Modified Active Contour Snake Model for Image Segmentation using Anisotropic Filtering

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Abstract - The role of segmentation is to partition a digital image into multiple regions to simplify or change the representation of an image into something more meaningful and easier to analyze. Active Contour Snake model is a very effective technique used for medical image segmentation. This method finds the curve that best separates objects in an image. It attempts to minimize an energy associated to the current contour as a sum of an internal and external energy. Despite, several advantages there are several limitations of this method. Major drawback taken into is that active contour models are difficult to use when the edges of a feature are weak, noisy, and diffuse; they tend to get distracted and trapped by nearby edges that do not belong to the desired feature.

The main purpose of this paper is to improve the smoothing efficiency of snakes to a very extent keeping in mind that less computation efforts would be made. In the proposed approach we have used anisotropic filters instead of Gaussian filters in the pre-processing phase of active contour snake model to suppress noise in the medical images. As anisotropic filters are more efficient in suppressing noise. Diffusion coefficient is chosen to be an appropriate function of the image gradient so that anisotropic diffusion enhances edges maintaining the stability of diffusions. The proposed algorithm when used to smoothen the noisy images gives efficient results when noisy medical images are segmented using active contour Snake model. Experimental results are shown on number of images.

Key Words: (Size 10 & Bold) Key word1, Key word2, Key word3, etc (Minimum 5 to 8 key words)...

1. INTRODUCTION

Active contours are a name for methods that find the curve that best separates objects in an image. This is known as segmentation. Snake is an energy minimizing deformable spline influenced by constraint and image forces that pull it towards object contours. Applications of Snakes are: Object Tracking, Shape Recognition Segmentation, Edge detection, Stereo matching. Snakes is a general technique of matching a deformable model to an image by means of energy minimization.[6] Snakes exhibit dynamic behavior as it always minimizes its energy functional. A simple elastic snake[5] is thus defined by a set of n points, an internal elastic energy term, an external edge based energy term Snake is placed near the object contour. It will dynamically move towards object contour by minimizing its energy iteratively. In Snakes, we use the technique of matching a deformable model[5] to an image by means of energy minimization. A snake initialized near the target gets refined iteratively and is attracted towards the salient contour. A snake in image can be represented as a set of n points.

vi=(xi,yi), where i=0…n-1

Parametric representation of curve:

\[ V(s)=(x(s),y(s)) \]

Energy Function

\[ E=\int\text{E}_{\text{snake}}(v(s))\,ds=\int\left(E_{\text{internal}}(v(s))+E_{\text{image}}(v(s))+E_{\text{con}}(v(s))\right)\,ds \]

Einternal is internal energy of the spline (snake) due to bending, Eimage is image forces acting on the spline Econ refers to external constraint forces introduced by the user[6].

Snakes are autonomous and self adapting[7] in their search for a minimal energy state. They can be easily manipulated using external image forces. They can be made sensitive to image scale by incorporating Gaussian smoothing in the image energy function[4]. They can be used to track dynamic objects in temporal as well as the spatial dimensions.

2. ACTIVE CONTOUR SNAKE MODEL

2.1 ACTIVE CONTOUR SNAKE MODEL

Snakes are active contour models. They lock onto nearby edges, localizing them accurately. Scale space continuation can be used to enlarge the capture region surrounding a feature. Snakes provide a unified account of a number of visual problems, including detection of edges, lines, and subjective contours; motion tracking; stereo matching. The snake provides a number of widely separated local minima to further levels of processing. Snakes have been successfully used for interactive interpretation, in which user-imposed constraint forces guide the snake near features of interest[5]. The problem domain they address is that of finding salient image contours- edges, lines, and subjective contours- as well as tracking those contours during motion...
and matching them in stereo sis. The variation approach differs from the traditional approach of detecting edges and then linking them.

Snakes do not try to solve the entire problem of finding salient image contours. They rely on other mechanisms to place them somewhere near the desired contour. Snakes are an example of a more general technique of matching a deformable model to an image by means of energy minimization. From, any starting point, the snake deforms itself into conformity with the nearest salient contour. Our basic snake model is a controlled continuity [7] spline under the influence of image forces and external constraint forces. The internal spline forces serve to impose a piecwise smoothness constraint. The image forces push the snake toward salient image features like lines, edges, and subjective contours. The external constraint forces are responsible for putting the snake near the desired local minimum. Parametric representation [5] of curve

\[ V(s) = (x(s), y(s)) \]

We can write its energy functional as:

\[ E = \int E_{\text{snake}}(v(s)) \, ds - \int \left[ E_{\text{internal}}(v(s)) + E_{\text{image}}(v(s)) + E_{\text{cont}}(v(s)) \right] \, ds \]

where \( E_{\text{internal}} \) is internal energy of the spline(s) due to bending, \( E_{\text{image}} \) is image forces acting on the spline and \( E_{\text{cont}} \) refers to external constraint forces.

**External Energy**

\[ E_{\text{external}} = E_{\text{image}} + E_{\text{cont}} \]

\[ E_{\text{external}} = \text{External energy acting on the spline} \]

**Internal energy**

\[ E_{\text{internal}} = E_{\text{cont}} + E_{\text{curv}}[5] \]

where \( E_{\text{cont}} \) is the energy of the snake contour, \( E_{\text{curv}} \) is the energy of the spline curvature,

\[ E_{\text{internal}} = (a(s) \cdot |v_\alpha(s)|^2 + \beta(s) \cdot |v_\alpha(s)|^2)/2 \]

Here, the first order term makes the snake act like a membrane. Large values of \( a(s) \) will increase the internal energy of the snake as it stretches more and more, whereas small values of \( a(s) \) will make the energy function insensitive to the amount of stretch. Second order term makes it act like a "thin plate" n term-minimum energy when the curve is smooth. Control \( \alpha \) and \( \beta \) to vary between extremes

**Image Energy**

Variety of terms gives different effects

e.g: \( E_{\text{image}} = \ |(x,y) \cdot \text{desired} | , \) minimizes energy at intensity \( \text{desired} \). \( E_{\text{image}} \) has three components: Lines, Edges and Terminations. The energy functional can be represented as follows:

\[ E_{\text{image}} = w_{\text{line}} \cdot E_{\text{line}} + w_{\text{edge}} \cdot E_{\text{edge}} + w_{\text{term}} \cdot E_{\text{term}}[5] \]

Adjusting the weights in the image will determine salient features in the image which will be considered by the snake

**Lines:** A line is the intensity of the image which can be represented as \( E_{\text{line}} = I(x,y) \). Depending on the sign of \( w_{\text{line}} \) the line will be attracted to either dark lines or light lines.

**Edges:** Gradient based [8]. Edges in the image can be found by the following energy function which will make the snake attract towards contours with large image gradients.

\[ E_{\text{image}} = -w \cdot \left( \nabla I(x,y) \right)^2 \]

It is common that a snake started very far from the object converges to the desired object contour. If a part of the snake finds a low energy feature, it pulls the other parts of the snake to continue to the contour.[5] Scale space continuation can be used in order to achieve desired results. One can allow the snake to come to equilibrium on a blurry energy edge functional and reduce the blurring as the calculation progresses. The energy functional is

\[ E_{\text{image}} = -w \cdot \left( \nabla^2 I(x,y) \right)^2 \]

where \( G_\sigma \) is a Gaussian standard deviation \( \sigma \). Minima of this functional lie on zero crossings of \( \nabla^2 I \) which define edges in [6] Marr-Hildreth theory. Thus the snake gets attracted towards zero crossing constrained by its own smoothness.

Laplacian-based

\[ E_{\text{image}} = w \cdot \left( \nabla^2 I(x,y) \right)^2 \]

It can also be smoothed with Gaussian.

**Termination:** Curvature of level lines in a slightly smoothed image is used to detect corners and terminations in an image. Let \( C(x,y) = G_\sigma I(x,y) \) be a slightly smoothed version of the \( I \) image. Let \( \theta = \arctan(C_x/C_y) \) be the gradient angle.[6] And let \( n = (\cos \theta, \sin \theta) \) and \( n = (-\sin \theta, \cos \theta) \) be the unit vectors along and perpendicular to the direction. The terminating functional of energy can be represented as

\[ E_{\text{term}} = \frac{\partial \varphi}{\partial n} - \frac{\partial \varphi}{\partial \varphi} = C_{xx} C_{y}^2 - 2 C_{xy} C_{x} C_{y} + C_{yy} C_{x}^2 \]

where \( G_\sigma \) is a Gaussian of standard deviation.

Minima of this functional lie on zero-crossings of \( \nabla^2 I \) which define edges in the Marr-Hildreth theory. Adding this energy term to a snake means that the snake is attracted to zero-crossings, but still constrained by its own smoothness.

**Constraint Energy**

The original snakes implementation and some other systems allowed for user interaction to guide the snakes in their initial placement as well as in their energy terms[5]. Such constraint energy \( E_{\text{cont}} \) can be used to interactively guide the snakes towards or away from particular features.
2.2 ANALYSIS OF IMAGE SEGMENTATION USING ACTIVE CONTOUR SNAKE MODEL ON NOISY IMAGES

The analysis is shown with the help of two different images. Figure 1 is a an original cortex image. Figure 2 is the original image in Figure 1 with added Gaussian noise. Figure 3 shows the segmentation results of noisy cortex image with active contour snake model with given parameters. Figure 4 is the original cardia image. Figure 5 is the original cardia image with added Gaussian noise. Figure 6 shows the segmentation results of noisy cardia image with active contour snake model with given parameters. For this experiment values are shown at the bottom of each figure.

\[ E_{\text{con}} = k \|v-x\|^2 \]
\[ E_{\text{rep}} = \frac{k}{\|u-x\|^2} \]

2.3 SHORTCOMINGS OF THE ACTIVE CONTOUR SNAKE MODEL

Snake Model has some advantages over other segmentation methods. However, it also has some disadvantages. Observing from figure 3 and 6, we can examine that Active Contour Models are difficult to use when the edges of a feature are weak, noisy, and diffuse; they tend to get distracted and trapped by nearby edges that do not belong to the desired feature.

3. MODIFIED ACTIVE CONTOUR SNAKE MODEL USING ANISOTROPIC DIFFUSION

In our proposed algorithm, we have used anisotropic filters[10] proposed by perona and malik instead of Gaussian filters in the preprocessing phase of active contour snake model[4]. This technique aims at reducing image noise without removing significant parts of the image content. Noised images are denoised with this proposed approach. It resembles the process that creates a scale space where an image generates a parametrized family of successively more and more blurred images based on a diffusion process. It is as defined as:

\[ \frac{\partial I}{\partial t} = \text{div}(c(x,y,t)\nabla I) = \nabla c \cdot \nabla I + c(x,y,t)\Delta I \]

where \( \Delta \) denotes the Laplacian, \( \nabla \) denotes the gradient, \( \text{div}(...) \) is the divergence operator and \( c(x,y,t) \) controls the rate of diffusion which is a diffusion co-efficient[10].

Among the two diffusion co-efficients proposed by perona and malik we have used

\[ c(\|\nabla I\|) = \frac{1}{1+(\frac{\|\nabla I\|}{k})^2} \]

where the constant \( k \) controls the sensitivity to edges and is usually known as a function of the noise in the image and apply to the image \( f(x,y) \).

Conduction coefficient is set to be 1 in the interior of each region and 0 at the boundaries. This would encourage smoothing within a region in preference to smoothing across
the boundaries. The blurring would then take place separately in each region with no interaction between regions. The region boundaries would remain sharp.

Taking an estimate \( E(x,y,t) \), the conduction coefficient \( c(x,y,t) \) can be chosen to be a function \( c = g(\|E\|) \) of the magnitude \( E \). According to the above definition, \( g \) has to be a nonnegative monotonically decreasing function with \( g(0) = 1 \). Thus, the diffusion process will mainly take place in the interior of regions, and will not affect the region boundaries where the magnitude of \( E \) is large. The gradient of the brightness function used is \( E(x,y,t) = \nabla I(x,y,t) \)

**Edge Enhancement**

With low pass filtering and linear diffusion, eliminating the noise and performing scale space, results in the blurring of edges. This causes their detection and localization to be difficult and thus the segmentation. Edge enhancement and reconstruction of blurry images can be achieved by high-pass filtering or running the diffusion equation backwards in time. This causes numerically unstable computational methods, unless the problem is appropriately constrained and reformulated. The algorithm is as follows:

1. Reading the image \( f(x,y) \)
2. Adding additive Gaussian noise to the image as:
   \[
   f(x,y) + \eta(x,y) = g(x,y)
   \]
   where \( f(x,y) \) = original image
   \( \eta(x,y) \) = additive noise
   \( g(x,y) \) = Noised image
3. Taking the concept of anisotropic diffusion proposed by Perona and Malik [10] and applying it to the noised image \( g(x,y) \), the conduction coefficient is chosen to be an appropriate function of the image gradient, the anisotropic diffusion enhance edges while running forward in time and stability of diffusions is also appropriate. An edge is modeled as a step function convolved with a Gaussian. Without loss of generality, assume that the edge is aligned with the y axis.

The expression of the divergence operator simplifies to

\[
\nabla \cdot (c(x,y,t) \nabla I) = \frac{\partial}{\partial x} (c(x,y,t) \frac{\partial I}{\partial x}) + \frac{\partial}{\partial y} (c(x,y,t) \frac{\partial I}{\partial y})
\]

Choose \( c \) to be a function of gradient of \( I \):

\[
c(x,y,t) = g(I_x(x,y,t)).\]

Let \( \partial I_x \cdot I_x \) denote the flux \( c \cdot I_x \)

Then the 1-D version of the diffusion equation (1) becomes

\[
\frac{\partial}{\partial t} (I_x) = \frac{\partial}{\partial x} (\nabla \cdot (c(x,y,t) \nabla I)) = \nabla' . I_{xx} + \nabla' . I_{xxx}.
\]

The edge is oriented in such a way that \( I_x > 0 \). At the point of inflection \( I_{xxx} \ll 0 \) since the point of inflection corresponds to the point with maximum slope. Then the neighbourhood of the point of inflection \( \frac{\partial}{\partial t} (I_x) \) has sign opposite to \( \partial I_x(x) \). If \( \partial I_x(x) > 0 \), the slope of the edge will decrease in time and if \( \partial I_x(x) < 0 \), the slope will increase with time. This increase in slope cannot be caused by scaling of the edge. The image is denoised and the edges becomes sharper.

The image \( g(x,y) \) is thus denoised with anisotropic diffusion and this denoised image \( f(x,y) \) is then segmented using the active contour snake model as follows:

Gradient descent minimization [11] is used to minimize the snake energy as follows:

\[
\begin{align*}
x_{t+1} & = x_t + \gamma \frac{df}{dx}(x_t) \\
y_{t+1} & = y_t + \gamma \frac{df}{dy}(y_t)
\end{align*}
\]

Vector representation

\( \vec{x}_{t+1} = \vec{x}_t + \gamma \nabla f(\vec{x}_t) \)

Approximate the energy function [5] of the snake by using the discrete points on the snake.

\[
E_{\text{snake}}^n \approx \sum_{i=1}^{n} E_{\text{snake}}(\vec{v}_i)
\]

Sum of derivatives

\[
\nabla E_{\text{snake}}^n \approx \sum_{i=1}^{n} \nabla E_{\text{snake}}(\vec{v}_i)
\]

Iteratively adjust the point vector \( \vec{v}_i \) by using gradient descent minimization:

\[
\vec{v}_i \leftarrow \vec{v}_i - \nabla E_{\text{snake}}(\vec{v}_i)
\]

Applying the derivative to energy function

Derivative of internal energy of the image can be solved as

\[
\begin{align*}
\nabla E_{\text{internal}}(s) & = \nabla \left( \int \left( \alpha(s) \| \nu_s(s) \|^2 + \beta(s) \| \nu_{ss}(s) \|^2 \right)^{\frac{1}{2}} ds \right) \\
E_{\text{internal}}(s) & = \int \left( \alpha(s) \nabla \| \nu_s(s) \|^2 + \beta(s) \nabla \| \nu_{ss}(s) \|^2 \right)^{\frac{1}{2}} ds
\end{align*}
\]

These can be approximated using finite differences: the second derivative w.r.t \( s \) can be calculated using three adjacent points on the snake, and the fourth derivative can be calculated using five adjacent points. Final equations are:

\[
\begin{align*}
\vec{v}_i & \leftarrow \vec{v}_i - \gamma \left( \left[ \alpha \nabla \| \nu_s(s) \|^2 + \beta \nabla \| \nu_{ss}(s) \|^2 \right] \nabla E_{\text{internal}}(\vec{v}_i) \right) \\
\nabla E_{\text{exc}}(\vec{v}_i)
\end{align*}
\]
\[ \bar{x}_i = \bar{x}_i - \gamma \alpha \left( \frac{\partial^2}{\partial x^2} (\bar{v}_i) + \beta \frac{\partial^2}{\partial y^2} (\bar{v}_i) \right) \]
\[ \bar{y}_i = \bar{y}_i - \gamma \alpha \left( \frac{\partial^2}{\partial x^2} (\bar{v}_i) + \beta \frac{\partial^2}{\partial y^2} (\bar{v}_i) \right) \]

where
\[ E_{\text{external}} = E_{\text{image}} + E_{\text{con}} \]

The control parameters for the snake are:
- Alpha: Specifies the elasticity of the snake. This controls the tension in the contour by combining with the first derivative term.
- Beta: Specifies the rigidity in the contour by combining with the second derivative term.
- Gamma: specifies the step size.
- Kappa: acts as the scaling factor for energy term.

**Pseudocode:**

1. Before entering the iteration calculate \( E_{\text{ext}}(\bar{v}_i) \) and the derivatives of this w.r.t x and y separately.
2. At the start of iteration, calculate \( \frac{\partial^2}{\partial x^2} (\bar{v}_i) \) and \( \frac{\partial^2}{\partial y^2} (\bar{v}_i) \) using the three adjacent points and \( \frac{\partial^2}{\partial x^2} (\bar{v}_i), \frac{\partial^2}{\partial y^2} (\bar{v}_i) \) using five adjacent points.
3. Then, calculate change in x and y for each point in \( \bar{v}_i \) use the precalculated \( E_{\text{ext}}(\bar{v}_i) \)
   In general, snake is placed near the object contour. It will dynamically move towards object contour by minimizing its energy iteratively.

Thus, we used the proposed algorithm to smoothen the noisy images and get efficient segmentation results with active contour snake model. Experimental Results are given below.

### 4. EXPERIMENTAL RESULTS

#### 4.1 RESULTS OF MODIFIED ALGORITHM

Figure 7 and 10 are the original cortex and cardia images respectively. Figure 8 is the original cortex image with added 0.25 gaussian noise and figure 11 is the original cardia image with 0.5 added noise. Figure 9 is the segmented cortex image using modified approach with parameters given below the figure. Figure 12 is the segmented cardia image using modified approach with parameters given below the figure.

**Fig -7:** Original Cortex Image
**Fig -8:** Cortex image with 0.25 added noise

**Fig -9:** Segmented noisy cortex image with proposed method with parameters alpha 0.1 & kappa and gamma 0.5

**Fig -10:** Original Cardia Image
**Fig -11:** Cardia image with 0.5 gaussian noise

**Fig -12:** Segmented noisy cardia image with proposed method with parameters alpha 0.5 & kappa and gamma 1.
4.2 COMPARING RESULTS OF PROPOSED METHOD WITH PREVIOUS ACTIVE CONTOUR SNAKE METHOD

Figure 13 shows the segmentation of noisy cortex image with active contour snake model. Figure 14 shows the segmentation of noisy cortex image with modified algorithm. Figure 15 shows the segmentation of noisy cardia image with active contour snake model. Figure 16 shows the segmentation of noisy cardia image with modified algorithm with certain parameters given below the figure.

**Fig -13:** Segmented noisy image with active contour snake model. Parameters alpha &beta=0.1 kappa &gamma=0.5

**Fig -14:** Segmented noisy cortex image with proposed method with parameters alpha and beta 0.1 & kappa and gamma 0.5

**Fig -15:** Segmented noisy cardia image with active contour snake model. Parameters alpha and beta=0.5 kappa and gamma=1.0

**Fig -16:** Segmented noisy cardia image with proposed method with parameters alpha and beta 0.5 & kappa and gamma 1.

Thus, from the above results it is clear that noisy images segmented using proposed approach in which concept of anisotropic filtering is used gives more effective results.

5. CONCLUSIONS

Active Contour Model is a very efficient technique of image segmentation. But then also there are certain drawbacks of this model as any other technique. Many modifications have been done in this model. Major drawback taken into is that active contour models are difficult to use when the edges of a feature are weak, noisy, and diffuse; they tend to get distracted and trapped by nearby edges that do not belong to the desired feature. In our work we have tried to segment the images with more smoothing efficiency to get good results. In this domain, we have proposed an algorithm in which anisotropic filtering is used in the preprocessing phase of active contour snake model instead of Gaussian filters to remove noise from the medical images as anisotropic filters worked more effectively on noised images in which diffusion coefficient is chosen to vary spatially so that the region boundaries remain sharp and obtain a high quality edge detector. Thus, the proposed algorithm when used to smoothen the noisy images and gives efficient segmentation results with active contour snake model.

REFERENCES

[3] Xiaolei Huang et al., “Medical Image Segmentation”.

