Power System

If there is interconnection exists between two control
areas through tie line than that is called a two area
interconnected power system. Fig-1 shows a two area
power system where each area supplies to its own area
and the power flow between the areas are allowed by
the tie line.

![Fig-1: Two Area Interconnected Power System](image)

In this case of two area power system an assumption is
taken that the individual areas are strong and the tie line
which connects the two area is weak. Here a single
frequency is characterized throughout a single area;
means the network area is 'strong' or 'rigid'. There may
be any numbers of control areas in an interconnected
power system.

2.3 Model of A Two Area LFC

The block diagram model of two area LFC is shown in
Figure 3.3.

where, the control command inputs are changed from
the requested deviation of generator output ΔPci to its
adjusting speed Δui. Using Δui, it is more convenient to
limit the damping rate of the units and to reflect the
wear and tear of the units in the control objective
function. We have the following linear model to describe
the system dynamics

\[ x(t) = A x_d(t) + B u_d(t) + E \Delta P_d(t) \]
system gain, the electric system time constant, the tie-line synchronizing coefficient, and the speed drop, respectively. Considering the intermittent energy in the system, the power perturbation $\Delta P_{di}$ is considered as the external perturbation input, consists of two parts as

$$\Delta P_{di} = \Delta P_{li} - \Delta P_{el}$$

where, $\Delta P_{li}$ and $\Delta P_{el}$ are the perturbations of loads and intermittent energy output, respectively. We assume that both as $\Delta P_{li}$ and $\Delta P_{el}$ are known constants during the control period obtained by some short-term forecasting technology. Hence $\Delta P_{di}$ is a known step perturbation.

3. CLASSIFICATION OF DIFFERENTIAL GAME

The cooperative differential game is used to control the control areas in the multi area power system. Based on the stability criterion the games can be classified as dynamic games, static games and state space games. The main aim of this game is to attain the stability of the system in a shorter duration, during the frequency deviation.

3.1 Cooperative Games

A fundamental element in the theory of cooperative games is the formulation of the optimal behavior for the players satisfying specific optimality principles. From the solution of the cooperative games, the control action can be performed among the players. In particular, the solution of a cooperative game is produced by a set of optimality principles [for instance, the Nash bargaining solution].

A cooperative solution is sub game consistent if an extension of the solution policy to a situation with a later starting time and any possible state brought about by the prior optimal behavior of the players remains optimal. In the field of cooperative stochastic differential games, little research has been published to date due to the inherent difficulties in deriving tractable sub game consistent solutions. A sub game consistent solution is developed for a class of cooperative stochastic differential games with nontransferable payoffs.

3.2 Dynamic Games

In the dynamic games a stringent condition on their solutions is required: the specific optimality principle must remain optimal at any instant of time throughout the game along the optimal state trajectory chosen at the outset. This condition is known as dynamic stability or time consistency. The issue of dynamic stability in differential games has been explored rigorously in the past three decades; In the presence of stochastic

\[
\begin{align*}
A &= \begin{bmatrix}
-1/T_p & K_p/T_p & 0 & 0 & 0 & 0 & -K_p/T_p & 0 & 0 \\
0 & -1/T_q & 1/T_q & 0 & 0 & 0 & 0 & 0 & 0 \\
-1/T_{qi} & 0 & -1/T_{qi} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1/T_{pi} & K_p/T_{pi} & 0 & K_p/T_{pi} & 0 & 0 \\
0 & 0 & 0 & 0 & -1/T_{qi} & 1/T_{qi} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1/T_{qi} & 0 & 0 & -1/T_{qi} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},

B &= \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}^T,

E &= \begin{bmatrix}
-K_p/T_p & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -K_p/T_p & 0 & 0 & 0
\end{bmatrix}
\]

With subscript $i$ representing the $i_{th}$ CA, $\Delta f_i$, $\Delta P_{gi}$, $\Delta P_{tie}$ and $\Delta P_{ei}$ are the deviation of frequency, generator mechanical output, valve position, tie-line power, and requested generator output, respectively as $T_p$, $T_q$, $K_p$, $T_{pi}$, $T_{qi}$ and $r$ are the time constant of the governor, the time constant of the turbine, the electric

\[
\begin{align*}
\text{Fig-2: Two Area LFC}
\end{align*}
\]
elements, an even more stringent condition that of subgame consistency is required.

The previously intractable problem of obtaining subgame consistent cooperative solution has now been rendered tractable for the first time. If the players in a game wish to make an agreement to share the benefits of cooperative behavior, the axiom states that no player is willing to accept an agreement that will give her less payoff than what she could obtain by rejecting to participate in the cooperative solution.

3.3 Static Games

In a static game, the issue of individual rationality can be resolved since one can determine what each player will be offered as her part of an agreed solution and what amount of payoff she could secure for herself in case of no agreement. Comparing these payoffs provides an answer to the individual rationality question. In games evolving overtime, resolving the problem of individual rationality may not be easy. The reason is that individual rationality may fail to apply when the game has reached a certain position, despite the fact that it was satisfied at the outset. This phenomenon is notable in differential, and instate-space games as such. Studies in repeated games have developed conditions under which individually rational outcomes can be sustained as equilibrium overtime.

An important aspect of this is that cooperative outcomes may be supported by non cooperative equilibrium strategies, but it raises the question if individually rational outcomes exist. In repeated games with complete information and perfect monitoring, the answer is yes since the players face the same game at every stage.

3.4 State-Space Games

In state-space games the situation is different. In a discrete-time setup, a stochastic game includes a state variable that evolves over time, as a product of the initial conditions, the players’ actions, and a transition law. Particular results exist for situations in which Pareto-optimal outcomes are supported by trigger strategies. Note that the punishments underlying trigger strategies are effective only if cooperation is individually rational throughout the game. For only in this case punishments will hurt a defector. Thus, one reason for studying inter temporal individual rationality is that it is an important component of a self-enforcing agreement.

Such instabilities arise from the simple reason that in general the position of the game at an intermediate instant of time will differ from the initial position. Therefore, the original solution may not be the solution of the subgame that starts out in an intermediate position of the game. The problem is that although individual rationality was satisfied initially, it may not be satisfied at intermediate positions of the game.

4. PROPOSED METHOD BASED COOPERATIVE CONTROL MODEL OF LFC

Consider a two-player differential game played on a prescribed time interval \([t_0, T]\) where the time horizon \(T\) may be infinite. The state equation is,

\[
\dot{x}(t) = f(x(t), u_1(t), u_2(t), t), x(t_0) = x_0 \quad (1)
\]

In which, \(x(t) \in X, u_i(t) \in U_i, i = 1, 2, \) Here \(X\) denotes the state space and \(U_i\) is the set of admissible controls of player \(i\). We refer to the pair \((t, x) \in [t_0, T] \times X\) as the position of the game at time \(t\). If the game is played non cooperatively, players use markovian strategies given by the pair \((s(t, x), x(t, x))\). For each \(t \in [t_0, T]\) and \(x \in X\), the resulting cooperative strategies are represented.

The payoff functional to be maximized by player \(i\) is

\[
I_i = I_i(u(\cdot); t_0, x_0) = \int_{t_0}^{T} g_i(x(t), u(t), t) dt + S_i(x(t), T) \quad (2)
\]

In equation \((2)\), the function \(g_i\) represents player’s instantaneous payoff and function \(S_i\) is terminal payoff. If the horizon is infinite, put the terminal payoffs equal to zero. The initial position \((t_0, x_0)\) is fixed.

The players negotiate at the initial position to establish an agreement on how to play the cooperative dynamic game (and how to distribute the resulting payoff) using equation \((2)\). An agreement must satisfy group rationality and thus we consider cooperative outcomes belonging to the Pareto optimal frontier. Such outcomes can be found by solving at \((t_0, x_0)\) the optimal control problem given by

\[
\max_{u_i\in U_i} \sum_{i=1}^{2} [\mu I_1 + (1 - \mu) I_2] \quad (3)
\]

The problem of time consistency arises due to the following fact Clearly, the problem disappears if agreements are binding or if the players have pre committed.

Players bargain at the initial instant of time and arrive at an agreement which is individually rational at this particular position. To present the scenario in a transparent way, we introduce three assumptions.
Assumption 1

Reconsidering the game at any instant \( T > t_0 \), the players have cooperated from time \( t_0 \) till time \( T \).

Assumption 2

At any position \((T, x_\mu^*(T))\) one of the two situations may occur:

- If the players agree, they reopen negotiations and may choose to replace the original agreement with a new one. If implemented, the new agreement is supposed to be used throughout the time interval \([t, T]\).
- Any player can reconsider her participation in the agreement, and decide to deviate from it. If a player deviates, she abandons her cooperative strategy throughout the time interval \([t, T]\).

Assumption 3

If a player at time \( T \) deviates from her cooperative control path, she switches to her disagreement strategy.

In the Cooperative control model of LFC, the different types of controllers are used to control the control areas of the LFC. The conventional controllers are PI controllers and the optimal controllers. These conventional controllers are combined with the Differential Game based controller to achieve the best control action as shown in Figure 1.

4.1 PI Controller

As the name suggests it is a combination of proportional and an integral controller the output (also called the actuating signal) is equal to the summation of proportional and integral of the error signal. Now let us analyze proportional and integral controller mathematically. As we know in a proportional and integral controller output is directly proportional to the summation of proportional of error and integration of the error signal, writing this mathematically we have,

\[
A(t) = \beta_1 + \Delta f(s) \cdot \Delta f(t) \cdot dt + \Delta(t) \cdot e(t)
\]  

(4)

Removing the sign of proportionality we have,

\[
A(t) = K_i \int_0^t e(t) \cdot dt + K_p e(t)
\]  

(5)

Where \( K_i \) and \( K_p \) are proportional constant and integral constant respectively.

\[\begin{align*}
\Delta f(s) & = K_i \int_0^t e(t) \cdot dt + K_p e(t) \\
\Delta P_c(s) & = \frac{K_i}{s} \Delta f(s) \\
\Delta P_d(s) & = \frac{\Delta P_d}{s}
\end{align*}\]

Fig-3: Block Diagrams of Three Kinds of Controllers

(a) PI Controller

(b) Controller Based on DGs or Optimal Control

Using the conventional control strategy, we can control the dynamic frequency response and also make the steady state error to zero with changes in load. An integral controller is added to the un-controlled system which actuates the speed changer by real power command signal \( \Delta P_c \).

\[
\Delta P_c = -K_i \int \Delta f dt
\]  

(6)

The negative polarity must be chosen so as the frequency error will give a negative control to reduce the value of command signal.

Area Control Error (ACE), is

\[
ACE = \Delta f
\]  

(7)

Taking Laplace transform of equation 4.6, we get \( \Delta P_c \)

\[
\Delta P_c(s) = \frac{K_i}{s} \Delta f(s)
\]  

(8)

for step input load

\[
\Delta P_d(s) = \frac{\Delta P_d}{s}
\]  

(9)
Using final value theorem, we readily obtain from the above equation the static frequency droops: 

\[ \Delta f(s) = \frac{K_p}{(1+sT_p+s^2)} \Delta P_d \]

\[ = \frac{R K_p s (1+sT_p)(1+sT_r)}{s(1+sT_p)(1+sT_r)} \Delta P_d \quad (10) \]

Using final value theorem, we readily obtain from the above equation the static frequency droops:

\[ \Delta f_{steady} = \lim_{s \to 0} [s \Delta f(s)] = 0 \text{ ie. No error.} \]

Thus, by using the PI controller, the steady state error can be made to zero.

4.2 Optimal Controller

The theory of optimal control is concerned with operating a dynamic system at minimum cost. The case where the system dynamics are described by a set of linear differential equations and the cost is described by a quadratic function is called the LQ problem. One of the main results in the theory is that the solution is provided by the Linear-Quadratic Regulator (LQR). The dynamics of a structure is represented in state space form as:

\[ \dot{x} = Ax + Bu; x \in \mathbb{R}^n, u \in \mathbb{R}^m \quad (11) \]

Where, A and B are the system matrices. The output equation can be written as,

\[ y = Cx \quad (12) \]

Now, assume a static output feedback of the form, \( U = Gy \)

Where, G is the gain matrix.

The objective is to design a controller by choosing a proper controller gain G, which is also optimal in the sense that it minimizes a performance index:

\[ J(G) = \int_{0}^{\infty} (x^TQx + u^TRu)dt \quad (13) \]

Here, \( x_0 = X_0 \)

The optimization results in a set of non-linear coupled matrix equations which is given by,

\[ KM + M'K + Q + C'RG = 0 \]

\[ LM' + ML + X_0 = 0 \]

\[ GCLC' + R^{-1}B'KLC = 0 \]

4.3 Controller Based on DG

The individual payoff function of the ith CA in LQDG based cooperative control is given by the following quadratic performance metric:

\[ \int_{t_0}^{\infty} \frac{1}{2} [x^{-T}(t)Q_i x(t) + u^{-T}(t)R_i u(t)]dt, \quad (14) \]

\[ i=1,2 \]

Where, \( Q_i \) is a 9 x 9 positive definite weight matrix that determines the penalty associated with the frequency error and tie-line power error, and \( R_i \) is a 2x2 positive definite weight matrix that represents the control costs. By setting \( Q_i \) and \( R_i \), each CA specifies the way it wishes its LFC units to perform.

However, the term \( E_i \Delta P_d \). To eliminate this term, we employ the method by redefining the state, control, and disturbance vectors by using their final steady state values,

\[ x(t) = \bar{x}(t) - \bar{u}u(t) = \bar{u} - \bar{u} \]

\[ \Delta P_d(t) = \Delta P_d(t) - \Delta P_d(t). \quad (15) \]

Under a step power perturbation, the deviations of steady state frequency, tie-line power, and adjusting speed of the unit’s power generation are required to be zero. The cooperative control for the two-area LFC then takes the following form:

\[ \min_{u} J_i = \int_{t_0}^{\infty} \frac{1}{2} [x^{-T}(t)Q_i x(t) + u^{-T}(t)R_i u(t)]dt \quad (16) \]

\[ s.t. \bar{x}(t) = Ax(t) + Bu(t) x(t) = x_0 \]

The objective of each CA is to minimize its own payoff function by choosing its appropriate control.

5. ANALYSIS OF RESULT

The simulation of the two area AGC system with the PI controller, Optimal controller and the control based on Differential Game is analyzed from the resulting waveform of frequency deviation. These are analyzed based on the aspect of the peak overshoot, settling time and steady state error.
5.1 Frequency Deviation in Area One with PI Controller

Fig-4: Frequency Deviation in Area One with PI Controller

5.2 Frequency Deviation in Area Two with PI Controller

Fig-5: Frequency Deviation in Area Two with PI Controller

5.3 Tie-Line Frequency Deviation Between Area One and Two with PI Controller

Fig-6: Tie-Line Frequency Deviation Between Area One and Two with PI Controller

5.4 Tie-Line Frequency Deviation Between Area One and Two with Optimal Controller

Fig-7: Tie-Line Frequency Deviation Between Area One and Two with Optimal Controller

5.5 Tie-Line Frequency Deviation Between Area One and Two with DG Technique Combining PI and Optimal controller

Fig-8: Tie-Line Frequency Deviation Between Area One and Two with DG Technique Combining PI and Optimal controller

5.6 Comparison and Evaluation of Results

Figure 5.6 shows the change in tie line power between area one and two of the power systems with respect to time, while employing PI controller. From the response the following observation are obtained.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak overshoot</td>
<td>0.025</td>
</tr>
<tr>
<td>Settling time</td>
<td>35 sec</td>
</tr>
<tr>
<td>Steady state error</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Figure 5.7 shows the change in tie line power between area one and two of the power systems with respect to time, while employing optimal controller. From the response the following observation are obtained.

| Peak over shoot | = 0.014 |
| Settling time | = 28 sec |
| Steady state error | = 0.005 |

Figure 5.8 shows the change in tie line power between area one and two of the power systems with respect to time, while employing Differential Game technique by combining the PI controller and Optimal Controller. From the response the following observation are obtained.

| Peak over shoot | = 0.029 |
| Settling time | = 21 sec |
| Steady state error | = 0.01 |

6. CONCLUSION

The two-area thermal LFC problem is investigated under the framework of DG-based cooperative control. Three kinds of equilibrium solutions have been derived using the DG theory, with several simplification and approximations to cope with the computational difficulties. Case studies are provided by using various LFC schemes. Simulation results show that unlike the conventional PI controllers and the optimal controllers, the DG-based cooperative controllers assign the persuasive amount of LFC regulation to each CA to guarantee the stable and faithful implementation of control command. Despite the challenge associated with the computational complexity, the DG theory is a powerful mathematical tool for solving the multiplayer multi objective optimization problems. The system is more stable, while using this Differential game technique instead of the conventional PI and Optimal controllers.

REFERENCES


