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Representation of Numerical Solution Proposal for Initial Value Problems.

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Abstract - *In this paper a new numerical proposal which focuses to solve some difficult initial value problems of ordinary differential equations. The complete representation of the new numerical scheme derivation is presented. In my research, I examined the main features of the proposal such as consistency, convergence and reliability. The extension of this new numerical proposal should be worked on and comparison is also made with some available methods.*

***Key Words*: Numerical proposal,Ordinary Differential Equation,Proposal extension**

1.INTRODUCTION *(Size 11 , cambria font)*

Many ordinary differential Equations are easily solved by the help of integration or by the use of some standard methods but some times many practical applications in various field like as Engineering and Science, the used differential equations can not be solve by the use of integration or standard method to given analytical solution, but rather need to be solved numerically. Some Numerical Analysts have developed schemes for the solution of some initial value problems of ordinary differential Equations. The efficiency of all their efforts where for stability, accuracy, convergence and consistency. The accuracy can be considered an order and convergence and truncating error coefficients.

In this paper ,a new proposal scheme is developed with some special characteristics to solve initial value problems of ordinary differential equations based on local representation of its form.

$$y' = \frac{dy}{dx} = f(x,y), \qquad y(x_0) = y_0 ------------ (1)$$

The required solution of above Equation (1) is

$$f(x) = A_0 + A_1x^2 + A_2e^{x^2} + B\sin x^2 --------------- (2)$$

Where A_0, A_1, A_2 & B are the real undetermined Coefficients .

2. Derivation of the New Proposal:

We shall assume that y_k is a numerical estimate to the theoretical solution $y(x) \& f_k = f(x_k, y_k)$

We define mesh points as follows.

$$x_k = A + kh, k = 0,1,2,3,4, \dots \dots \dots$$

Here we will process to the new proposal for derivation as follows.

$$f'(x) = 2A_1x + 2xA_2e^{x^2} + 2Bx\cos x^2 ----------------- (3)$$

$$f''(x) = 2A_1 + 2A_2\left(2x^2e^{x^2} + e^{x^2}\right) - 2B\left(2x^2\sin x^2 - \cos x^2\right) -------- (4)$$

Similarly

$$f'''(x) = 2A_2\left(4x^3e^{x^2} + 6xe^{x^2}\right) - 2B\left(4x^3\cos x^2 + 6x\sin x^2\right) --------- (5)$$

$$f^{iv}(x) = 2A_2\left(8x^4e^{x^2} + 24x^2e^{x^2} + 6x^2\right) - 2B\left(-8x^4\sin x^2 + 24x^2\cos x^2 + 6\sin x^2\right) -- \quad (6)$$

From Equation (2)

$$f(x) = A_0 + A_1x^2 + A_2e^{x^2} + B\sin x^2$$

$$A_0 = f(x) - A_1x^2 - A_2e^{x^2} - B\sin x^2 --------------------- (7)$$

From Equation (3)

$$A_1 = \frac{f'(x) - 2xA_2e^{x^2} - 2Bx\cos x^2}{2x} --------------------- (8)$$

From Equation (4) we get

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$$A_2 = \frac{f''(x) - 2A_1 + 2B(2x^2 \sin x^2 - \cos x^2)}{2(2x^2 e^{x^2} + e^{x^2})} ------------------ (9)$$

From Equation (5)

$$B = \frac{f'' - 2A_2(4x^3 e^{x^2} + 6x e^{x^2})}{-2(4x^3 \cos x^2 + 6x \sin x^2)} ----------------------(10)$$

By putting Equation (8) in to the Equation (9) we get

$$A_2 = \frac{x f''(x) - f'(x) + 4Bx^3 \sin x^2}{2x^3 e^{x^2}} ------------------(11)$$

Now putting Equation (11) in to the Equation (10)

$$B = \frac{x^2 e^{x^2} f'''(x) - (4x^3 + 6x) f''(x) e^{x^2} + (4x^2 + 6) f'(x) e^{x^2}}{8x^5(2\sin x^2 - \cos x^2) e^{x^2} + 12x^3 \sin x^2 e^{x^2}} --------(12)$$

Putting Equation (12) in to Equation no (11)

$$A_2 = \frac{4x^3 \sin x^2 (4x^2+6) e^{x^2} f'(x) + (-8x^6 \cos x^2 + 12x^5 \sin x^2 - 24x^4 \sin x^2) e^{x^2} f''(x) + 4x^5 \sin x^2 e^{x^2} e^{x^2} f'''(x)}{\{16x^8(2\sin x^2 - \cos x^2) + 24x^6 \sin x^2\}(e^{x^2})^2}$$

-------------------- (13)

Putting Equation no (12) & Equation no (13) in to the equation no (8) ,we get

$$A_1 = \frac{1}{2x} f'(x) - A_2 e^{x^2} - B \cos x^2$$

$$= \frac{1}{2x} f'(x) - e^{x^2} \left\{ \frac{4x^3 \sin x^2 (4x^2+6) e^{x^2} f(x) + (-8x^6 \cos x^2 + 12x^5 \sin x^2 - 24x^4 \sin x^2)}{\{16x^8(2\sin x^2 - \cos x^2) + 24x^6 \sin x^2\}(e^{x^2})^2} \right\} - \cos x^2 \left\{ \frac{x^2 e^{x^2} f'''(x) - (4x^3 + 6x) e^{x^2} f''(x) + (4x^2+6) f'(x) e^{x^2}}{8x^5(2\sin x^2 - \cos x^2) e^{x^2} + 12x^3 \sin x^2 e^{x^2}} \right\}$$

$Let X =$

$$e^{x^2} \left\{ \frac{4x^3 \sin x^2 (4x^2+6) e^{x^2} f(x) + (-8x^6 \cos x^2 + 12x^5 \sin x^2 - 24x^4 \sin x^2)}{\{16x^8(2\sin x^2 - \cos x^2) + 24x^6 \sin x^2\}(e^{x^2})^2} \right\}$$

And

$Y =$

$$\cos x^2 \left\{ \frac{x^2 e^{x^2} f'''(x) - (4x^3 + 6x) e^{x^2} f''(x) + (4x^2+6) f'(x) e^{x^2}}{8x^5(2\sin x^2 - \cos x^2) e^{x^2} + 12x^3 \sin x^2 e^{x^2}} \right\}$$

Hence the above equation become

$$A_1 = \frac{1}{2x} f'(x) - X - Y$$

..............................(15)

Now we applying the following on the interpolating function (2) in the following pattern:

(1) The interpolating function (2) must coincide with the theoretical solution at

$x = x_k \& x = x_k + 1$,

Such That

$$F(x_k) = f(x) = A_0 + A_1 x_k^2 + A_2 e^{x_k^2} + B \sin x_k^2$$

$$F(x_{k+1}) = A_0 + A_1 x_{k+1}^2 + A_2 e^{x_{k+1}^2} + B \sin x_{k+1}^2$$

(2) The derivatives f'(x) ,f''(x) and fk(x) are coincide with f(x), $f'(x)$& fk-1(x) Respectively that is

$$f'(x) = f_k$$

$$f''(x) = f_k'$$

$$f'''(x) = f_k''$$

$$f^4(x) = f_k'''$$

If $f(x_{k+1}) - f(x_k) = y_{k+1} - y_k$

From the Condition (1)&(2) above, it follows that,

$$\Rightarrow A_0 + A_1 x_{k+1}^2 + A_2 e^{x_{k+1}^2} + B \sin x_{k+1}^2 - (A_0 + A_1 x_k^2 + A_2 e^{x_k^2} + B \sin x_k^2) = y_{k+1} - y_k$$

$$A_1(x_{k_{+1}}^2 - x_k^2) + A_2\left(e^{x_{k+1}^2} - x_k^2\right) + B(\sin x_{k_{+1}}^2 - \sin x_k^2) = y_{k+1} - y_k$$

$$\Longrightarrow y_{k+1} = y_k + A_1(x_{k_{+1}}^2 - x_k^2) + A_2\left(e^{x_{k+1}^2} - x_k^2\right) + B(\sin x_{k_{+1}}^2 - \sin x_k^2) \quad \text{------}(16)$$

$Let x_k = A + kh then$

$$x_k^2 = (A+kh)^2 = A^2 + 2Akh + (kh)^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(17)$$

And also $x_{k_{+1}} = A + (k+1)h$

So

$$x_{k_{+1}}^2 = A^2 + 2Akh + 2Akh + 2Ah + (kh)^2 + 2kh^2 + h^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(18)$$

From Equation no (16)

$$x_{k+1}^2 - x_k^2 = 2h(A+kh) + h^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(19)$$

Similarly

$$e^{x_{k+1}{}^2} - e^{x_{k+1}{}^2} = e^{(A^2+2Akh+(kh)^2}\{e^{2h(A+kh)+h^2}\} \quad \ldots\ldots\ldots\ldots\ldots(20)$$

$$Sinx_{k_{+1}}^2 - Sinx_k^2 = Sin(A^2 + 2Akh + 2Ah + (kh)^2 + 2kh^2 + h^2) - Sin(A^2 + 2Akh + (kh)^2) \ldots\ldots(21)$$

Putting Equation (17) through (21) in to Equation (16), we get the new proposal as follows

$$y_{k_{+1}} = y_k + \left[\frac{1}{2x_k} f'(x_k) - X - Y\right]\{2h(A+kh) + h^2\}$$

$$+\left[\frac{4x^3\sin x(4x^2+6)e^{x_k^2}f'(x) - \{8x^6\cos x^2 - 12x^5\sin x^2 + 24x^4\sin x^2\}e^{x_k^2}f''(x) + 4x^5\sin x^2 e^{x_k^2}f'''(x)}{\{16x^8(2\sin x^2 - \cos x^2) + 24x^6\sin x^2\}\{e^{x_k^2}\}}\right]$$

$$\left[e^{(A^2+2Akh+}(kh)^2\right]\left[e^{2h(A+kh)+h^2}\right]$$

$$+ \left\{\frac{x^2 e^{x_k^2} f'''(x) - (4x^3+6x)e^{x_k^2} f''(x) + (4x^2+6)f'(x)e^{x_k^2}}{8x^5(2\sin x_k^2 - \cos x_k^2)e^{x_k^2} + 12x^3 \sin x^2 e^{x_k^2}}\right\}[sin(A^2 + 2Akh + 2Ah + (kh)^2 + 2kh^2 + h^2) - sin(A^2 + 2Akh + (kh)^2] \quad \ldots\ldots\ldots\ldots\ldots\ldots(22)$$

Equation (22) in the new proposal of the numerical solution of ordinary differential equation with the given initial values.

3. Conclusion:

We wish to introduce a new proposal which can consistently true with the existing ones for the numerical solution of some standard initial value problems of ordinary differential equations.So this paper has been able to introduce the new proposal as proposed.In our research . we shall give more attention on the implementation of this new proposal to solve some standard initial value problems of the form of equation(1),and also match the result with the existing method and after that we test some standard properties such as the accuracy, consistency, reliability,stability of the new numerical proposal.

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