Pythagorean triangle with Area/Perimeter as a Jarasandha number of orders 2 & 4

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Abstract - *We present patterns of Pythagorean triangles, in each of which the ratio Area/Perimeter is represented by the Jarasandha number. A few interesting relations among the sides are also given.*

Key Words: Pythagorean triangles, Jarasandha numbers.

1.INTRODUCTION

Mathematics is the language of patterns and relationships, and is used to describe anything that can be quantified. Number theory is one of the largest and oldest branches of mathematics. The main goal of number theory is to discover interesting and unexpected relationships. It is devoted primarily to the study of natural numbers and integers. In number theory, Pythagorean triangles have been a matter of interest to various mathematicians, because it is a treasure house in which the search for many hidden connection is a treasure hunt. For an extensive variety of fascinating problems, one may refer [1-5]. Apart from the polygonal numbers, we have some more fascinating patterns of numbers namely Jarasandha numbers, nasty numbers and dhuruva numbers. These numbers have been presented in [6-9].

In [10], special pythagorean triangles connected with nasty numbers are obtained. In [11], special pythagorean triangles connected with Jarasandha numbers are obtained. In [12], special pairs of pythagorean triangles and Jarasandha numbers are presented. Recently in [13] &[14], rectangles in connection with Jarasandha numbers are obtained.

In this communication, we search for patterns of Pythagorean triangles, in each of which the ratio Area/Perimeter is represented by the Jarasandha number. Also, a few interesting relations among the sides are given.

2.BASIC DEFINITIONS

Definition 1:

The ternary quadratic Diophantine equation given by $x^2+y^2=z^2$ is known as Pythagorean equation where x, y and z are natural numbers. The above equation is also referred to as Pythagorean triangle and denote it by $T(x, y, z)$

Also, in Pythagorean triangle $T(x, y, z)$: $x^2+y^2=z^2$, x and y are called its legs and z its hypotenuse.

Definition 2:

Most cited solution of the Pythagorean equation is $x=m^2-n^2, y=2mn, z=m^2+n^2$, where $m>n>0$. This solution is called primitive, if m, n are of opposite parity and $\gcd(m,n)=1$.

Definition 3:

An n-digit number that is the sum of nth powers of its digits is called narcissistic number.

3. Jarasandha Numbers

In our indian epic mahabharatha, we come across a person named 'JARASANDHA'. He had a boon that if he was split into two parts and thrown apart, the parts would rejoin and return to life. In fact, he was given life by the two halves of his body. In the field of mathematics, we have numbers exhibiting the same property as Jarasandha. Consider a number of the form XC. This may split as two numbers X and C and if these numbers are added and squared we get the same number XC.

(i.e) $XC=(X+C)^2=XC$

Note: If C is an n-digit number, then $(X+C)^2=(10^n)(X)+C$.

4.Method of Analysis

Denoting the Area & Perimeter of the triangle by A & P respectively, the assumption

$$\frac{A}{P}= \text{Jarasandha number.}$$

The above relation leads to the equation

$\frac{n\,(m-n)}{2} =$ Jarasandha number. (1)

Case 1:

When $\frac{n\,(m-n)}{2} = 81$(2-digit Jarasandha number) (2)

$\Rightarrow n\,(m-n) = 162$ (3)

On evaluation, the values of the generators m, n satisfying(3) are given in the following Table1:

Table 1:

S.NO	n	$m-n$	m	x	y	z	A	P	$\frac{A}{P}$
1.	1	162	163	26568	326	26570	4330584	53464	81
2.	2	81	83	6885	332	6893	1142910	14110	81
3.	3	54	57	3240	342	3258	554040	6840	81
4.	6	27	33	1053	396	1125	208494	2574	81
5.	9	18	27	648	486	810	157464	1944	81
6.	18	9	27	405	972	1053	196830	2430	81
7.	27	6	33	360	1782	1818	320760	3960	81
8.	54	3	57	333	6156	6165	1024974	12654	81
9.	81	2	83	328	13446	13450	2205144	27224	81
10.	162	1	163	325	52812	52813	8581950	105950	81

Thus it is seen that there are 10 Pythagorean triangles. Of these 10 Pythagorean triangles, 2 triangles are Primitive and remaining 8 triangles are non-primitive triangles.

Case 2:

Consider the 4-digit Jarasandha number 2025,

In this case, $n\,(m-n) = 4050$ (4)

Following the same procedure as in case1, we have 30 distinct values for m, n satisfying (4) are given below:

Table 2:

S.NO	n	$m-n$	m	x	y	z	A	P	$\frac{A}{P}$
1.	1	4050	4051	16410600	8102	16410602	66479340600	3282930400	2025
2.	2	2025	2027	4108725	8108	4108733	16656771150	8225566	2025
3.	3	1350	1353	1830600	8118	1830618	7430405400	3669336	2025
4.	5	810	815	664200	8150	664250	2706615000	1336600	2025
5.	6	675	681	463725	8172	463797	1894780350	935694	2025

6.	9	450	459	210600	8262	210762	869988600	429624	2025
7.	10	405	415	172125	8300	172325	714318750	352750	2025
8.	15	270	285	81000	8550	81450	346275000	171000	2025
9.	18	225	243	58725	8748	59373	256863150	126846	2025
10.	25	162	187	34344	9350	35594	160558200	79288	2025
11.	27	150	177	30600	9558	32058	146237400	72216	2025
12.	30	135	165	26325	9900	28125	130308750	64350	2025
13.	45	90	135	16200	12150	20250	98415000	48600	2025
14.	50	81	131	14661	13100	19661	96029550	47422	2025
15.	54	75	129	13725	13932	19557	95608350	47214	2025
16.	75	54	129	11016	19350	22266	106579800	52632	2025
17.	81	50	131	10600	21222	23722	112476600	55544	2025
18.	90	45	135	10125	24300	26325	123018750	60750	2025
19.	135	30	165	9000	44550	45450	200475000	99000	2025
20.	150	27	177	8829	53100	53829	234409950	115758	2025
21.	162	25	187	8725	60588	61213	264315150	130526	2025
22.	225	18	243	8424	109350	109674	460582200	227448	2025
23.	270	15	285	8325	153900	154125	640608750	316350	2025
24.	405	10	415	8200	336150	336250	1378215000	680600	2025
25.	450	9	459	8181	413100	413181	1689785550	834462	2025
26.	675	6	681	8136	919350	919386	3739915800	1846872	2025
27.	810	5	815	8125	1320300	1320325	5363718750	2648750	2025
28.	1350	3	1353	8109	3653100	3653109	14811493950	7314318	2025
29.	2025	2	2027	8104	8209350	8209354	33264286200	16426808	2025
30.	4050	1	4051	8101	32813100	32813101	132909461600	65634302	2025

Thus it is seen that there are 30 Pythagorean triangles. Of these 30 Pythagorean triangles, 4 triangles are Primitive and remaining 26 triangles are non-primitive triangles.

Case 3: For the 4-digit Jarasandha number 3025,

For this choice, we have, $n\,(m-n) = 6050$ (5)

Following the same procedure as in case1, we have 18 distinct values for m, n satisfying (5) are given below:

Table 3:

S.NO	n	$m-n$	m	x	y	z	A	P	$\frac{A}{P}$
1.	1	6050	6051	36614600	12102	36614602	221554944600	73241304	3025
2.	2	3025	3027	9162725	12108	9162733	55471137150	18337566	3025
3.	5	1210	1215	1476200	12150	1476250	8967915000	2964600	3025
4.	10	605	615	378125	12300	378325	2325468750	768750	3025
5.	11	550	561	314600	12342	314842	1941396600	641784	3025
6.	22	275	297	87725	13068	88693	573195150	189486	3025
7.	25	242	267	70664	13350	71914	471682200	155928	3025
8.	50	121	171	26741	17100	31741	228635550	75582	3025
9.	55	110	165	24200	18150	30250	219615000	72600	3025
10.	110	55	165	15125	36300	39325	274518750	90750	3025
11.	121	50	171	14600	41382	43882	302088600	99864	3025
12.	242	25	267	12725	129228	129853	822213150	271806	3025
13.	275	22	297	12584	163350	163834	1027798200	339768	3025
14.	550	11	561	12221	617100	617221	3770789550	1246542	3025
15.	605	10	615	12200	744150	744250	4539315000	1500600	3025
16.	1210	5	1215	12125	2940300	2940325	17825568750	5892750	3025
17.	3025	2	3027	12104	18313350	18313354	110832394200	36638808	3025
18.	6050	1	6051	12101	73217100	73217101	443000063600	146446302	3025

Thus it is seen that there are 18 Pythagorean triangles. Out of these 18 Pythagorean triangles, 4 triangles are Primitive and remaining 14 triangles are non-primitive triangles.

Case 4:

Consider the 4-digit Jarasandha number 9801,

Following the same procedure as in case1, we have 30 distinct values for m, n.

Table 4:

S.NO	n	$m-n$	m	x	y	z	A	P	$\frac{A}{P}$
1.	1	19602	19603	384277608	39206	384277610	7532993950000	768594424	9801
2.	2	9801	9803	96098805	39212	96098813	1884113171000	192236830	9801
3.	3	6534	6537	42732360	39222	42732378	838024312000	85503960	9801

4.	6	3267	3273	10712493	39276	10712565	210371937500	21464334	9801
5.	9	2178	2187	4782888	39366	4783050	94141584500	9605304	9801
6.	11	1782	1793	3214728	39446	3214970	63404080340	6469144	9801
7.	18	1089	1107	1225125	39852	1225773	24411840750	2490750	9801
8.	22	891	913	833085	40172	834053	16733345310	1707310	9801
9.	27	726	753	566280	40662	567738	11513038680	1174680	9801
10.	33	594	627	392040	41382	394218	8111699640	827640	9801
11.	54	363	417	170973	45036	176805	3849970014	392814	9801
12.	66	297	363	127413	47916	136125	3052560654	311454	9801
13.	81	242	323	97768	52326	110890	2557904184	260984	9801
14.	99	198	297	78408	58806	98010	2305430424	235224	9801
15.	121	162	283	65448	68486	94730	2241135864	228664	9801
16.	162	121	283	53845	91692	106333	2468577870	251870	9801
17.	198	99	297	49005	117612	127413	2881788030	294030	9801
18.	242	81	323	45765	156332	162893	3577266990	364990	9801
19.	297	66	363	43560	215622	219978	4696247160	479160	9801
20.	363	54	417	42120	302742	305658	6375746520	650520	9801
21.	594	33	627	40293	744876	745965	15006644330	1531134	9801
22.	726	27	753	39933	1093356	1094085	21830492570	2227374	9801
23.	891	22	913	39688	1626966	1627450	32285513300	3294104	9801
24.	1089	18	1107	39528	2411046	2411370	47651913140	4861944	9801
25.	1782	11	1793	39325	6390252	6390373	125648330000	12819950	9801
26.	2178	9	2187	39285	9526572	9526653	187125690500	19092510	9801
27.	3267	6	3273	39240	21385782	21385818	419589042800	42810840	9801
28.	6534	3	6537	39213	85425516	85425525	1674895379000	170890254	9801
29.	9801	2	9803	39208	192158406	192158410	3767073391000	384356024	9801
30.	19602	1	19603	39205	768516012	768516013	15064835130000	1537071230	9801

Thus it is seen that there are 30 Pythagorean triangles. Of these 30 Pythagorean triangles, 4 triangles are Primitive and remaining 26 triangles are non-primitive triangles.

5.Observations

1. $x+y-z,\ \ y+z,\ 2(x+z),\ 2(z-x)$ are perfect squares.

2. $6(x+y-z),\ \ 6(y+z),\ 3(x+z),\ 27(z+x)$ are nasty numbers.

3. $\frac{x+y-z}{4}$ is a Jarasandha number.

4. It is worth to note that for one of the triangles in all the cases, m, n are consecutive & y, z are also consective.

5. For the Jarasandha number 81, $x+y-z+46=$ Narcissistic number.

6.Conclusion

To conclude, One may search for the connections between the Pythagorean triangles and other Jarasandha numbers of higher order and other number patterns.

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