

Finite Element Analysis of Funicular Shells with Rectangular plan ratio 1:0.7 under Concentrated Load using SAP2000

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Abstract - A shell structure is a three dimensional structure, thin in one direction and long in the other two directions. Such structures are abundantly found in nature, although thin and light, they span over relatively large areas, and hold applied loads in a very effective way. It seems that with shell structures, nature have maximized the ability to span over large areas with a minimum amount of material. The shell of an egg is an impressive example. Also, of course, shell structures in nature can be very beautiful and have indeed inspired many artists. All buildings are meant to enclose spaces. Most of the different types of superstructures we commonly used for present day building are only a modification of the age old system of column, beam and roof covering arrangements. They fulfill their function by two separate systems. One is the space covering system to cover the space, such as concrete slab or roof covering sheets in steel building. These are supported by a second system of beams and columns which we may call the supporting system. In many steel buildings they are obviously separate and in R.C buildings also, they are treated as two separate systems. In reinforced concrete shells, however, the two functions of covering the space and supporting the covering system are integrated into one. The structure covers the space without beams and columns within the buildings. In this study doubly curved thin shells are analyzed using Finite Element software SAP 2000 with new version. Doubly curved shells which are in rectangular plan having 1mX0.7m are considered. The behavior of shells under concentrated load varying from 1to5KN is studied and compared with the slabs of same dimension and thickness. In this case study deflection curves, membrane stress and stress contour diagram are obtained. It is observed that with the increase in rise and thickness of funicular shell the deflection are reduced. The membrane stresses decreases with the increase in rise and thickness of concrete funicular shell.

Key Words: Edge beam, Funicular shell, Rise, Thickness, Stress contour, Deflection.

1. INTRODUCTION

The shells of double curvature are stronger when compared to shells of single curvature, equivalent to cylindrical shells. Further, the arch distributes the load in all paths equally and resists the impact of loading at any point.

These shells are used as floors and also as roofs. Funicular shells are a class of doubly curved shells, the form of which satisfies the desired state of stress in its body for the given loading and boundary conditions. The state of stress favored in an unreinforced concrete thin shell will be pure compression unaccompanied by shear and bending stresses. Under different conditions of loading, bending moments would strengthen and the shell will not behave only as a funicular element. Analytically, it's possible to compute the funicular surface of any ground plan for the given loading conditions.

The shapes of these shells are chosen that, under uniformly distributed vertical loads, in a membrane state of stress, they develop only pure compression unaccompanied by shear stresses. Thus theoretically no reinforcement will be necessary except in the edge members. Small precast funicular shells without any reinforcement except in edge beams are suitable for roofs and floors of residential, industrial, and institutional buildings. For roofs of larger size in situ construction may be resorted, in such shells provision of reinforcement is necessary to take care of the effects of shrinkage, temperature and bending.

In the lower portion steel reinforcements are required to counter the tensile stresses. It can be converted as compression structure by inverting it, with a considerable reduction in the quantity of steel & cement. Compression structures are used in the form of vaults, arches, catenaries, domes and doubly curved shells also known as funicular shells have been used extensively in the construction of forts and temples. These structures are the best proof for the durable performance of shell structures. The funicular shell roof structure is one such compression structure, which can be built by utilizing waste materials and natural resources can be conserved effectively and use of expensive steel and cement are optimized. Further, point load is distributed equally in all direction by arch thus it is able to withstand the impact of loading at any point. Diagonal grid of funicular shell gives the illusion of a larger space.

A typical view of funicular shells used for a roof construction is shown in figure 1.



Fig-1: Typical view of Funicular Shell roof.

2. Literature review

Abolhassan vafai, Massoud mofid and Homayoon E.Estekanchi (1997) have done an experimental study on prefabricated funicular shells. Along diagonal of the sections experimental values and theoretical values of membrane stresses are compared. The results were similar to the theoretical results. Experimental results of vertical deflections along the transverse sections & longitudinal sections of the shells also favorable with the theory. Experimental failure and crack loads are found & empirical equations, expressing the relation between rise & failure crack loads, are given. Based on the design forty five models were constructed. First eight samples of them are loaded to a specified supported at four edges load within elastic region. Both inside and outside Electrical resistance gauges were mounted at several locations along the diagonal on the surface of the shell on two different specimens with rise of 6cm & 9cm, respectively. Also in other six specimens having different rise & type of reinforcement dial gauges are installed at several locations on the surface. Following these non-destructive tests all 45 samples subjected to concentrated load at centre and are loaded to failure. The finite element technique is used to analyze a similar model in the elastic range to relate experimental results to theory.

P.Sachithanantham, S.Elavenil and S.Sankaran (2011) explained about the funicular shells with square plan with different rises and analyzed for concentrated central force. Shell units of size 1m x 1m in plan with 0.04m X 0.04m edge beam are prepared by M20 grade cement concrete with mix design by IS method. For casting the shells a form made up of square steel frame and foam leather rexine is used. Specimens with different rises of 8cm, 12cm and 16cm are prepared and moist cured for 28 days. On the centre of the shell specimen a concentrated force is applied and within the elastic range corresponding deflections are observed. All the shell specimens are subjected to failure after the elastic range to obtain the ultimate loads. Finite element models of shell specimens are developed and by using Total station coordinates are determined. Using standard software

Analysis and computation of stresses are done. A relation between span to rise ratio and ultimate loads is arrived. By comparing the experimental and analytical results conclusions are made.

P.Sachithanantham (2012) has done the work on funicular shells having rectangular plan and various rises under concentrated central force on its apex. The Specimens of measurement 600mm x 1000mm in plan having edge beam of 40mm x 40mm are prepared by cement concrete of grade M30. The specimens consist of varying rises(R) of 0.052m, 0.069m, and 0.097m. Deflections are measured under the concentrated force on the center of the shell. Finite element models of the shell specimens are developed and the coordinates are decided using total station. Using standard software Analysis and computation of stresses are done for the modeled shells. A relation between span to rise ratio & ultimate loads is arrived.

Tongbram Tarunkumar and P.Sachithanantham (2012) Carried out work on the Concrete shallow funicular shells of rectangular plan, by considering the specimens with three various rises(R) of 3.15 in. [80 mm], 3.546 in. [90 mm], and 4.88 in. [124 mm]. The concentrated force is applied over the centre of the specimen and corresponding deflections are noted. Finite element models of the shell specimens are developed and the coordinates are decided using total station. Analysis and computation of stresses are carried for the modeled shells using standard software. A relation between span to rise ratio & ultimate loads is arrived. Conclusions are made by comparing the analytical results.

Dr. S. Sankaran, P. Sachithanantham, Dr. S. Elavenil (2014) described about the funicular shells which rectangular ground plan. Specimens of size a 1m x 0.9m in plan with 4cm X 4cm edge beam are ready with M30 grade concrete, specimens are made with different rises & moist cured. The deflections & strains of Specimens are noted under corresponding ultimate loads. With different rises failure patterns of shells are observed. From the experimental outcomes a relation between ultimate load and span to rise ratio is arrived & hence concluded that ultimate loads are functions of the rise of the shells.

P. Sachithanantham, S. Sankaran, S. Elavenil(2015) has done work funicular shells with rectangular ground plan of dimension 1m x 0.5m in plan with 0.04m X 0.04m edge beam with M30 grade concrete which is designed by IS method. With different rises specimens are prepared & moist cured. These specimens are subjected to ultimate loads & the corresponding deflections & strains are noted. Failure patterns of shells were observed with various rises. Using these experimental values a relation between ultimate load & span to rise ratio is arrived & hence concluded that the ultimate loads are functions of the rise of the shells.

3. Objectives of the work

Concrete funicular shells of rectangular in ground plan, with doubly curved surfaces and various rises and thickness are analyzed by using finite element method. To study the behavior of funicular shells under concentrated load and comparing with the slab of same dimension. In this work an analytical investigation on doubly curved funicular shell with ground plan ratio 1:0.7 subjected to concentrated load with distinct rise at L/10 and L/20 with thickness of 20mm, 40mm and 50mm is offered. The dimensions of the slabs and shells with their rise and thickness are shown in Table 1.

Table -1: Size of shells and slabs.

SHELLS WITH VARYING RISES & THICKNESS				
Geometry of the shells	Designation of the shell	Plan dimensions in mm	Rise (R) in mm	Thickness in mm
Rectangle	FS I	1000x700	100	50
	FS II	1000x700	100	40
	FS III	1000x700	100	20
	FS IV	1000x700	50	50
	FS V	1000x700	50	40
	FSV VI	1000x700	50	20
SLABS WITH VARYING THICKNESS				
Geometry of the slab	Designation of the slab	Plan dimensions in mm	Thickness in mm	Rise (R) in mm
Rectangle	SLAB I	1000x700	50	0
	SLAB II	1000x700	40	0
	SLAB III	1000x700	20	0

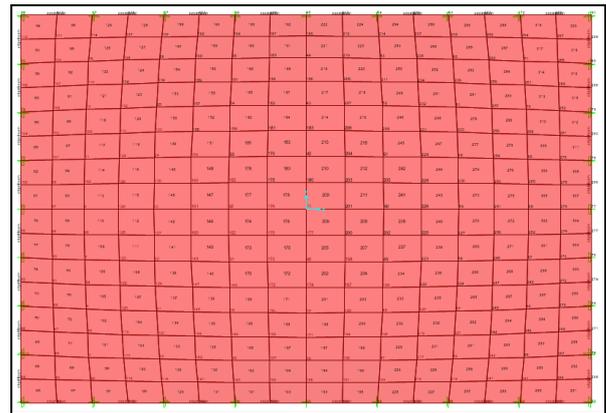


Fig-3: Discretized model of shell FS I

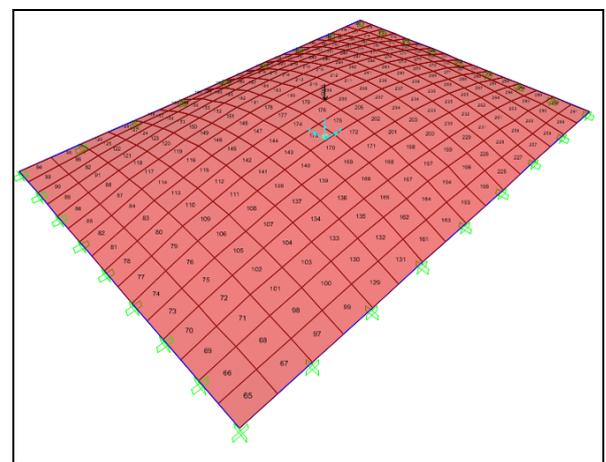


Fig-4: FS I Shell model after loading

4. Finite Element Modeling and Analysis

The shells were modeled with dimensions as mentioned in "Table 1", the rise of shell is taken at L/10 and L/20 where L is the span 1000mm and the edge beam thickness is taken as two to three times the thickness of shell, and all models are fixed supported at the edges. For material properties, M20 grade concrete properties are considered. The models were discretized and subjected concentrated load varying from 1-5kN .The 3D model of FS I is shown in Figure2, discretized model is shown in Figure3 and model after applying load is shown in Figure 4.

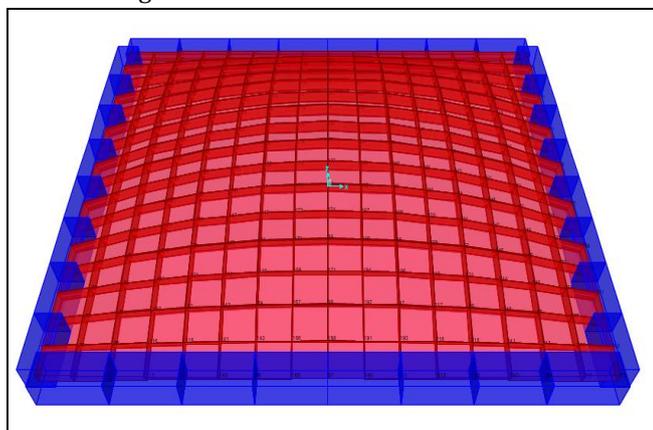


Fig -2: 3D model of shell FS I

After applying load the corresponding deflections and stresses at nodes were noted and graph is plotted with corresponding load and distance. The stress contour and deflected model is shown in Figure 5 and Figure 6 respectively and the graph of stress and deflection values for shell FS I is shown in Chart 1 and Chart 2 respectively. Similarly analysis is carried out for every shell and the values are obtained.

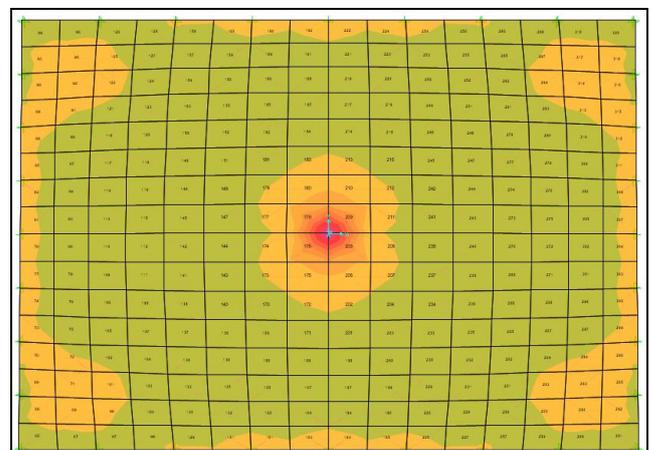


Fig-5: Stress contour of shell FS I

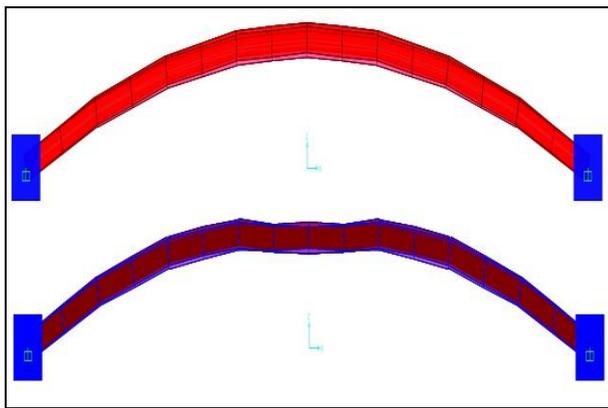


Fig-6: Deflected shape of shell FS I

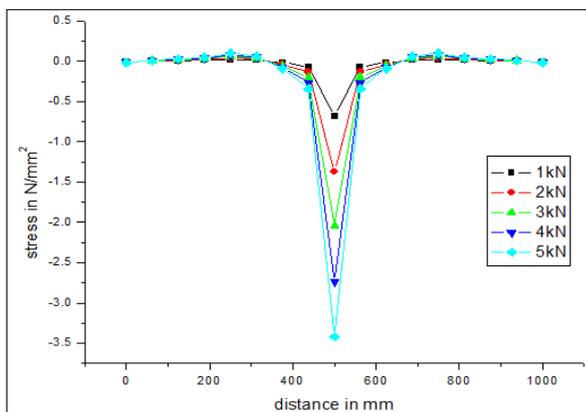


Chart-1: Graph showing Stress v/s distance for FS I

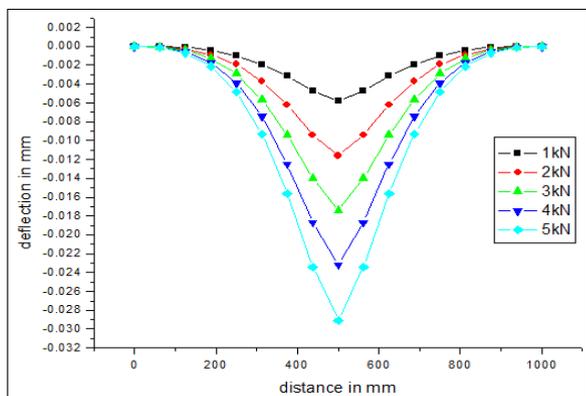


Chart-2: Graph showing Deflection v/s distance for FS I

The maximum deflection and stress are obtained at the center of the shell, values are tabulated in Table 2 with corresponding loadings.

Table-2: Stress and Deflection values of FS I

Load in kN	Stress in N/mm ²	Deflection in mm
1	-0.6815	0.0058
2	-1.3704	0.0116
3	-2.0447	0.0174
4	-2.7408	0.0232
5	-3.4260	0.0291

The values of maximum deflection and membrane stress for FS II, FS III, FS IV, FS V and FS VI are shown in Table 3.

Table-3: Stress and Deflection values of FS II, FS III, FS IV, FS V and FS VI.

FS II			FS III		
Load in KN	Stress in N/mm ²	Deflection in mm	Load in KN	Stress in N/mm ²	Deflection in mm
1	-1.0315	0.0092	1	-3.6011	0.0391
2	-2.0518	0.0185	2	-7.2023	0.0783
3	-3.0775	0.0277	3	-10.8034	0.1174
4	-4.0791	0.0369	4	-14.320	0.1566
5	-5.1295	0.0461	5	-18.0057	0.1957
FS IV			FS V		
Load in KN	Stress in N/mm ²	Deflection in mm	Load in KN	Stress in N/mm ²	Deflection in mm
1	-0.7403	0.0096	1	-1.1335	0.0161
2	-1.4878	0.0193	2	-2.2671	0.0322
3	-2.2042	0.0289	3	-3.4006	0.0483
4	-2.9756	0.0385	4	-4.5342	0.0645
5	-3.7016	0.0482	5	-5.6678	0.0806
FS VI					
Load in KN	Stress in N/mm ²	Deflection in mm			
1	-4.0663	0.0709			
2	-8.1771	0.1419			
3	-12.1260	0.2128			
4	-16.2652	0.2837			
5	-20.4427	0.3546			

Slabs are analyzed to compare their results with the shells of the same dimension and thickness. Slabs of the dimension 1000mm X 700mm with the thickness of 20mm, 40mm and 50mm are analyzed under concentrated load. Slab before and after loading is shown in Figure 7. Results of the analysis are shown below.

Under concentrated load Slabs are analyzed and the corresponding deflection and stress values are shown in Table 4.

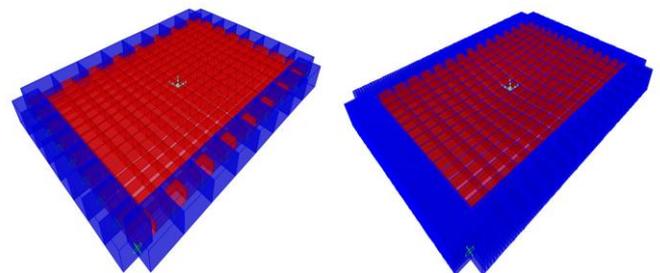


Fig-7: Deflected shape of Slab before and after loading.

Table-4: Deflection and Stress values of Slab I, Slab II and Slab III

Deflection of Slabs in mm			Membrane stress of Slabs in N/mm ²				
Load in KN	Slab I	Slab II	Slab III	Load in KN	Slab I	Slab II	Slab III
1	0.0146	0.0285	0.2278	1	-0.755	-1.1810	-4.704
2	0.0292	0.0571	0.4556	2	-1.507	-2.3621	-9.422
3	0.0437	0.0856	0.6835	3	-2.252	-3.5431	-14.13
4	0.0583	0.1141	0.9113	4	-3.015	-4.7242	-18.82
5	0.0729	0.1427	1.1391	5	-3.768	-5.9053	-23.55

The analysis results of each shell are compared between one another shells and slabs of same dimensions under concentrated load. The maximum deflection of shells and

slabs under concentrated load are tabulated in Table 5 and variations of values are shown in Chart 3.

Table-5: Maximum deflection of shells and Slab in mm.

Thickness in mm	R100	R50	Slab
20	0.1957	0.3546	1.1391
40	0.0461	0.0806	0.1427
50	0.0291	0.0482	0.0729

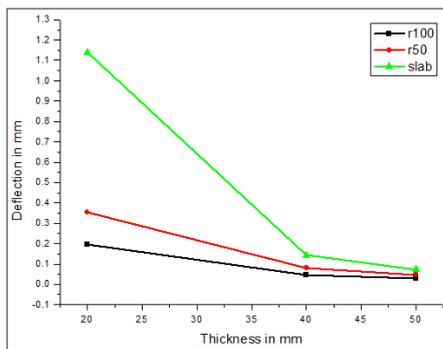


Chart-3: Deflection of shells and slab under concentrated load

The maximum membrane stresses of shells and slabs under UDL are tabulated in Table 6 and the variations are shown in Chart 4.

Table 6: Membrane stress of shells and slab in N/mm²

Thickness in mm	R100	R50	Slab
20	-18.0057	-20.4427	-23.5507
40	-5.1295	-5.6678	-5.9053
50	-3.4260	-3.7016	-3.7689

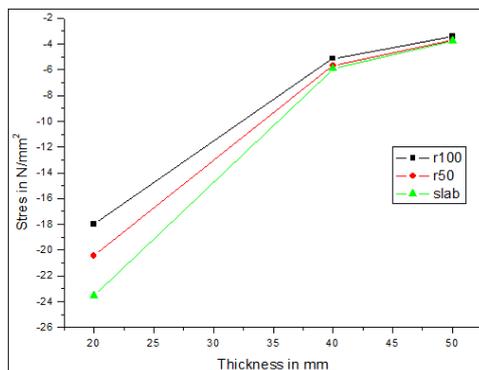


Chart-4: Membrane stress of shells and slab under concentrated load

From comparison of shells with slabs it is clear that Slab deflects more when compared to shells, membrane stress also more in case of slabs. The rate of increase in deflection is high in case of slab when compared to shells.

5. Conclusion

The following conclusions are obtained from the outcomes of the investigation:

1. There is decrease in deflection with increase in rise and thickness of shell.

2. There is decrease in membrane stresses with increase in rise and thickness of shell.

3. In case of concentrated load there is a maximum tension in between the edge beam and centre region of shell and maximum compression is at the center of the shell.

4. There is a 36% more deflection under concentrated load in FS II when compared to FS I due to the 10mm lesser thickness.

5. In case of FS III it deflects 85% more than FS I and 76% more than FS II under concentrated load.

6. There is 40% more deflection under concentrated load in FS V when compared to FS IV due to the 10mm lesser thickness than FS IV.

7. In case of FS VI it deflects 86% more than FS IV and 77% more than FS V under concentrated load.

8. Rate of deflection is more in case of FS VI when compared to FS IV and FS V due to the lesser thickness.

9. There is 33% more membrane stress under concentrated load in FS II when compared to FS I.

10. In case of FS III 80% more stress than FS I and 71% more stress than FS II under concentrated load.

11. There is 35% more stress under concentrated load in FS V when compared to FS IV.

12. In case of FS VI 82% more stress than FS IV and 72% more than FS V under concentrated load.

13. When compared with slabs, FS I deflects 60% lesser and FS IV deflects 34% lesser than slab I under concentrated load.

14. FS II deflects 67% lesser and FS V deflects 44% lesser than slab II under concentrated load.

15. FS III deflects 83% lesser and FS VI deflects 69% lesser than slab III under concentrated load.

16. It is clear that the rate of deflection is more in case of slabs when compared to shells.

17. It is clear that the rate of membrane stresses is more in case of slabs when compared to shells.

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