

# **Quanser Rotary Inverted Pendulum Stabilization with Fuzzy Logic** Controller

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**Abstract** - *The inverted pendulum problem is a standard* control problem which has got wide applications in aerospace, robotics and many other control engineering fields. It is also used for testing and studying various controllers. An inverted pendulum is a highly nonlinear unstable under actuated system. This paper presents the stabilization of a Quanser rotary inverted pendulum at its upright position using Fuzzy Logic Controller. The mathematical model of the system has been established. Matlab simulation of Fuzzy Logic Controller shows that the system can be stabilized.

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Key Words: (Size 10 & Bold) Key word1, Key word2, Key word3, etc (Minimum 5 to 8 key words) ...

# **1. INTRODUCTION**

The inverted pendulum is a classic experiment used to teach dynamics and control systems. There are two types of inverted pendulum. Rotary inverted pendulum and cart type inverted pendulum. The rotary inverted pendulum has a pendulum attached to a rotary arm instead of moving cart. The advantage of this system over cart system is that there is no end point. For controlling an inverted pendulum there are several tasks to be solved, such as swinging up and catching the pendulum from its stable pending position to the upright unstable position, and then balancing the pendulum at the upright position during disturbances. Many control strategies for swinging and stabilization of an inverted pendulum have been proposed in various literatures.

Nguyen et al. [1] presented the physical structure, the dynamic model of rotary inverted pendulum system and the method of identifying, controlling this system by an artificial neural network (ANN). Vishwa Nath and R. Mitra [2] described steps to design controllers for swing up and stabilization of RIP using counter based and pole placement with integrator respectively. Yung-Feng et al. [3] proposed an adaptive controller to balance a rotary inverted pendulum with time-varying uncertainties. Mohamed I. El-Hawwary et al. [4] presented the adaptive fuzzy control of the inverted-pendulum on a cart problem as an under actuated mechanical system. Haohua Qin [5] established dynamical equation of single rotary inverted pendulum,

which based on Lagrange equation for the swing-up control problem of single rotary inverted pendulum. Ahangar-Asr. H. et al. [6] presented a hybrid strategy for the control of rotary inverted pendulum. Using their proposed hybrid control structure they took the advantage of useful properties of both sliding mode controllers and fuzzy logic controllers at the same time.

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# 2. SYSTEM DESCRIPTION

The Quanser Rotary Pendulum, pictured in Fig. 1, consists of a flat arm, or hub, with a pivot at one end and a metal shaft on the other end [7]. The pivot-end can be mounted on top of the SRV02 load gear shaft and fastened with screws. The actual pendulum link is fastened onto the metal shaft and the shaft is instrumented with a sensor to measure its angle. The result is a horizontally rotating arm with a pendulum at the end. The ROTPEN is instrumented with an encoder (1024 line) to obtain a digital measurement of the pendulum and is free to rotate 360 degrees. The encoder used to measure the pendulum angle on the ROTPEN module is a US Digital S1 single-ended optical shaft encoder. It offers a high resolution of 4096 counts per revolution in quadrature mode (1024 lines per revolution).



Fig -1: Quanser rotary inverted pendulum module

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# **2.1 System Components**

The components of the Rotary Pendulum module are listed in Table.1 below and labeled in Fig 2.



Fig -2: ROTPEN components

### Table -1: List of ROTPEN components

ROTPEN Components	
No.	Components
1	SRV02
2	Thumbscrews
3	Rotary arm
4	Shaft housing
5	Shaft
6	Pendulum T fitting
7	Pendulum link
8	Pendulum encoder connector
9	Pendulum encoder

The Rotary Inverted Pendulum Module is attached to the SRV02 load gear by two thumbscrews [8]. The Pendulum Arm is attached to the module body by a set screw. The Inverted Pendulum experiment is a classical example of how the use of control may be employed to stabilize an inherently unstable system. The Inverted pendulum is also an accurate model in the pitch and yaw of a rocket in flight and can be used as a benchmark for many control methodologies.

# 2.2 System Modeling

Fig. 3 below depicts the rotary inverted pendulum in model [8]. The rotary arm pivot is attached to the SRV02 system and is actuated. The arm has a length of  $L_r$ , a moment of inertia of J<sub>r</sub>, and its angle,  $\theta$ , increases positively when it

rotates counter-clockwise (CCW). The servo (and thus the arm) should turn in the CCW direction when the control voltage is positive, i.e.,  $V_m > 0$ .



Fig -3: Rotary inverted pendulum conventions

The pendulum link is connected to the end of the rotary arm. It has a total length of  $L_p$  and it center of mass is  $L_p/2$ . The moment of inertia about its center of mass is  $J_p$ . The inverted pendulum angle,  $\alpha$ , is zero when it is perfectly upright in the vertical position and increases positively when rotated CCW.

Instead of using classical mechanics, the Lagrange method is used to find the equations of motion of the system. The Euler- Lagrange equation is given as:

$$\frac{\partial^2 L}{\partial t \partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = Q_1$$

$$\frac{\partial^2 L}{\partial t \partial \dot{\alpha}} - \frac{\partial L}{\partial \alpha} = Q_2$$
(1)

The Lagrangian of the system is described as:

Where, T is the total kinetic energy of the system and V is the total potential energy of the system.

The generalized forces  $Q_i$  are used to describe the nonconservative forces (e.g., friction) applied to a system with respect to the generalized coordinates. In this case, the generalized force acting on the rotary arm and pendulum is:

$$Q_{1} = \tau - B_{r} \theta$$

$$Q_{2} = -B_{p} \dot{\alpha}$$
(3)

The description of the corresponding SRV02 parameters are given in [8]. The control variable is the input servo motor voltage,  $V_m$ . Opposing the applied torque is the viscous friction torque, or viscous damping, corresponding to the term  $B_r$ . Since the pendulum is not actuated, the only force

acting on the link is the damping. The viscous damping coefficient of the pendulum is denoted by  $B_p$ .

The nonlinear equations of motion for the SRV02 rotary inverted pendulum are:

$$\begin{pmatrix} m_{p}L_{r}^{2} + \frac{1}{4}m_{p}L_{p}^{2} - \frac{1}{4}m_{p}L_{p}^{2}\cos(\alpha)^{2} + J_{r} \end{pmatrix}\ddot{\theta} - \left(\frac{1}{2}m_{p}L_{p}L_{r}\cos(\alpha)\right)\ddot{\alpha} + \left(\frac{1}{2}m_{p}L_{p}L_{r}\cos(\alpha)\right)\ddot{\alpha} + \left(\frac{1}{2}m_{p}L_{p}L_{r}\sin(\alpha)\right)\dot{\alpha}^{2} = \tau - B_{r}\dot{\theta}$$

$$-\frac{1}{2}m_{p}L_{p}L_{r}\cos(\alpha)\ddot{\theta} + \left(J_{p} + \frac{1}{4}m_{p}L_{p}^{2}\right)\ddot{\alpha} - \frac{1}{4}m_{p}L_{p}^{2}\cos(\alpha)\sin(\alpha)\dot{\theta}^{2} \qquad (5)$$

$$-\frac{1}{2}m_{p}L_{p}g\sin(\alpha) = -B_{p}\dot{\alpha}$$

The torque applied at the base of the rotary arm (i.e., at the load gear) is generated by the servo motor as described by the equation:

$$\tau = \frac{\eta_m \eta_s K_{\iota} K_s (V_m - K_s K_m \theta)}{R_m}$$
(6)

The nonlinear equations can be linearized using the below method:

$$f_{lin} = f(z_0) + \left(\frac{\partial f(z)}{\partial z_1}\right)|_{z=z_0} (z_1 - a) + \left(\frac{\partial f(z)}{\partial z_2}\right)|_{z=z_0} (z_2 - b)$$
(7)

With all initial state variables as zero, the linearized equation can be obtained as:

$$\left(m_{p}L_{r}^{2}+J_{r}\right)\ddot{\theta}-\frac{1}{2}m_{p}L_{p}L_{r}\overset{\cdots}{\alpha}=\tau-B_{r}\overset{\cdot}{\theta}$$

$$\left(8\right)$$

$$\left(\frac{1}{2}m_{p}L_{p}L_{r}\overset{\cdots}{\theta}+\left(J_{p}+\frac{1}{4}m_{p}L_{p}^{2}\right)\ddot{\alpha}-\left(\frac{1}{2}m_{p}L_{p}g\alpha\right)=-B_{p}\overset{\cdot}{\alpha}$$

$$(9)$$

After substituting the values of parameters [7] equation (8) can be written in state space model as:

$$\begin{bmatrix} \dot{\theta} \\ \dot{\alpha} \\ \ddot{\theta} \\ \ddot{\alpha} \\ \ddot{\theta} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 80.3 & -45.8 & -0.930 \\ 0 & 122 & -44.1 & -1.40 \end{bmatrix} \begin{bmatrix} \theta \\ \alpha \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 83.4 \\ 80.3 \end{bmatrix} V_m (10)$$

# **3. CONTROLLER DESIGN**

Balancing an inverted pendulum is like balancing a vertical stick with your hand moving it back and forth [9]. The pendulum is balanced by applying torque to the arm. The balance controller supplies a motor voltage that supplies a torque to the pendulum pivot. The amount of voltage depends on angular position and speed of both the arm and the pendulum.

Here, a fuzzy logic controller (FLC) is designed for controlling the pendulum. A fuzzy control system is one based on fuzzy logic, a mathematical system that analyzes analog input values in terms of logical variables. It takes on continuous values between 0 and 1. Whereas in classical or digital logic, it operates on discrete values of either 1 or 0 (true or false, respectively).

The fuzzy logic control comprise of an input stage, a processing stage, and an output stage. The input stage maps sensor or different inputs, for example, switches, thumbwheels, etc, to the appropriate membership functions and truth values. The processing stage summons each suitable rule and creates an outcome for each, then consolidates the results of the rules. At last, the output stage changes over the joined result once more into a particular control output value.

In the case of rotary inverted pendulum, input to the fuzzy controller is the pendulum angle and output is the control voltage. The membership functions of them are shown in the figures below.



Fig -4: Membership function for angle of pendulum



Fig -5: Membership function for velocity of pendulum



Trapezoidal and triangular membership functions are used for input. While output voltage consists of only triangular membership function.

- The rule base for the fuzzy controller is given as
- If the angle is positive voltage is positive
- If the angle is zero voltage is zero
- If the angle is negative voltage is negative

### **4. SIMULATION RESULTS**

In designing FL controller, first we need to create a fis file having all the inputs, outputs their range, membership functions and rules. Then sufficient modifications can be made to meet the required specifications. The simulated response of the pendulum angle, arm angle, pendulum velocity and arm velocity are shown in the below figures.



**Fig -6**: .Output response of pendulum angle (deg) Vs time (sec)



**Fig -7**: Output response of pendulum velocity (deg/sec) Vs time (sec)



**Fig -9**: Output response of arm velocity (deg/sec) Vs time (sec)

The simulation results shows that the output states pendulum angle, pendulum velocity and arm velocity can be attained zero within a time of 0.5 sec. Arm angle cannot be attained zero because at any position of the arm the pendulum can be stabilized vertically above its pivot. The results shows that the controller performs well in this system.

### **5. CONCLUSION**

This paper presents the simulation and the results of a fuzzy logic control problem which deals with the balancing of a rotary inverted pendulum driven by a motor with the voltage of the DC servo motor as control input. By using Fuzzy and Simulink tools we have obtained the control of the problem. The mathematical model of the system using Euler-Lagrange equation is established. Finally, the responses of pendulum angle, velocity of arm and pendulum show that the controller effectively stabilizes the system.

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