

Mathematical Modeling of buildings towards the design strategy of the Green buildings with MATLAB

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Abstract - Maintaining of the indoor climate conditions so that they keep compatible with the occupants comfort is a key issue for control of heating, ventilation along with air conditioning systems (HVAC systems). Computer modelling offer a virtual environment similar to real climatic conditions indoors and outdoors. It aims basically to devise solutions for control of indoor climatic conditions. This process requires understanding of these environments from physical and mathematical perspective, so that physical processes of these environments can be represented using relationships and equations which can reflect the influence of different environmental parameters. Here different physical models associated in the construction of building are modelled and a new control strategy is developed to limit the effect of temperature rise using MATLAB/SIMULINK and the output results are shown which results in better performance.

Key Words: HVAC system, mathematical modeling, temperature, control.

1. INTRODUCTION

It is must to have a better knowledge about the heat, air and moisture balance of the whole building and how the indoor environment is affected due to it. And it in turn impacts the thermal comfort of the occupants which invariably reduces pollution and energy consumption. There is no such tool or computer application in existence which covers these issues. Hence, integrated mathematical models which cover every aspects of building design are required. Modeling is defined as obtaining the mathematical equations which shows the dynamic response of the physical processes. It is basically derived from the known laws. The physical system, once its modeling is completed [1]; it can be simulated by a particular software tool. It just takes few seconds to simulate the process, as compared to the real system as it takes hours. It helps in saving a great deal of money and we can correct the errors before the real system is implemented. For predicting the energy being consumed and what are the indoor climate parameters, some popular methods can be used[2][3]. The results which are obtained from these methods are used to design the buildings, which meets every aspect of the energy consumption and indoor

climate. The principles of energy simulation are to determine the energy flow across the building. Energy flow is found out in terms of heat loss through the building envelopes with also the ventilation system and infiltration. It is quite evident that the actual indoor temperature is the representation of complex interaction between losses and gains. The variables used vary continuously as outdoor temperature, solar heat gain and internal heat gain, vary with time [4].

All the parameters which directly or indirectly affect the energy balance must be included in the model. This in term says that every wall, windows, ventilation system and all the components sub-model must be designed. With so many sub-models being designed come so many factors, which further increase the complexity of the whole model. And with many factors, a detailed result will be obtained. But we may have problems in running it. Accuracy and complexity must be balanced in such way that, we get higher accuracy with less complexity.

The mathematical model is again run in various simulation processes. After the initial implementation of the model, validation of the model is important as to check whether the results obtained are near to real parameters. There is a big advantage of simulation, as we can vary the parameters and can further improve the design of the building [5]. There is only one software that has advanced capabilities for simulation of indoor thermal process, and it is MATLAB.SIMULINK is a software package which is basically used for modeling, simulating and analyzing dynamical systems. Using ordinary differential equations, the indoor environment modeling can be implemented in SIMULINK.

Here in this work a closed room is considered, where an external wall and windows separate it from the outdoors, internal partitions divide it from the adjoining rooms, and from above by a ceiling and a floor slab at below and it is shown below in figure 1.

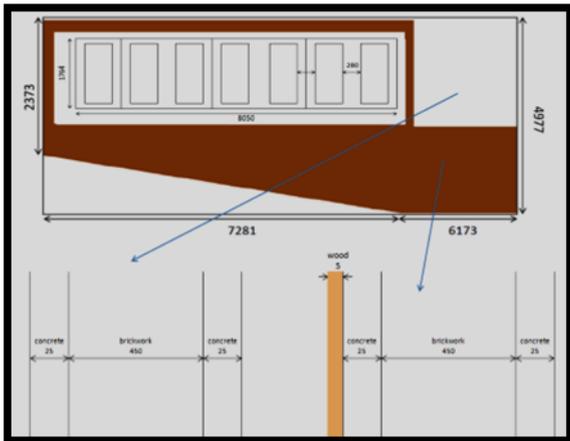


Figure 1: External walls

2. MATHEMATICAL MODEL FOR EXTERNAL WALLS

The method used to model the interactions between the instantaneous loads and system models is the method of heat equilibrium, where, it requires solving a set of equations for heat transfer and temperature of the indoor air in order to determine the loads indoors[6]. Where, the indoor temperature can be calculated based on the indoor environmental conditions, such as internal loads, the interactions between the surfaces, infiltration, ventilation, and air handling system. And then, the concept of energy balance is used for determining the temperature inside at which the indoor space become balanced with the heating or cooling provided by the system, so that can maintain indoor climate conditions comfortable for occupants. Here four different layers are considered to design the model of external walls and temperatures of all four layers is expressed by T1, T2, T3 and T4 which is as shown in figure 2.

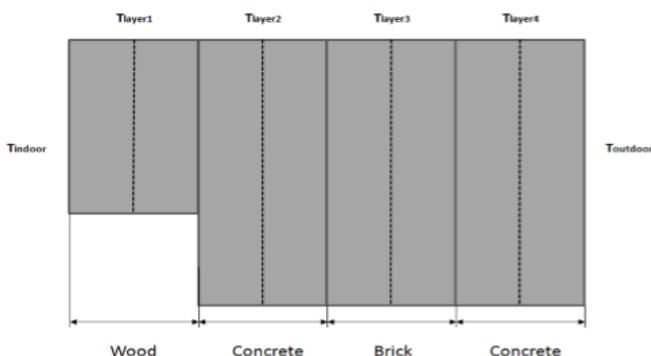


Figure 2 Different layers of external walls

Solar radiation plays an important role for the heat transfer through the external walls. The conduction heat transfer through an external wall is given by equation (1) and it is the effect of the convective heat that the surface at each side of

the element is exchanging with the surrounding air. The heat transfer through the building envelopes can be done by taking the mathematical representations of the individual layers and there by investigating the temperatures

Temperature of the wood layer can be calculated from the following equation,

$$T_1 = \frac{\int \left(\frac{1}{\frac{1}{h_{inside}} + \frac{l_1}{k_1}} A_1 (T_{indoor} - T_1) - \frac{2 \cdot k_1 \cdot k_2}{l_1 \cdot k_1 + l_2 \cdot k_2} A_2 (T_1 - T_2) \right)}{\rho_1 C_{p1} V_1} \dots (1)$$

Where,

T_1 (°C) is the Temperature of wood layer (1st layer inside)

h_{inside} (W/m².K) is the Heat transfer co-efficient inside

l_1 (m) is the thickness of wood layer (1st layer inside)

k_1 (W/m.K) is the thermal conductivity of wood layer (1st layer inside)

A_1 (m²) is the area of wood layer (1st layer inside)

T_{indoor} (°C) is the indoor temperature

k_2 (W/m.K) is thermal conductivity of 1st concrete layer (2nd layer inside)

l_2 (m) is the thickness of the 1st concrete layer (2nd layer inside)

A_2 (m²) is the area of the 1st concrete layer (2nd layer inside)

T_2 (°C) is the temperature of the 1st concrete layer (2nd layer inside)

ρ_1 (kg/m³) is the density of wood layer (1st layer inside)

C_{p1} (J/kg.K) is the specific heat capacity of wood layer (1st layer inside)

V_1 (m³) is the volume of the wood layer (1st layer inside)

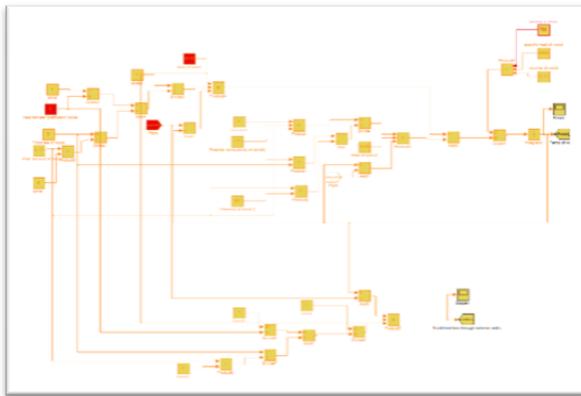


Figure 3: Model for the wood layer

Temperature of the first concrete layer is obtained from the following,

$$T_2 = \frac{\int (\frac{2 \cdot k_1 \cdot k_2}{l_1 \cdot k_2 + l_2 \cdot k_1} A_2 (T_1 - T_2) - \frac{2 \cdot k_3 \cdot k_2}{l_3 \cdot k_2 + l_2 \cdot k_3} A_3 (T_2 - T_3))}{\rho_2 C_{p2} V_2} \dots (2)$$

Where,

$k_3 (W/m.K)$ is the thermal conductivity of the brick layer (3rd layer inside)

$A_3 (m^2)$ is the area of the brick layer (3rd layer inside)

$T_3 (^\circ C)$ is the temperature of the brick layer (3rd layer inside)

$\rho_2 (kg/m^3)$ is the density of first concrete layer (2nd layer inside)

$C_{p2} (J/kg.K)$ is the specific heat capacity of first concrete layer (2nd layer inside)

$V_2 (m^3)$ is the volume of the first concrete layer (2nd layer inside)

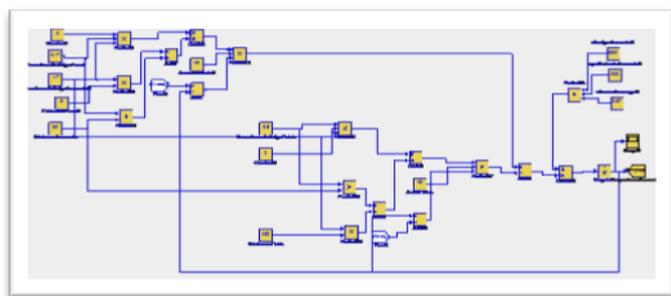


Figure 4: Model for the first concrete layer

The following equation gives us the temperature of the brick layer,

$$T_3 = \frac{\int (\frac{2 \cdot k_2 \cdot k_3}{l_2 \cdot k_3 + l_3 \cdot k_2} A_3 (T_2 - T_3) - \frac{2 \cdot k_3 \cdot k_4}{l_4 \cdot k_3 + l_3 \cdot k_4} A_4 (T_3 - T_4))}{\rho_3 C_{p3} V_3} \dots (3)$$

Where,

$k_4 (W/m.K)$ is the thermal conductivity of the second concrete layer (4th layer)

$A_4 (m^2)$ is the area of second concrete layer (4th layer)

$T_4 (^\circ C)$ is the temperature of the second concrete layer (4th layer)

$\rho_3 (kg/m^3)$ is the density of brick layer (3rd layer inside)

$C_{p3} (J/kg.K)$ is the specific heat capacity of brick layer (3rd layer inside)

$V_3 (m^3)$ is the volume of brick layer (3rd layer inside)

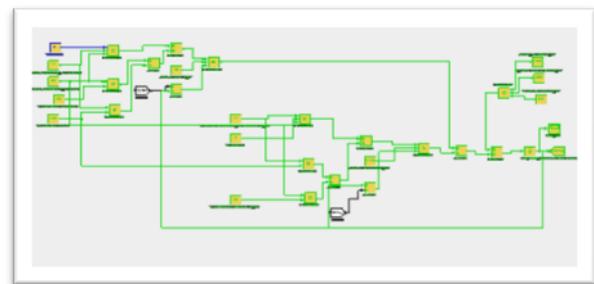


Figure 5: Model for the brick layer

By the equation given below, temperature of the second concrete layer is calculated,

$$T_4 = \frac{\int \left(\frac{2 \cdot k_4 \cdot k_2}{l_4 \cdot k_2 + l_2 \cdot k_4} A_4 (T_2 - T_4) - \frac{1}{\frac{1}{h_{outside}} + \frac{l_4}{2 \cdot k_4}} A_{outside} (T_4 - T_{so,walls}) \right)}{\rho_4 C_{p4} V_4} \dots (4)$$

Where,

$A_{outside} (m^2)$ is the area of outside surface

$h_{outside} (W/m^2.K)$ is the heat transfer co-efficient outside

$T_{so,walls} (^\circ C)$ is the solar air temp for external walls

$\rho_4 (kg/m^3)$ is the density of second concrete layer (4th layer)

$C_{p4} (J/kg.K)$ is the specific heat capacity of second concrete layer (4th layer)

$V_4 (m^3)$ is the volume of the second concrete layer (4th layer)

Area of frame – 4.35 m^2

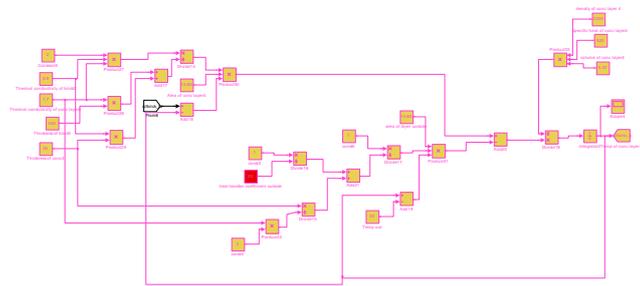


Figure 6: Model for the second concrete layer

After individually designing each layer, the all four layers are linked together and the designed parameters are shown below

Volume of indoor space – 910 m^3

Every wall of the room is covered with wood.

Area of wood covered part – 28.63 m^2

Area of concrete part – 14.65 m^2

External wall from inside to outside direction	Thickness [mm]	Thermal conductivity [W/m.K]	Specific heat capacity [J/kg.K]	Density [kg/m^3]
Wood Layer	5	0.17	2000	700
Concrete layer	25	1.7	920	2300
Brick layer	450	0.8	800	1700
Concrete layer	25	1.7	920	2300

Table 1. Dimensions and properties of materials of all layers in external wall

3. MATHEMATICAL MODEL FOR WINDOWS FRAME

Every window in each wall has a steel frame[7].

Area of glass – 9.64 m^2

Window frame	Thermal conductivity in [W/m.K]	Specific heat capacity in [J/kg.K]	Density in [kg/m^3]
Steel	16	490	7820

Table 2. Properties of window frame

Therefore temperature of the windows frame is calculated from the given equation,

$$T_{frame} = \int \frac{\left(\frac{\frac{1}{h_{inside}} + \frac{l_{frame}}{2k_{frame}}}{\frac{1}{h_{outside}} + \frac{l_{frame}}{2k_{frame}}} \right) \frac{A_{frame}(T_{indoor} - T_{frame}) - A_{frame}(T_{frame} - T_{so,frame})}{P_{frame} \cdot C_{P,frame} \cdot V_{frame}}}{..(5)}$$

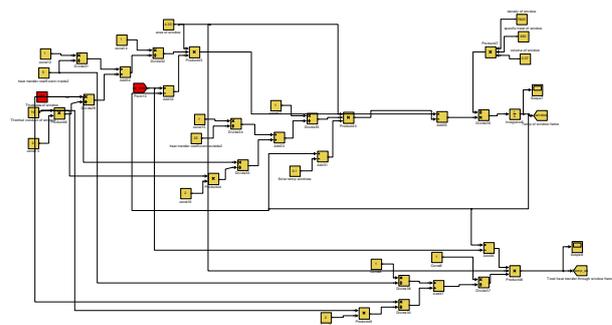


Figure 7: Model for the window frame

4. MATHEMATICAL MODEL FOR WINDOWS

A part of solar radiation is incident on the glass pane. It increases the temperature of the glass pane, and heat flows in both inside and outside direction.

Heat transferred through the window is expressed as follows,

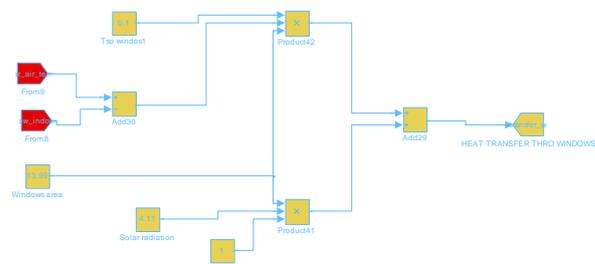


Figure 8: Model for the windows

5. MATHEMATICAL MODEL FOR ROOF

The following equation gives the temperature of the roof,

$$T_{roof} = \int \left(\frac{\frac{1}{h_{inside}} + \frac{l_{roof}}{2 \cdot K_{roof}} A_{roof} (T_{indoor} - T_{roof}) - \frac{1}{h_{outside}} + \frac{l_{roof}}{2 \cdot K_{roof}} A_{roof} (T_{roof} - T_{so,roof})}{P_{roof} \cdot C_{P_{roof}} \cdot V_{roof}} \right)$$

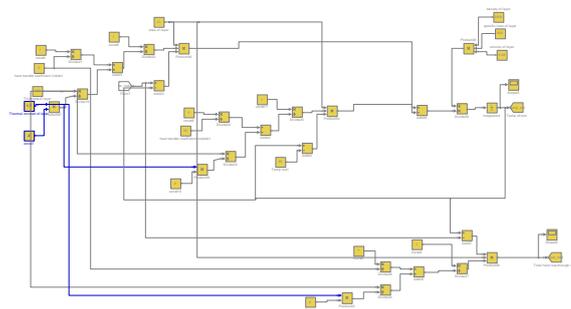


Figure9: Model for the roof

6. MATHEMATICAL MODEL FOR VENTILATION

Ventilation heat loss/gain is calculated from the following,

$$Q_{ventilation} = \frac{N \cdot P_{air} \cdot V_{indoor} \cdot C_{P_{air}} \cdot (T_{outdoor} - T_{indoor})}{3600}$$

Where:

$Q_{ventilation}$ [W]: heat loss/gain due to ventilation.

N [1/h]: air change rate = (air change rate due to infiltration + air change rate due to ventilation).

V_{indoor} [m³]: indoor space volume.

$C_{P_{air}}$ [J/kg.K]: specific heat capacity of air.

$T_{outdoor}$ [K]: outdoor temperature.

T_{indoor} [K]: indoor temperature.

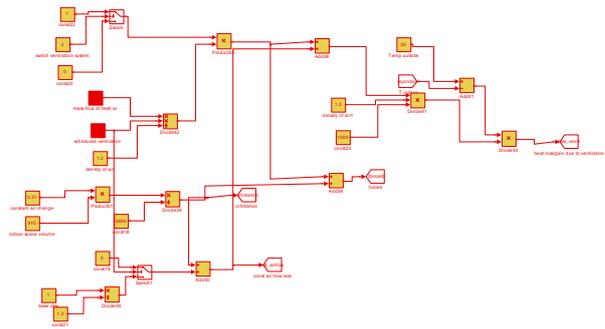


Figure 10: Model for the ventilation

7. MATHEMATICAL MODEL FOR INTERNAL GAINS

Internal gains is basically depends upon the activity inside the room, appliances used and power consumed by each application[8].

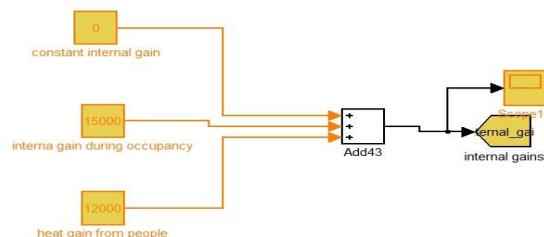


Figure 11: Model for the internal gains

8. CONTROL STRATEGY

The control is the method of starting, stopping and regulation of HVAC scheme as well as the implement that can make tools work efficiently, and now and then gives the ability to change their proceedings over time, while conditions and occupancies may modify. Controls are tools that have the ability to observe the inputs and regulate the output of systems. Scheming of HVAC system include measure the variables and obtain data, as well as process the data with other in sequence, then cause a control action[9].

Here, the closed loop organize includes a sensor which sample a measurable variable. Base on the set point value and measured value, and according to the control algorithm, the controller calculate the control signal that feed the actuator. ON-OFF controller has been worn to control indoor temperature, so that it works as controller for heating process in the direct track of control signal, as shown in figure below

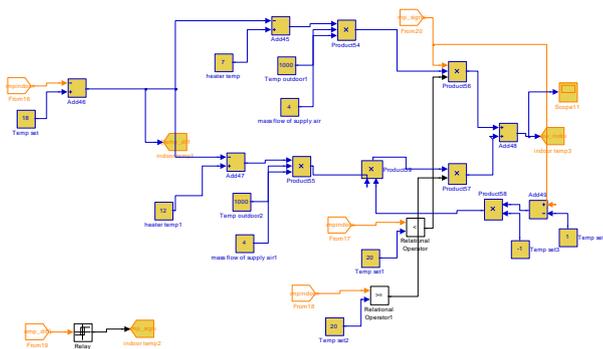


Figure12 control design of Heating/Cooling

9. SIMULATION RESULTS

The below shown figures from 12-18 shows the temperature rise at each and every layer of the wall, roof, windows and ventilation. Here a HVAC system is developed with the proper control such that the room temperature is every time adjusted and kept automatically to 18 degrees and the result is shown in figure 13 and figure 14.

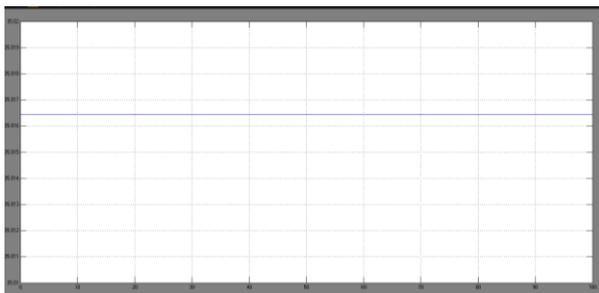


Figure 13 Temperature rise due to solar radiation

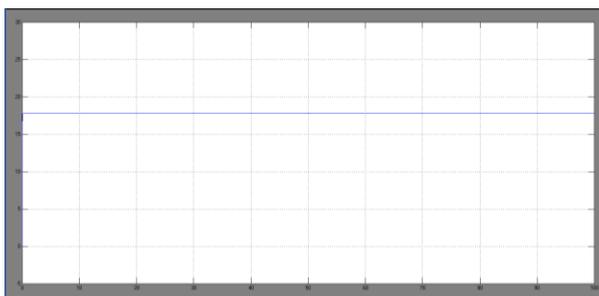


Figure 14 Total Indoor temperature after controller is placed

3. CONCLUSION

The basic physical process within classrooms with dissimilar sizes of occupancy has been modeled using MATLAB/SIMULINK software. Modeling the indoor temperature is base on the principle of conservation of energy. The main flows of force to and from the building from side to side the envelope has been modeled, captivating

into account the internal thermal loads. Then, control strategy to control temperature using an ON-OFF controller has been planned, so that it works for heating/cooling process, as it is requisite, at the same time. In direct signal, it reins heating process. After reversing of control signal, it controls cooling process, taking into account that the principle to begin to control temperature inside the room are conditions after using supply air, but in absence of heating or cooling.

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