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Dufour effect on unsteady free convection MHD flow past an exponentially accelerated plate through porous media with variable temperature and constant mass diffusion in an inclined magnetic field

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Abstract - *Dufour effect on unsteady free convection MHD flow past an exponentially accelerated plate through porous media with variable temperature and constant mass diffusion in an inclined magnetic field is studied here. Fluid Considered is electrically conducting. The Laplace transform technique has been used to find the solutions for the velocity, temperature and concentration profiles. The results obtained are discussed with the help of graphs drawn for different parameters like thermal Grashof number, mass Grashof number, Prandtl number, permeability parameter, the Hartmann number, Schmidt number, Dufour number, time, inclination of magnetic field and acceleration parameter.*

Key Words: MHD, Dufour effect, free convection, Heat transfer, Mass transfer, Porous media.

1. INTRODUCTION

The effect of magnetic field on viscous, incompressible and electrically conducting fluid plays important role in many applications such as glass manufacturing, control processing, paper industry, textile industry, magnetic materials processing and purification of crude oil etc. Nelson and Wood [2] have studied combined heat and mass transfer natural convection between vertical parallel plates with uniform heat flux boundary conditions. Mass transfer effects on the flow past an exponentially accelerated vertical plate with constant heat flux have been studied by basant et al [3]. Rajput and Kumar [10] have studied radiation effects on MHD flow through porous media past an impulsively started vertical oscillating plate with variable mass diffusion. Kesavaiah and Satyanarayana [13] have discussed radiation absorption and Dufour effects to MHD flow in vertical surface. V M Soundalgekar [1] have studied free convection effects on the oscillatory flow an infinite, vertical porous, plate with constant suction. Chemical reaction effect on unsteady MHD flow through porous medium past an exponentially accelerated inclined plate with variable temperature and mass diffusion in the presence of Hall current have been studied by Rajput and Kumar [14]. Das and Jana [8] have studied heat and mass transfer effects on unsteady MHD free convection flow near a moving vertical plate in porous medium. Rajesh [9] has studied MHD effects on free convection and mass transform flow through a porous medium with variable temperature. Sandeep and Sugunamma [12] have disscussed effect of inclined magnetic field on unsteady free convection flow of a dusty viscous fluid between two infinite flat plates filled by a porous medium. Postelnicu [5] has studied influence of chemical reaction on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects. Alam et al [4] have studied Dufour and Soret effects on unsteady MHD free convection and mass transfer flow past a vertical porous plate in a porous medium. Reddy [7] have discussed Soret and Dufour effects on steady MHD free convection flow past a semi-infinite moving vertical plate in a porous medium with viscous dissipation. Dipak et al. [11] have studied Soret and Dufour effects on steady MHD convective flow past a continuously moving porous vertical plate. Muthucumaraswamy et al [6] has discussed heat transfer effects on flow past an exponentially accelerated vertical plate with variable temperature.
In this paper we are analyzing Dufour effect on unsteady free convection MHD flow past an exponentially accelerated plate through porous media with variable temperature and constant mass diffusion in an inclined magnetic field.

2. MATHEMATICAL FORMULATION

In this paper we have considered the flow of unsteady viscous incompressible fluid. The plate taken is electrically non-conducting. The *x*- axis is taken along the plate in the upward direction and *y*- axis is taken normal to it. A uniform inclined magnetic field B_0 is applied on the plate with angle α from vertical. Initially the fluid and plate are at the same temperature T_∞ and the concentration of the fluid is C_∞. At time $t > 0$, the plate starts moving exponentially in its own plane with velocity $u = u_0 e^{bt}$, temperature of the plate is raised to T_w, and the concentration of the fluid is raised to C_w.
The governing equations under the usual Boussinesq's approximations are as follows:

$$\frac{\partial u}{\partial t} = \upsilon \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma B_0^2}{\rho}\sin^2(\alpha)u - \frac{\upsilon}{K}u, \quad (1)$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_P}\frac{\partial^2 T}{\partial y^2} + \frac{D_m K_T}{C_S C_P}\frac{\partial^2 C}{\partial y^2}, \quad (2)$$

$$\frac{\partial C}{\partial t}=D\frac{\partial^2 C}{\partial y^2}. \quad (3)$$

The initial and boundary conditions are given as:

$$\left.\begin{array}{l} t\le 0; u=0, T=T_\infty, C=C_\infty \text{ for each value of } y, \\ t>0; u=u_0e^{bt}, T=T_\infty+(T_w-T_\infty)\dfrac{u_0^2 t}{\upsilon}, C=C_w \text{ at } y=0, \\ u\to 0, T\to T_\infty, C\to C_\infty \text{ as } y\to\infty. \end{array}\right\} \quad (4)$$

Here u is the velocity of the fluid, g – the acceleration due to gravity, β – volumetric coefficient of thermal expansion, β^* – volumetric coefficient of concentration expansion, t – time, T – the temperature of the fluid, T_∞ – the temperature of the plate at $y\to\infty$, C – species concentration in the fluid, C_∞ – species concentration at $y\to\infty$, υ – the kinematic viscosity, ρ – the density, C_P – the specific heat at constant pressure, k – thermal conductivity of the fluid, K_T – thermal diffusion ratio, D – the mass diffusion constant, D_m – the effective mass diffusivity rate, T_w – the temperature of the plate at $y=0$, C_w – species concentration at the plate at $y=0$, C_S – Concentration susceptibility B_0 – the uniform magnetic field, σ – electrical conductivity, K – permeability of the porous medium and α – angle of inclination from vertical.

By using the following dimensionless quantities, the above equations (1), (2), and (3) can be transformed into dimensionless form.

$$\left.\begin{array}{l} \bar y=\dfrac{yu_0}{\upsilon}, \bar t=\dfrac{tu_0^2}{\upsilon}, \bar u=\dfrac{u}{u_0}, \theta=\dfrac{T-T_\infty}{T_w-T_\infty}, \bar C=\dfrac{C-C_\infty}{C_w-C_\infty}, \\ Sc=\dfrac{\upsilon}{D}, Ha^2=\dfrac{\sigma B_0^2\upsilon}{\rho u_0^2}, \bar K=\dfrac{Ku_0^2}{\upsilon^2}, Gr=\dfrac{g\beta\nu(T_w-T_\infty)}{u_0^3}, \\ \Pr=\dfrac{\mu C_p}{k}, M=Ha^2*\sin^2(\alpha), Gm=\dfrac{g\beta^*\upsilon(C_w-C_\infty)}{u_0^3}, \\ D_f=\dfrac{D_mK_T(C_w-C_\infty)}{\upsilon C_SC_P(T_w-T_\infty)}, \bar b=\dfrac{b\upsilon}{u_0^2}, \mu=\upsilon\rho \end{array}\right\} \quad (6)$$

Here $\bar u$ is dimensionless velocity, $\bar t$ – dimensionless time, *Pr*- Prandtl number, *Sc*- Schmidt number, *Gr*- thermal Grashof number, *Gm*- mass Grashof number, θ- dimensionless temperature, $\bar C$- dimensionless concentration, *Ha*- the Hartmann number, μ- the coefficient of viscosity, $\bar b$ – dimensionless acceleration parameter, $\bar K$ – permeability parameter and D_f - Dufour number.

Then model is transformed into the following non dimensional form of equations:

(6)

$$\frac{\partial \bar u}{\partial \bar t}=\frac{\partial^2 \bar u}{\partial \bar y^2}+Gr\theta+Gm\bar C-A\bar u,$$

$$\frac{\partial \theta}{\partial \bar t}=\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial \bar y^2}+D_f\frac{\partial^2 \bar C}{\partial \bar y^2},$$

(7)

$$\frac{\partial \bar C}{\partial \bar t}=\frac{1}{Sc}\frac{\partial^2 \bar C}{\partial \bar y^2}. \quad (8)$$

Here $A=M+\dfrac{1}{\bar K}$.

The initial and boundary conditions become:

$$\left.\begin{array}{l} \bar t\le 0; \bar u=0, \theta=0, \bar C=0 \text{ for each value of } \bar y, \\ \bar t>0; \bar u=e^{\bar b\bar t}, \theta=\bar t, \bar C=1 \text{ at } \bar y=0, \\ \bar u\to 0, \theta\to 0, \bar C\to 0 \text{ as } \bar y\to\infty. \end{array}\right\} \quad (9)$$

Dropping bars in the above equations, we get:

(10)

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}+Gr\theta+GmC-Au,$$

$$\frac{\partial \theta}{\partial t}=\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2}+D_f\frac{\partial^2 C}{\partial y^2},$$

(11)

$$\frac{\partial C}{\partial t}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}. \quad (12)$$

Here $A=M+\dfrac{1}{K}$.

The initial and boundary condition become:

$$\left.\begin{array}{l} t\le 0; u=0, \theta=0, C=0 \text{ for each value of } y, \\ t>0; u=e^{bt}, \theta=t, C=1 \text{ at } y=0, \\ u\to 0, \theta\to 0, C\to 0 \text{ as } y\to\infty. \end{array}\right\} \quad (13)$$

Now the solutions of equations (10), (11) and (12) under the boundary conditions (13) are obtained by the Laplace - transform technique. The exact solutions for species concentration *C*, fluid temperature θ and fluid velocity *u* are respectively:

$$C=Erfc\left[\frac{\sqrt{Sc}y}{2\sqrt{t}}\right] \quad (14)$$

$$\theta=-\frac{e^{\frac{-\Pr y^2}{4t}}\sqrt{\Pr t}\,y}{\sqrt{\pi}}-\frac{A_{25}}{2}\left(t+\frac{\Pr y^2}{2}+\frac{\Pr ScD_f}{Sc-\Pr}\right)+\frac{\Pr ScD_f}{2(Sc-\Pr)}A_{27} \quad (15)$$

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$$u = \frac{1}{4A^2} Gr \begin{bmatrix} 2A_9 B_{16} B_{13} + y\sqrt{A} A_9 B_{17} + A_{13} B_{18} (1-\Pr) \\ + \frac{A \Pr Sc D_f}{Sc - \Pr} (-2A_9 B_{19} + A_{13} B_{20}) \end{bmatrix} +$$

$$\frac{(A_{18} B_{11} - A_9 B_{12})}{2A} \left[\frac{Gr \Pr Sc D_f}{Sc - \Pr} + Gm \right] - Gr$$

$$\begin{bmatrix} \frac{1}{2A^2} \{-2A_{22} B_{13} + \frac{A\sqrt{\Pr} y}{\sqrt{\pi}} (2e^{\frac{-\Pr y^2}{4t}} \sqrt{t} + \sqrt{\pi} \Pr y A_{22}) \\ + \frac{1}{2} A_{13} B_{21} (\Pr - 1)\} + \frac{\Pr Sc}{2A(Sc - \Pr)} \\ (A_{25} + \frac{A_{13}}{2} (1 + A_{23} + A_{14} A_{26})) D_f \end{bmatrix} -$$

$$\frac{Gr \Pr Sc D_f B_{14}}{2A(Sc - \Pr)} - \frac{1}{2A} Gm B_{15} + \frac{1}{2} e^{bt} C_1 \left[1 + C_2 + C_1^{-2} C_3 \right]$$

The expressions for the constants involved in the above equations are given in the appendix.

3. SKIN- FRICTION

We calculate the non-dimensional form of skin friction (τ) from the velocity field as:

$$\tau = \left(-\frac{\partial u}{\partial y} \right)_{y=0}$$

4. RESULTS AND DISCUSSION

The numerical values of velocity, concentration and temperature are computed for different parameters like thermal Grashof number (*Gr*), mass Grashof number (*Gm*), Hartmann number (*Ha*), Prandtl number (*Pr*), Schmidt number (*Sc*), inclination (α), permeability parameter (*K*), acceleration parameter (*b*), Dufour number (D_f) and time (*t*). The values of the parameters considered are Gr = 5, 10, 15, Ha = 2, 4, 6, Gm = 50, 60, 70, $\alpha = 15°, 30°, 60°$, Pr = 7, 10, Sc = 2.01, 2.10, 2.20, K = 0.2, 0.4, 0.6, b = 1, 3, 5, D_f = 0.15, 0.23, 0.5 and t = 0.15, 0.18, 0.2. Figures 3, 6, 7, 9 and 10 show that velocity increases when *Gm, K, Pr, t* and *b* are increased. Figures 1, 2, 4, 5, and 8 show that velocity decreases when α, D_f, *Gr, Ha,* and *Sc* are increased.

Skin- friction is given in table-1. The value of skin-friction increases with increasing the values of *Ha, Sc, Gr, b*, α and D_f and decreases with increasing the values of *Gm, t, Pr,* and *K.*

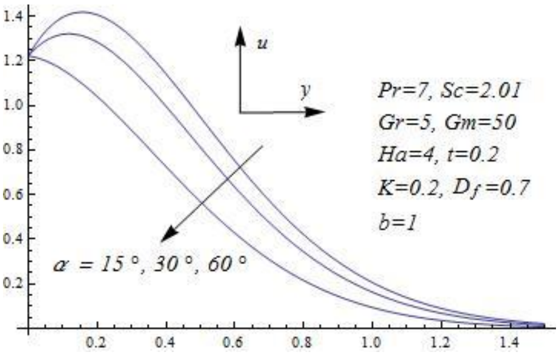

Figure 1: velocity profile for different values of α

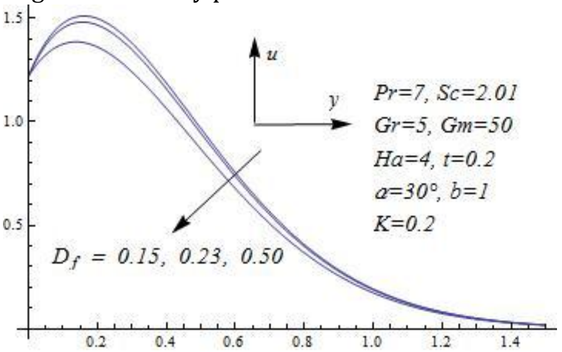

Figure 2: velocity profile for different values of D_f

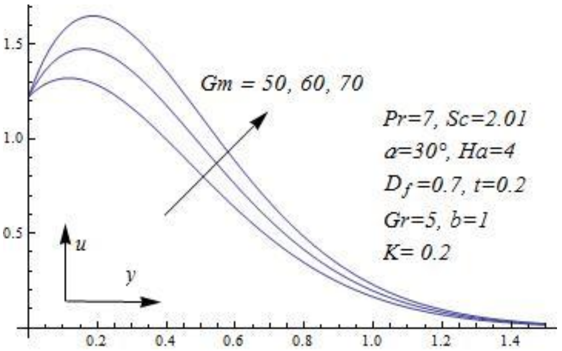

Figure 3: velocity profile for different values of *Gm*

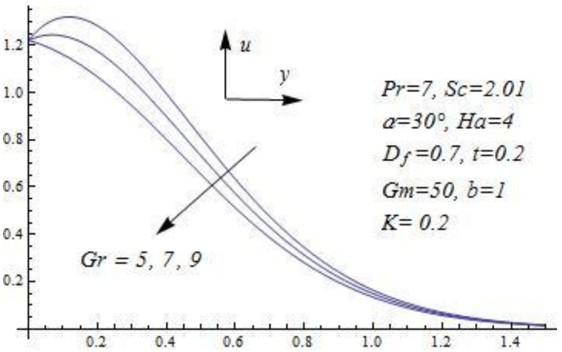

Figure 4: velocity profile for different values of *Gr*

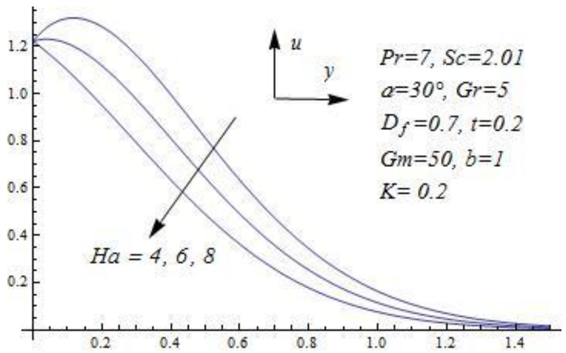

Figure 5 velocity profile for different values of *Ha*

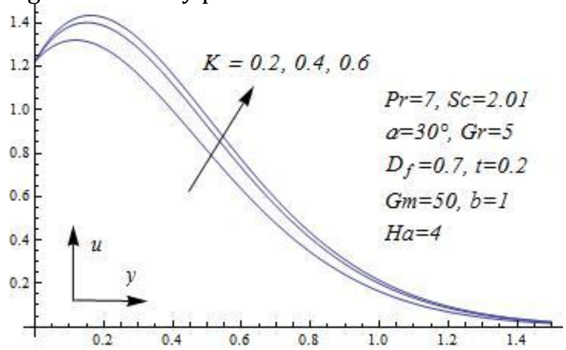

Figure 6 : velocity profile for different values of *K*

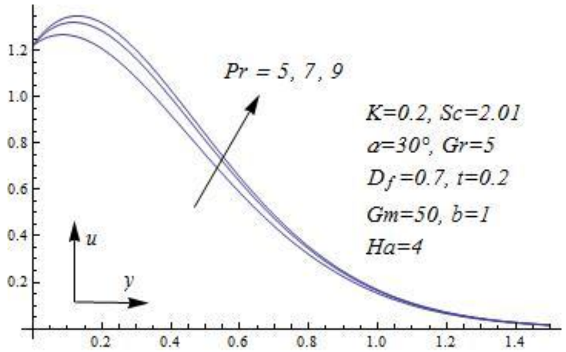

Figure 7: velocity profile for different values of *Pr*

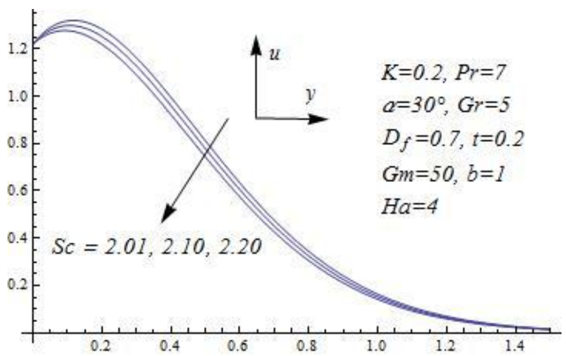

Figure 8: velocity profile for different values of *Sc*

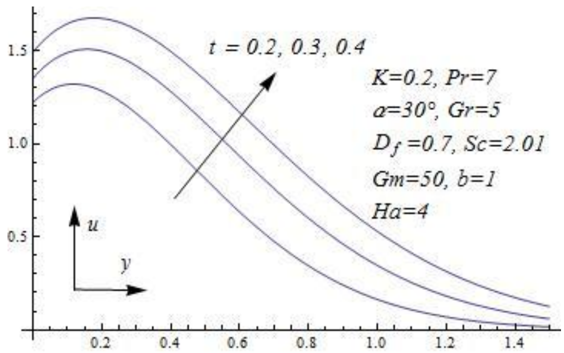

Figure 9 : velocity profile for different values of *t*

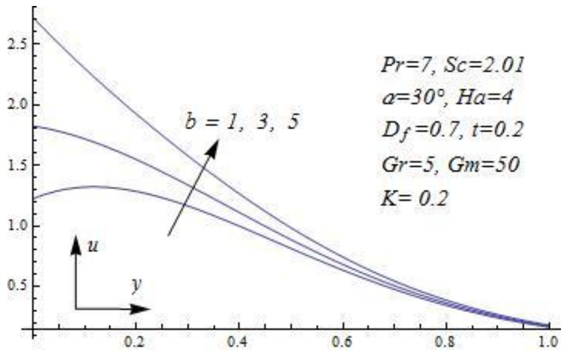

Figure 10 : velocity profile for different values of *b*

5. CONCLUSIONS

Some conclusions of study are as below:

- The velocity of the fluid increases with increasing the values of *K, b, Gm, t* and *Pr.*
- The velocity of the fluid decreases with increasing the values of *Ha, Gr, Sc,* α and D_f.
- The skin-Friction of the fluid increases with increasing the values of *Ha, Sc, Gr ,b,* α and D_f.
- The skin-Friction of the fluid decreases with increasing the values of *Gm, t , Pr,* and *K.*

APPENDIX

$$A_9 = e^{-\sqrt{A}y},\quad A_{10} = Erfc\left[\frac{2\sqrt{At}+y}{2\sqrt{t}}\right],\quad A_{11} = Erf\left[\frac{2\sqrt{At}-y}{2\sqrt{t}}\right],$$

$$A_{12} = Erf\left[\frac{2\sqrt{At}+y}{2\sqrt{t}}\right],\quad A_{13} = 2e^{\frac{At}{\Pr-1}-\sqrt{\frac{A\Pr}{\Pr-1}}y},\quad A_{14} = e^{2\sqrt{\frac{A\Pr}{\Pr-1}}y},$$

$$A_{15} = Erf\left[\frac{2\sqrt{\frac{A\Pr}{\Pr-1}}t-y}{2\sqrt{t}}\right],\quad A_{16} = Erf\left[\frac{2\sqrt{\frac{A\Pr}{\Pr-1}}t+y}{2\sqrt{t}}\right],\quad A_{17} = Erfc\left[\frac{2\sqrt{\frac{A\Pr}{\Pr-1}}t+y}{2\sqrt{t}}\right],$$

$$A_{18} = e^{\frac{At}{Sc-1}-\sqrt{\frac{ASc}{Sc-1}}y},\quad A_{19} = e^{2\sqrt{\frac{ASc}{Sc-1}}y},$$

$$A_{20} = Erf\left[\frac{2\sqrt{\frac{ASc}{Sc-1}}t-y}{2\sqrt{t}}\right],\quad A_{21} = Erf\left[\frac{2\sqrt{\frac{ASc}{Sc-1}}t+y}{2\sqrt{t}}\right],\quad A_{22} = -1+Erf\left[\frac{\sqrt{\Pr}y}{2\sqrt{t}}\right],$$

$$A_{23} = Erf\left[\frac{2\sqrt{\frac{A}{\Pr-1}}t - \sqrt{\Pr}y}{2\sqrt{t}}\right],\ A_{24} = Erf\left[\frac{2\sqrt{\frac{A}{\Pr-1}}t + \sqrt{\Pr}y}{2\sqrt{t}}\right],\ A_{25} = -2Erfc\left[\frac{\sqrt{\Pr}y}{2\sqrt{t}}\right],$$

$$A_{26} = Erfc\left[\frac{2\sqrt{\frac{A}{\Pr-1}}t + \sqrt{\Pr}y}{2\sqrt{t}}\right],\ A_{27} = -2Erfc\left[\frac{\sqrt{Sc}y}{2\sqrt{t}}\right],$$

$$A_{28} = Erf\left[\frac{2\sqrt{\frac{A}{Sc-1}}t - \sqrt{Sc}y}{2\sqrt{t}}\right],\ A_{29} = Erfc\left[\frac{2\sqrt{\frac{A}{Sc-1}}t + \sqrt{Sc}y}{2\sqrt{t}}\right]$$

$$A_{30} = Erf\left[\frac{2\sqrt{\frac{A}{Sc-1}}t + \sqrt{Sc}y}{2\sqrt{t}}\right],\ B_{11} = 1 + A_{19} + A_{20} - A_{19}A_{21},$$

$$B_{12} = 1 + A_{11} + A_9^{-2}A_{10},\ B_{13} = 1 - \Pr - At,$$

$$B_{15} = A_{18}(1 + A_{19} + A_{28} - A_{19}A_{30}) + A_{27}$$

$$B_{16} = 1 + A_9^{-2} + A_{11} - A_9^{-2}A_{12},\ B_{17} = 1 - A_9^{-2} + A_{11} + A_9^{-2}A_{12},$$

$$B_{18} = -1 - A_{14} - A_{15} + A_{14}A_{17},\ B_{19} = 1 + A_{11} + A_9^{-2}A_{10},$$

$$B_{20} = 1 + A_{15} + A_{14}A_{17},\ B_{21} = 1 + A_{14} + A_{23} - A_{14}A_{24},\ C_1 = e^{-\sqrt{A+b}y}$$

$$C_2 = Erf\left[\frac{2t\sqrt{A+b} - y}{2\sqrt{t}}\right],\ C_3 = Erfc\left[\frac{y + 2t\sqrt{A+b}}{2\sqrt{t}}\right]$$

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Table -1: Skin friction for different Parameters.

α (In degree)	Ha	Pr	D_f	t	K	Gm	Gr	Sc	b	τ
15	4	7	0.70	0.2	0.2	50	5	2.01	1	-2.7576
30	4	7	0.70	0.2	0.2	50	5	2.01	1	-1.8174
60	4	7	0.70	0.2	0.2	50	5	2.01	1	+0.2205
30	**6**	7	0.70	0.2	0.2	50	5	2.01	1	-0.4713
30	**8**	7	0.70	0.2	0.2	50	5	2.01	1	+1.0432
30	4	**5**	0.70	0.2	0.2	50	5	2.01	1	-1.1240
30	4	**9**	0.70	0.2	0.2	50	5	2.01	1	-2.1307
30	4	7	**0.15**	0.2	0.2	50	5	2.01	1	-4.0696
30	4	7	**0.23**	0.2	0.2	50	5	2.01	1	-3.7420
30	4	7	**0.50**	0.2	0.2	50	5	2.01	1	-2.6364
30	4	7	0.70	**0.3**	0.2	50	5	2.01	1	-2.2096
30	4	7	0.70	**0.4**	0.2	50	5	2.01	1	-2.3096
30	4	7	0.70	0.2	**0.4**	50	5	2.01	1	-2.6114
30	4	7	0.70	0.2	**0.6**	50	5	2.01	1	-2.8990
30	4	7	0.70	0.2	0.2	**60**	5	2.01	1	-3.5162
30	4	7	0.70	0.2	0.2	**70**	5	2.01	1	-5.2151
30	4	7	0.70	0.2	0.2	50	**7**	2.01	1	-0.7047
30	4	7	0.70	0.2	0.2	50	**9**	2.01	1	+0.4080
30	4	7	0.70	0.2	0.2	50	5	**2.10**	1	-1.5649
30	4	7	0.70	0.2	0.2	50	5	**2.20**	1	-1.2823
30	4	7	0.70	0.2	0.2	50	5	2.01	**3**	+0.6283
30	4	7	0.70	0.2	0.2	50	5	2.01	**5**	+4.4841