A Nonparametric Approach for Validation of Chain Ladder Methods in Claims Reserving

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Abstract - *The chain-ladder method was considered as a deterministic model for predicting claim amounts in non-life insurance. The results obtained using chain-ladder methods does not have provision for conducting diagnostic test and to measure the important statistical measures such as confidence interval. This paper provides a statistical non-parametric procedure to validate the results of the chain-ladder methods and also able to compute the vital statistical measures through kernel density estimation with suitable illustration.*

***Key Words*: Case Reserving, Chain Ladder, Kernel Density Estimation**

1.INTRODUCTION

The challenge of general insurance is to finalize the necessary reserves for liabilities that are not fully known, and it has got great importance when considering the risk of insolvency and the capital requirements for general insurers. The concept of case reserving is introduced by Richard. J. Verrall. The insurance regulation plays a vital role in determining the expected profit or loss in a general insurance business. Usually in general insurance business, the ultimate claims amount of an accident year may not know at the end of that year because of its nature. For eg. the claims settlement process may involve several years because of bodily injuries and/or long legal processes. Also it may take place a delay of possible time lag between the occurrence of the accident and the realization of the consequences of the event. Estimation of the outstanding claims and thereby enable the company to set its reserves is the main objective of claim reserving. The loss reserving methods are classified in to two categories, deterministic and stochastic. Usually for determining a reserve estimate, the insurer will choose several methods of claim reserving. Chain-ladder method and the Bornhuetter-Ferguson method are the two most popular methods of case reserving. For a comprehensive review of loss reserving methods, the reader is referred to Taylor (2000), Foundations of Casualty Actuarial Science (2001), Brown and Gottlieb (2001), and England and Verrall (2002). Renshow and Verrall (1994) defined the generalized linear model (GLM) underlying the chain ladder technique and suggested some other GLMs which might be useful in claim reserving. Richard Verrall (1996) showed that nonparametric smoothing can provide more stable reserve estimates, and is an alternative to other smoothing methods suggested for this purpose such as the Kalman filter.

The chain-ladder method also known as the development method trusts completely on the data contained in the run-off triangle. Past experience is an indicator of future experience is the assumption underlying the chain-ladder method. To estimate how claim amounts will increase (or decrease) in the future, use the loss development patterns in the past. The chain-ladder technique will produce accurate results only when the patterns of loss development in the past can be assumed to continue in the future. The Bornhuetter-Ferguson method restricts the use of the run-off triangle to the estimation of the percentage of the outstanding loss and uses the product of the earned premium and an expected loss ratio to estimate the expected ultimate loss. When there are changes to an insurer's operations, such as a change in claims settlement times, changes in claims staffing, or changes to case reserve practices, the chain-ladder method will not produce an accurate estimate without adjustments. The chain-ladder method is also very responsive to changes in experience, and as a result, it may be unsuitable for very volatile lines of business. The Bornhuetter-Ferguson method uses both past loss development as well as an independently derived prior estimate of ultimate expected losses. But it depends on expected loss ratio or a priori pure premium and requires development factors; therefore it may not be always suitable for case reserving.

Also the application of nonparametric methods in case reserving such as the classical chain ladder method consists of the construction of a structured histogram on a triangle. The histogram separates the data into distinct non-overlapping bins, and constructs bars (hypercube) with heights defined as the proportion (or the number) of observations falling into each bin. This proportion gives an estimate of the probability density function at the midpoint of the bin. Histograms for the graphical presentation of bivariate or trivariate data present several difficulties and in all cases, the histogram still requires a choice of the amount of smoothing. And also the shape of the histogram can potentially be influenced by where the bin centers are placed. Kernel smoothing method overcomes the drawbacks of this histogram method. Apart from the histogram, the kernel estimator is probably the most commonly used estimator and is certainly the most studied mathematically. But fitting a kernel density estimator for a discrete random variable does not make sense. Therefore one can use data recorded in the continuous time.

Density estimation based on kernel density estimation was first introduced by Rosenblatt (1956). For a detailed treatment of kernel density estimation, see the book of Silverman (1986), as well as Scott (1992), Wand and Jones (1995) and Bowman and Azzalini (1997). Catalina Bolance, Montserrat Guillen, Jens Perch Nielsen (2003) estimate actuarial loss functions based on a symmetrized version of the semi parametric transformation approach to kernel smoothing. Catalina Bolance, Montserrat Guillen and David Pitt (2014) illustrated the benefits of applying transformations to data prior to employing kernel based methods. Arthur Charpentier and Emmanuel Flachaire (2014) showed that a preliminary logarithmic transformation of the data, combined with standard kernel density estimation methods, can provide a much better fit of the overall density estimation.

2. Chain-ladder method

Let us assume that the data consist of a triangle of incremental claims. This is the simplest shape of data that can be obtained, and it is often the case that data from early origin years are considered fully run-off or that other parts of the triangle are missing. We assume that we have the following set of incremental claims data:

$$\left\{C_{ij} : i = 1, 2, \ldots, n;\ j = 1, 2, \ldots, n-i+1\right\}$$

The suffix *i* refers to the row, and could indicate accident year or underwriting year. The suffix *j* refers to the column, and indicates the delay, here assumed also to be measured in years. The skeleton of the table is presented as follows

Origin Period	*Development Period* 1	2	3	...	n
1	c_{11}	c_{12}	c_{13}	...	c_{1n}
2	c_{21}	c_{22}	c_{23}	...	c_{2n}
3	c_{31}	c_{32}	c_{33}	...	c_{3n}
...	...	... ...	...	...	...
n	c_{n1}	c_{n2}	c_{n3}	...	c_{nn}

The classical chain-ladder is a deterministic algorithm to forecast claims based on historical data. It assumes that the proportional developments of claims from one development period to the next are the same for all origin years.

The age-to-age link ratios are calculated as the volume weighted average development ratios of a cumulative loss development triangle from one development period to the next $C_{ik}\,; i, k = 1, 2, \ldots, n$.

$$f_k = \frac{\sum_{i=1}^{n-k} C_{i.k+1}}{\sum_{i=1}^{n-k} C_{i.k}} \qquad (1)$$

The link ratios are then applied to the latest known cumulative claims amount to forecast the next development period.

3. Kernel density estimation

Kernel density estimation is a fundamental data smoothing problem where inferences about the population are made, based on a finite data sample. It is a nonparametric way to estimate the probability density function of a random variable. For a univariate random variable *X* with unknown density *f*(*x*), if we draw a sample of *n* independent and identically distributed observations x_1, $x_2, \ldots x_n$, the kernel density estimator is given by (Wand and Jones, 1995)

$$\hat{f}_{kde}(x) = \frac{1}{n}\sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right) \qquad (2)$$

where *K* is the kernel and *h* is the bandwidth (Scott, 1992).

3.1 Selection of Kernel

The choice of kernel and the selection of bandwidth are closely related. The kernel *k (.)* is only well defined up to a scale. The optimal kernel in density estimation was extensively discussed in the literature. In practice, the Epanechnikov kernel is referred as the optimal kernel; nevertheless, the choice of kernel is not critical. The key message is that the suboptimal kernels lose very little in performance. And, these results suggest that most unimodal kernel densities perform about the same as each other. Further, we have used R software to perform density estimation and dpik method is used for bandwidth.

4. Illustration

Consider the following data from the Reinsurance Association of America(RAA)

Development year

Origin	1	2	3	4	5	6	7	8	9	10
1981	5012	8269	10907	11805	13539	16181	18009	18608	18662	18834
1982	106	4285	5396	10666	13782	15599	15496	16169	16704	
1983	3410	8992	13873	16141	18735	22214	22863	23466		
1984	5655	11555	15766	21266	23425	26083	27067			
1985	1092	9565	15836	22169	25955	26180				
1986	1513	6445	11702	12935	15852					
1987	557	4020	10946	12314						
1988	1351	6947	13112							
1989	3133	5395								
1990	2063									

Source: Historical Loss Development, Reinsurance Association of America(1991)

Table 1

The loss development factor (LDF) is calculated for the above is given below :

1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	tail
2.999	1.624	1.271	1.172	1.113	1.042	1.033	1.017	1.009	1.050

Table 2

The squaring of the run-off triangle is calculated below, where an ultimate claims is given in table 3 since the expected development beyond the oldest age (10) of the triangle due to the tail factor (1.05) being greater than unity.

The ultimate losses is defined as total sum the insured, its insurers and reinsurers (if any) pay for a fully developed loss. (i.e.,) Paid losses plus outstanding losses and incurred but not reported (IBNR) losses. It may not be possible to know the exact value of ultimate losses for a long time after the end of a policy period.

1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
18928	16942	24204	28847	29072	19599	17838	24139	16125	18495

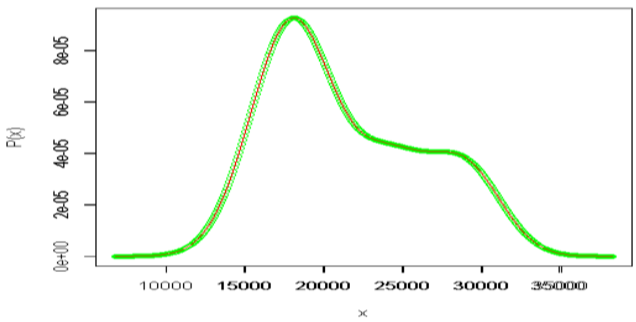
Table 3

The full run-off triangle is computed using loss development factor methods is given below:

Origin	1	2	3	4	5	6	7	8	9	10
1981	5012	8269	10907	11805	13539	16181	18009	18608	18662	18834
1982	106	4285	5396	10666	13782	15599	15496	16169	16704	*16858*
1983	3410	8992	13873	16141	18735	22214	22863	23466	*23863*	*24083*
1984	5655	11555	15766	21266	23425	26083	27067	27967	*28441*	*28703*
1985	1092	9565	15836	22169	25955	26180	27278	28185	*28663*	*28927*
1986	1513	6445	11702	12935	15852	17649	18389	19001	*19323*	*19501*
1987	557	4020	10946	12314	14428	16064	16738	17294	*17587*	*17749*
1988	1351	6947	13112	16664	19525	21738	22650	23403	*23800*	*24019*
1989	3133	5395	8759	11132	13043	14521	15130	15634	*15898*	*16045*
1990	2063	6188	10046	12767	14959	16655	17353	17931	*18234*	*18402*

Table 4

The kernel density plot for the total expected claims reserves (Given in Column 10 of the table 2) shown in green color and the ultimate total claim losses shown in red color is given below:



The summary of probability distribution of expected and ultimate claims reserving is given below:

Probability distribution function	Mean	Standard Deviation	Empirical Characteristic Function
Expected Claims	272	2459	1.318-4.61i
Ultimate Claims	272	2466	-0.081+2.54i

Table 5

5. Conclusion

It is observed that the probability distributions obtained through kernel density estimation is asymmetric and does follow any standard probability distribution. Though the kernel density estimation does not provide the function form of the probability distribution but it is possible to get the shape of the distribution and probability values for each value of the variable thereby we can compute all the required statistical measures. This paper paved the way for the validation of the results obtained by Mack chain-ladder methods. Further, the kernel density estimation helps us to obtain the necessary statistical measures of the claim loss reserve of expected and ultimate total claims and compare the probability distributions of the both expected and ultimate cumulative claims sizes.

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