Digital Technique of fault study of power system

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Abstract - In huge complex power system networks, faults are inevitable. It’s very important to analyze the fault level for avoiding the severe consequences and proving the conditions thus quality power supply need not to face any kind of discontinuity. The objective of this paper is to study the common fault types which are balanced and unbalanced fault of transmission line in power system using a systematic procedure of digital techniques and to perform the analysis and obtain the result from simulation on that type of fault using MATLAB.

Key Words: Power system fault analysis, Asymmetrical fault, Symmetrical components, MATLAB, Z-parameters.

1. INTRODUCTION

Power system works in normal balanced condition but whenever some fault arises it becomes unbalanced. Any unexpected disturbance in the normal working condition is termed as fault. Fault may arise due to various factors such as Lightening stroke, high wind which leads to falling of trees on line, wind and ice loading resulting in failure of insulators. Fault can be symmetrical or unsymmetrical, when the fault occurs at all the three phases at a time, it is said to be symmetrical fault and where one or more phases may be involved is unsymmetrical fault. Though the symmetric fault is rare, but this type of faults shows most severe effects to our power systems. Unsymmetrical fault can be treated as a regular problem of the power system and has to be analyzed more precisely thus the system can remain in its stable condition and can lead its operation without breaking continuity. The point at which the fault occurs behaves as sink point and the voltage at that point tends to become zero. Thus all the points have potential higher than faulty point starts to send current to these faulty point and thus the fault level rises to very higher magnitude than normal operating level. Therefore, it is important to determine the values of system voltage and current during fault conditions so that the protective devices may be set to detect the fault and isolate the faulty portion of the system.

1.1 Faults in Three phase system

1. Symmetrical three-phase fault
2. Single line-to-ground fault
3. Line-to-line fault
4. Double line-to-ground fault

Symmetrical fault is defined as the simultaneous short circuit across all the three phases. It is most infrequent fault but the most severe type of fault encountered. Symmetrical fault is the rarest one and calculation of fault level is much simpler, but in case of unsymmetrical symmetry faulted power system does not have three phase symmetry, so it cannot be solved by per phase analysis. To find fault current and voltage, it is first transformed into their symmetrical component, use of which help to reduce the complexity of transmission line and components are by large and symmetrical, although the fault may be unsymmetrical. This can be done by replacing three phase fault current by the sum of three phases zero sequence sources, a three phase positive sequence source, a three phase negative sequence.

1.2 BUS IMPEDENCE CALCULATION

Fig. 1 shows an n bus passive linear network. The voltage of the ith bus (with respect to reference) is \( V_i \) and the current entering the ith bus is \( I_i \). The knowledge of network theory tells us that this network can be described by...
\[ [V_{bus}]_{n \times 1} = [Z_{bus}]_{n \times n}[I_{bus}]_{n \times 1} \]

Where \( V_{bus} \) is the bus voltage matrix, \( I_{bus} \) is the bus current matrix and \( Z_{bus} \) is the bus impedance matrix. The general entry \( Z_{ik} \) of \( Z_{bus} \) can be obtained

\[ Z_{ik} = \frac{V_i}{I_k} \]

\( I_1 = I_2 = \ldots = I_n = 0 \)

\( I_k \neq 0 \)

When an existing \( Z_{bus} \) is added with new element having self-impedance \( Z_s \), a new bus may be created or a new bus may not be created. This leads to the addition of \( Z_s \) in the following ways:

1. Injection of \( Z_s \) from a new bus to reference.
2. Injection of \( Z_s \) from a new bus to old bus.
3. Injection of \( Z_s \) from an old bus to reference
4. Injection of \( Z_s \) between two old buses.

**TYPE 1 MODIFICATION**

Fig. 2 shows a branch \( Z_s \) connected between reference bus and a new bus \( q \). For this new branch we can write

\[ V_q = z_s I_q \]

\[ V_i = [Z_{11} Z_{12} \ldots Z_{1n} 0] [I_1] \]

\[ V_f = [Z_{n1} Z_{n2} \ldots Z_{nn} 0] [I_n] \]

\[ V_q = [0 0 \ldots 0 z_s] [I_q] \]

Or

\[ [Z_{bus}]_{new} = \begin{bmatrix} [Z_{bus}]_{old} & 0 \\ 0 & z_s \end{bmatrix} \]

**TYPE 2 MODIFICATION**

Fig. 3 Type 2 modification

\[ V_q = z_s I_q + V_k \]

\[ = Z_{s1} I_1 + Z_{s2} I_2 + \ldots + Z_{sk} I_k + \ldots + Z_{sn} I_n + (Z_{kk} + z_s) I_q \]

The equation for \( V_1 \) can be written by considering that new current at kth bus is \( I_k + I_q \).

\[ V_i = Z_{11} I_1 + Z_{12} I_2 + \ldots + Z_{1k} I_k + \ldots + Z_{1n} I_n + Z_{1q} I_q \]

The equations for \( V_2, V_3, \ldots, V_n \) also get modified in a similar manner and is given by
The new set of the equation is

\[ V_{new} = V_{old} - Z_s + Z_i + Z_{sk} - Z_{1i} - Z_{2i} - \ldots - Z_{nk} \]

\[ = V_{old} - Z_s + Z_i + Z_{sk} - Z_{1i} - Z_{2i} - \ldots - Z_{nk} \]

1.3 SEQUENCE MATRICES

The sequence impedance matrices required for the short circuit studies are:

- \([Z_{0-bus}] = \) zero sequence bus impedance matrix \((n \times n)\), generally entry \(Z_{ik0}\)

- \([Z_{1-bus}] = \) positive sequence bus impedance matrix \((n \times n)\), generally entry \(Z_{ik1}\)

- \([Z_{2-bus}] = \) negative sequence bus impedance matrix \((n \times n)\), generally entry \(Z_{ik2}\)

The equations relating the sequence quantities are:

\[ V_{0-bus} = -[Z_{0-bus}] V_{0-bus} \]

\[ V_{1-bus} = E_{bus} - [Z_{1-bus}] V_{1-bus} \]
\[ V_{2-bus} = -[Z_{2-bus}]I_{2-bus} \]

The prefault currents are neglected, vector E contains \( 1 \leq 0 \) in all entries. The currents are all zero till the network is terminated externally. At a time only one bus (i.e. faulted bus k) is terminated. Thus, only \( I_{k0}, I_{k1}, I_{k2} \) have non-zero entry.

### 1.4 SHORT CIRCUIT STUDIES

**Symmetrical fault:** For a symmetrical fault the negative sequence and zero sequence are absent, i.e., \( V_{0-bus}, V_{2-bus}, I_{0-bus} \) and \( I_{2-bus} \) are zero.

\[ V_{k1} = E - (Z_{k11}I_{11} + Z_{k21}I_{21} + \cdots + Z_{kk1}I_{k1} + \cdots + Z_{kn1}I_{n1}) \]  

(1)

But all the currents except the current at the faulted bus, i.e., \( I_{k1} \) are zero. Therefore,

\[ V_{k1} = E - Z_{kk1}I_{k1} \]  

(2)

If \( Z_f \) is the fault impedance

\[ V_{k1} = I_{k1}Z_f \]  

(3)

From (2 and 3)

\[ I_{k1} = \frac{E}{Z_{kk1} + Z_f} \]  

(4)

The voltage at \( i \)th bus is

\[ V_{i1} = E - Z_{kk1}I_{k1} = E \left( 1 - \frac{Z_{kk1}}{Z_{kk1} + Z_f} \right) \]  

(5)

For \( i = 1, 2 \ldots \ldots \ n \)

**Single Line to Ground Fault**

Assuming that fault current \( I_f \) occurred on the phase with fault impedance \( Z_f \). The boundary conditions are:
\[ V_{ag} = Z_f I_a \]
\[ I_b = 0, I_c = 0 \]

The symmetrical component of current are, with \( I_b = I_c = 0 \):

\[
\begin{bmatrix}
I_{a0} \\
I_{a1} \\
I_{a2}
\end{bmatrix} = \frac{1}{3} \begin{bmatrix}
1 & 1 & 1 \\
1 & a & a^2 \\
1 & a^2 & a
\end{bmatrix} \begin{bmatrix}
I_a = I_f \\
0 \\
0
\end{bmatrix}
\]

\[ I_{a0} = I_{a1} = I_{a2} = \frac{I_f}{3} \]

\[ I_f = I_a = I_{a1} + I_{a2} = \frac{3V_f}{Z_0 + Z_1 + Z_2 + 3Z_f} \]

Where, \( Z_0, Z_1, Z_2 \) is a zero, positive, negative sequence impedance.

**Using Z-parameters analysis (Digital approach)**

We know that

\[ I_{i0} = I_{i1} = I_{i2} = 0 \quad \text{For } i = 1, 2 \ldots n \] (6a)

\[ i \neq K \] (6b)

And \( I_{k0} = I_{k1} = I_{k2} \)

\[ V_{k0} + V_{k1} + V_{k2} = 3Z_f I_{k1} \] (7)

Also \( V_{k0} = -Z_{kk0} I_{k0} \)

\[ V_{k1} = E - Z_{kk1} I_{k1} \] (8)

\[ V_{k2} = -Z_{kk2} I_{k2} \]

Combining equation (6, 7 and 8)

\[ I_{k1} = \frac{E}{Z_{kk0} + Z_{kk1} + Z_{kk2} + 3Z_f} \] (9)

The sequence components of bus voltages at ith bus \( (i = 1, 2, 3 \ldots n) \) are

\[ V_{i0} = -Z_{ik0} I_{k0} = -Z_{ik0} I_{k1} \] (10a)

\[ V_{i1} = E - Z_{ik1} I_{k1} \] (11a)

\[ V_{i2} = Z_f I_{k2} \]

\[ = \frac{-Z_{ik2} E}{Z_{kk0} + Z_{kk1} + Z_{kk2} + 3Z_f} \] (10b)

\[ V_{i1} = E - Z_{ik1} I_{k1} \]

\[ = E \left[ \frac{Z_{ik1}}{Z_{kk0} + Z_{kk1} + Z_{kk2} + 3Z_f} \right] \] (11b)

\[ V_{i2} = -Z_{ik2} I_{k2} = -Z_{ik2} I_{k1} \] (12a)

\[ V_{i2} = \frac{-Z_{ik2} E}{Z_{kk0} + Z_{kk1} + Z_{kk2} + 3Z_f} \] (12b)

**Line to Line Fault**

![Line to Line Fault Diagram](Image)

**Fig.8: Line-to-Line fault**

Line-to-Line fault takes place on phase b and c. The boundary conditions are:

\[ I_a = 0 \]

\[ I_b + I_c = 0 \]

\[ V_b = V_c \]

The symmetrical component of current are:

\[
\begin{bmatrix}
I_{a0} \\
I_{a1} \\
I_{a2}
\end{bmatrix} = \frac{1}{3} \begin{bmatrix}
1 & 1 & 1 \\
1 & a & a^2 \\
1 & a^2 & a
\end{bmatrix} \begin{bmatrix}
I_a = I_f \\
0 \\
0
\end{bmatrix}
\]

\[
\begin{bmatrix}
I_{a0} \\
I_{a1} \\
I_{a2}
\end{bmatrix} = \begin{bmatrix}
0 \\
\frac{1}{3} (a - a^2) I_b \\
-\frac{1}{3} (a - a^2) I_b
\end{bmatrix}
\]

Therefore, \( I_{a1} = -I_{a2} \)

Also \( V_{a1} = V_{a2} \)

Or \( I_{a1} = \frac{V_f}{Z_1 + Z_2} \)

Where, \( Z_1, Z_2 \) is positive, negative sequence impedances.

**Using Z-parameters analysis (Digital approach)**

For a line to line fault

\[ V_{0-bus} = I_{0-bus} = 0 \] (13)
(6a and 8) are still valid

\[ V_{k1} = V_{k2} + I_{k1}Z_f \]  \hspace{1cm} (14)

Substituting the values for \( V_{k1} \) and \( V_{k2} \) from (7) into (13)

\[ E - Z_{kk1}I_{k1} = -Z_{kk2}I_{k2} + I_{k1}Z_f \]  \hspace{1cm} (15)

We know that for a line to line fault \( I_{k2} = -I_{k1} \). Making this substitution in (14)

\[ E - Z_{kk1}I_{k1} = Z_{kk1}I_{k2} + I_{k1}Z_f \]

Or \( I_{k1} = \frac{E}{Z_{kk1} + Z_{kk2} + Z_f} \) \hspace{1cm} (16)

The positive and negative sequence voltages at ith bus (i = 1, 2, ..., n) are

\[ V_{f1} = E \cdot Z_{kk1}I_{k1} \]  \hspace{1cm} (17a)

\[ = \frac{E}{Z_{kk1} + Z_{kk2} + Z_f} \]  \hspace{1cm} (17b)

\[ = \left[ \frac{Z_{kk1} + Z_{kk2} + Z_f - Z_{kk1}}{Z_{kk1} + Z_{kk2} + Z_f} \right] \]  \hspace{1cm} (17c)

\[ V_{f2} = -Z_{kk2}I_{k2} \]  \hspace{1cm} (18a)

\[ = +Z_{kk2}I_{k1} \]  \hspace{1cm} (18b)

\[ = \frac{Z_{kk2}E}{Z_{kk1} + Z_{kk2} + Z_f} \]  \hspace{1cm} (18c)

**Double Line to Ground Fault**

![Double Line to Ground Fault Diagram](image)

Fig. 9: - Double-line-to-ground fault

The boundary conditions for Double Line-to-Ground:

\[ I_a = 0 \]

\[ V_b = V_c = (I_b + I_c)Z_f \]

The symmetrical components of voltage are:

\[
\begin{bmatrix}
V_{a0} \\
V_{a1} \\
V_{a2}
\end{bmatrix}
= \frac{1}{3}
\begin{bmatrix}
1 & 1 & 1 \\
1 & a & a^2 \\
1 & a^2 & a
\end{bmatrix}
\begin{bmatrix}
V_b \\
V_c
\end{bmatrix}
\]

Also \( V_{a1} = V_{a2} = V_{a0} - 3Z_f I_{a0} \)

And Since \( I_a = 0 \)

Therefore, \( I_{a0} + I_{a1} + I_{a2} = 0 \)

Or \( I_{a1} = \frac{V_f}{Z_{kk1} + Z_{kk2} + Z_f} \)

Where, \( Z_0, Z_1, Z_2 \) is a Zero, positive, negative sequence impedance.

**Using Z-parameters analysis (Digital approach)**

We know \( V_{k1} = V_{k2} \) \hspace{1cm} (19a)

\[ V_{k0} - V_{k1} = 3I_{k0}Z_f \]  \hspace{1cm} (19b)

And \( I_{k0} + I_{k1} + I_{k2} = 0 \) \hspace{1cm} (20)

Equations (6a and 8) are still valid. From (8, 19 and 20)

\[ I_{k1} = \frac{E - V_{k1}}{Z_{kk1}} \]  \hspace{1cm} (21a)

\[ I_{k2} = \frac{V_{k2}}{Z_{kk2}} = V_{k1} \]  \hspace{1cm} (21b)

\[ I_{k0} = -\frac{V_{k0}}{Z_{kk0}} = \frac{-V_{k1}}{Z_{kk0} + 3Z_f} \]  \hspace{1cm} (21c)

Substituting (21) into (20) we have

\[-\frac{V_{k1}}{Z_{kk0} + 3Z_f} + \frac{E - V_{k1}}{Z_{kk1}} - \frac{V_{k1}}{Z_{kk2}} = 0\]

Or \[ V_{k1} = \frac{E(Z_{kk0} + 3Z_f)Z_{kk2}}{Z_{kk1}(Z_{kk2} + 3Z_f) + Z_{kk2}(Z_{kk0} + 3Z_f)} \]  \hspace{1cm} (22)

\[ Z_{kk1}(Z_{kk2} + Z_{kk0} + 3Z_f) + Z_{kk2}(Z_{kk0} + 3Z_f) = \Delta \]  \hspace{1cm} (23)

From (21)

\[ I_{k1} = (Z_{kk2} + Z_{kk0} + 3Z_f)E/\Delta \]  \hspace{1cm} (24a)

\[ I_{k2} = -E(Z_{kk0} + 3Z_f)/\Delta \]  \hspace{1cm} (24b)
\[ I_{k0} = -\frac{E Z_{k2}}{\Delta} \] \hspace{1cm} (24c)

The sequence components of voltages at ith bus \((i = 1, 2 \ldots n)\) are

\[ V_{i0} = -Z_{ik0} I_{k0} = \frac{E Z_{ik0} Z_{k2}}{\Delta} \] \hspace{1cm} (25a)

\[ V_{i1} = E Z_{ik1} I_{k1} = E \left[ Z_{kk2} Z_{k0} + 3Z_f \right] / \Delta \] \hspace{1cm} (25b)

\[ V_{i2} = -Z_{ik2} I_{k2} = \frac{E Z_{ik2} Z_{k0} + 3Z_f}{\Delta} \] \hspace{1cm} (25c)

Test Problem:

A 3-phase fault occurs at the bus 2 of the system shown in fig. below. Find (a) sequence current and voltage (b) fault current of each phases (c) fault voltage (line to neutral) of each phases at bus 3. Assume that prefault voltages at all buses are 1 p.u. Given:

\[ Z_{1-bus} = \begin{bmatrix} 0.1507937 & j0.1269841 & j0.1071429 & j0.0793651 \\ j0.1269841 & j0.1574603 & j0.1328571 & j0.0984127 \\ j0.1071429 & j0.1328571 & j0.1542857 & j0.1142857 \\ j0.0793651 & j0.0984127 & j0.1142857 & j0.1365079 \end{bmatrix} \]

and

\[ Z_{0-bus} = \begin{bmatrix} j0.05 & 0 & 0 & 0 \\ 0 & j0.0514286 & j0.03 & j0.02 \\ 0 & j0.03 & j0.105 & j0.07 \\ 0 & j0.02 & j0.07 & j0.0933333 \end{bmatrix} \]

Since all the negative sequence reactance are the same as the positive sequence reactance,

\[ [Z_{1-bus}] = [Z_{2-bus}] \]

\[ [Z_{0-bus}] = \left[ \begin{bmatrix} j0.05 & 0 & 0 & 0 \\ 0 & j0.0514286 & j0.03 & j0.02 \\ 0 & j0.03 & j0.105 & j0.07 \\ 0 & j0.02 & j0.07 & j0.0933333 \end{bmatrix} \right] \]

RESULT

Using MATLAB tools, the result of the problem is

<table>
<thead>
<tr>
<th>Parameters of justification</th>
<th>For symmetrical fault</th>
<th>For single line-to-ground fault</th>
<th>For line-to-ground fault</th>
<th>For double line-to-ground fault</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_{a0} )</td>
<td>0 amp.</td>
<td>0-j716.36 amp.</td>
<td>0 amp.</td>
<td>0+J3.8415 amp.</td>
</tr>
<tr>
<td>( I_{a1} )</td>
<td>0-j1666.7 amp.</td>
<td>0-j716.36 amp.</td>
<td>0-j3.1754 amp.</td>
<td>0-j5.0961 amp.</td>
</tr>
<tr>
<td>( I_{a2} )</td>
<td>0 amp.</td>
<td>0-j716.36 amp.</td>
<td>j3.1754 amp.</td>
<td>j1.2547 amp.</td>
</tr>
<tr>
<td>( I_a )</td>
<td>0-j1666.7 amp.</td>
<td>0-j2149.1 amp.</td>
<td>0 amp.</td>
<td>0 amp.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters of justification</th>
<th>For symmetrical fault</th>
<th>For single line-to-ground fault</th>
<th>For line-to-ground fault</th>
<th>For double line-to-ground fault</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_b )</td>
<td>(-1443.3+j8 33.25) amp.</td>
<td>0-j0.0315 amp.</td>
<td>(-5.4998-j0.0001) amp.</td>
<td>(-5.4998+j 5.7620) amp.</td>
</tr>
<tr>
<td>( I_c )</td>
<td>1443.3+j8 33.33 amp.</td>
<td>0-j0.0315 amp.</td>
<td>5.4998+j0.001 amp.</td>
<td>5.4998+j 5.7622 amp.</td>
</tr>
</tbody>
</table>

| \( V_{a0} \)                | 19.8464 KV            | 0-0.0819KV                    | 0 KV                     | 0.1152 KV                      |
| \( V_{a1} \)                | 0 KV                  | 0.6373 KV                     | 0.4219 KV                | 1.0000-0.0262 KV               |
| \( V_{a2} \)                | 0 KV                  | -0.3627                      | 0.4219 KV                | 1.0000-0.0262 KV               |
RESULT ANALYSIS

It is evident from the result that a 3-phase fault is the most dangerous one and it has larger magnitude of current than other types of fault. Also among the unsymmetrical, line-to-line fault because of absence of zero sequence network as there is no path to ground, it has higher magnitude of current with respect to single line-to-ground fault and double line-to-ground fault. This could be matched with theoretical equation (16) as well, where zero sequence networks is not there in denominator. The magnitude of Double line-to-ground fault lies between single line-to-ground fault and line -to-line fault i.e. less than line-to-line fault and more than single line-to-ground fault. As far as voltage is concerned line-to-line voltage has least voltage among the unsymmetrical faults.

Also, data and analysis came same when the fault occurred at bus 3 and parameters calculated at bus 2.

CONCLUSION

This paper is basically related with finding the fault that occurs in power system using digital approach and through MATLAB programming and it is concluded that the evaluation of fault is very important part of power system analysis for stable and economic operations of power system network and also because it provide data such as voltage and currents which are necessary in designing the protective schemes of power system. The calculations are done theoretically and also by MATLAB programming. Both results are found equal, so this type of digital fault calculation is very useful.

REFERENCES


BIOGRAPHIES

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