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Performance of Tomlinson-Harashima Precoding and Dirty Paper coding for Broadcast Channels in MU-MIMO

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Abstract - *In this paper, two methods Dirty paper coding (DPC) and Tomlinson-Harashima Precoding are used to cancel interference. Dirty paper coding (DPC) is a method of precoding the data such that the effect of the interference can be canceled subject to some interference that is known to the transmitter side. Tomlinson-Harashima Precoding (THP) in Decision Feedback Equalization (DFE) is to cancel the post-cursor ISI in the transmitter, where the past transmits symbols are known without possibility of errors. This paper discusses the performance of dirty paper coding (DPC) and Tomlinson–Harashima Precoding in term of bit error rate (BER).*

Key Words- Decision Feedback Equalization (DFE), Dirty Paper Coding (DPC), Tomlinson–Harashima Precoding (THP), Bit error rate (BER) and InterSymbol Interference (ISI).

1. INTRODUCTION

In multi-user MIMO downlink typically refers to situations where a base station simultaneously communicates with several co-channel users. The multi-user MIMO downlink has concerned huge research interests because of its possible of increasing the system capacity [1].

In the multi-user MIMO major limitation are interference and channel fading. There can be minimized by precoding the signals before transmission which requires the knowledge of channel state information at the base station [2].

In multi-user MIMO system one base station communicate with multiple users. In multi-user MIMO systems are exposed to strong co- channel interference. To solve the problem of interference in MU-MIMO systems, several approaches have been proposed for interference [3].

This paper, Dirty Paper Coding is a technique of precoding the data effect the interference can be canceled to some interference that is known to the transmitter. Tomlinson-Harashima Precoding (THP) in Decision Feedback Equalization (DFE) is to cancel the post-cursor ISI in the transmitter, where the past transmits symbols are known without possibility of errors.

2. System Model

2.1 Dirty Paper Coding

Dirty paper coding was introduced by M. Costa in [4]. Dirty Paper Coding is a technique of precoding the data effect the interference can be canceled interference that is known to the transmitter side.

$$y = s + i + w \qquad (1)$$

Where i is the interference, w is Gaussian noise, the received signal is y and the signal s is used to transmit a signal [5].

The capacity of the MIMO multiple-user channel has been analyzed using coding techniques referred to as “writing on dirty paper.”

The Costa's result shows that the capacity of this channel is the same as if the interference were not present. If the signal has power constraint $|s|^2 \le P$, then the capacity of this system is [6],

$$C = \frac{1}{2}\log\left(1 + \frac{P}{N}\right) \qquad (2)$$

In this technique of precoding the data effect the interference can be canceled to some interference that is known to the transmitter. The interferences due to the first up to $(k\text{-}1)^{th}$ user signals are canceled in the course of precoding the k^{th} user signal.

We just consider the case of N_B=3, K=3, and $N_{M,\ u}$ = 1, u=1,2,3. The u^{th} user signal is given by $\tilde{x}_u$, then the received signal is given as,

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} H_1^{DL} \\ H_2^{DL} \\ H_3^{DL} \end{bmatrix} \begin{bmatrix} \tilde{X}_1 \\ \tilde{X}_2 \\ \tilde{X}_3 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad (3)$$

Where H_u^{DL} is channel matrix between base station and the u^{th} user. The channel matrix H_u^{DL} can be LQ-decomposed as

$$H^{DL} = \underbrace{\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}}_{Q} \quad (4)$$

Where $\{q\}_{i=1}^{3}$ are orthonormal row vectors. Let $\mathrm{x} = [x_1 x_2 x_3]^T$ indicate a precoded signal for $\mathrm{x} = [\tilde{x}_1\ \tilde{x}_2\ \tilde{x}_3]^T$. By transmitting $Q^H x$, the effect of Q in equation (4) is eliminated through the channel. The received signal is given as,

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} H_1^{DL} \\ H_2^{DL} \\ H_3^{DL} \end{bmatrix} Q^H X + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$= \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad (5)$$

From equation (5), the received signal of the first user is given as,

$$y_1 = l_{11} x_1 + b_1 \quad (6)$$

From the first user side therefore, the following condition needs to be met for the interference free data transmission.

$$\mathrm{x}_1 = \tilde{\mathrm{x}}_1 \quad (7)$$

From equation (5), the received signal of the second user is given as

$$y_2 = l_{21} x_1 + l_{22} x_2 + b_2$$

$$= l_{21} \tilde{x}_1 + l_{22} x_2 + b_2 \quad (8)$$

From equation (8) it can be seen that the following precoding cancels the interference component, $l_{21} x_1$ on the transmitter side:

$$x_2 = \tilde{x}_1 - \frac{l_{21}}{l_{22}} x_1 \quad (9)$$

The received signal of the third user is given as

$$y_3 = l_{31} x_1 + l_{32} x_2 + l_{33} x_3 + b_3 \quad (10)$$

From the third user the precoded signal x_1 and x_2 are interference component in equation (10) which can be canceled by the following precoding on the transmitter side:

$$x_3 = \tilde{x}_3 - \frac{l_{31}}{l_{33}} - \frac{l_{32}}{l_{33}} x_2 \quad (11)$$

The received signal is given as

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$= \begin{bmatrix} l_{11} & 0 & 0 \\ 0 & l_{22} & 0 \\ 0 & 0 & l_{33} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ x_2 \\ \tilde{x}_3 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad (12)$$

From above Equation (12) it is the interference free detection can be made for each user.

2.2 TOMLINSON-HARASHIMA PRECODING

Dirty Paper Coding on the transmitter side is very similar to decision feedback equalization (DFE) on the receiver side. Tomlinson-Harashima (TH) precoding is the combination of Dirty Paper Coding and modulo operation. Decision-feedback equalization is a non-linear equalization approach at the receiver side [7],[8],[9],[10].

The original idea of TH precoding in DFE is to Cancel the post-cursor ISI in the transmitter, where the past transmit symbols are known without possibility of errors.

The data symbol x is drawn from the M-ary PAM constellation, an expanded symbol c can be defined as

$$c = x + 2A.m \quad (13)$$

Where A is an even integer given by$A = \sqrt{M}$. By adding $2A \cdot m$ to the data symbol x, where m is an integer.

To reduce the peak or average power, m in Equation (13) must be chosen to minimize the magnitude of the expanded symbol c in the transmitter. The original data symbol x can be recovered from the expanded symbol c by the symmetric modulo operation defined as,

$$x = mod_A(c) \triangleq c - 2A\lfloor (c + A)/2A \rfloor \quad (14)$$

TH precoding for the multi-user MIMO system, we discuss the symmetric modulo operation for M-ary QAM modulated symbols, which is an extension of Equation (14) to the two dimensional case. In M-ary QAM with a square constellation, the real and imaginary parts of a symbol are bounded by *[-A, A]* with $A = \sqrt{M}$.

The symmetric modulo operation is defined as

$$mod_A(x) = x - 2A\lfloor (x + A + jA)/2A \rfloor \quad (15)$$

The above modulo operation using find integer values *m* and *n*.

$$-A - jA \leq mod_A(x) = x + 2A.m + j2A. < A + jA \quad (16)$$

The difference of complex numbers x_1 and x_2 in equation (16) is defined as

$$x_1 < x_2 \Leftrightarrow Re\{x_1\} < Re\{x_2\} < and\ Im\{x_1\} < Im\{x_2\} \quad (17)$$

Then the modulo operation in equation (17) can be expressed as,

$$mod_A(x) = x + 2A.m + j2A.n \quad (18)$$

Let us take an example of TH precoding for *K*=3. Let $\{x_u^{TH}\}_{u=1}^{3}$ indicate the TH precoded signal for the u^{th} user. Referring to equation (7), (9), and (11) by the above modulo operation, TH-precoded data symbol are represented as

$$x_1^{TH} = mod_A(\tilde{x}_1) = \tilde{x}_1 \quad (19)$$

$$x_2^{TH} = mod_A\left(\tilde{x}_2 - \frac{l_{21}}{l_{22}} x_1^{TH}\right) \quad (20)$$

$$x_3^{TH} = mod_A\left(\tilde{x}_3 - \frac{l_{31}}{l_{33}} x_1^{TH} - \frac{l_{32}}{l_{33}} x_2^{TH}\right) \quad (21)$$

The analysis in Equation (18) gives the following terms for TH-precoded signals

$$x_1^{TH} = \tilde{x}_1 \quad (22)$$

$$x_2^{TH} = \tilde{x}_2 - \frac{l_{21}}{l_{22}} \tilde{x}_1 + 2A.m_2 + j2A.n_2 \quad (23)$$

$$x_3^{TH} = \tilde{x}_3 - \frac{l_{31}}{l_{33}} x_1^{TH} - \frac{l_{32}}{l_{33}} x_2^{TH} + 2A.m_3 + j2A.n_3 \quad (24)$$

For the transmitted signal $Q^H x^{TH} = Q^H [x_1^{TH} x_2^{TH} x_3^{TH}]^T$ the received signal is given as

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} H_1^{DL} \\ H_2^{DL} \\ H_3^{DL} \end{bmatrix} Q^H x^{TH} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$= \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} x_1^{TH} \\ x_2^{TH} \\ x_3^{TH} \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad (25)$$

Since $x_1^{TH} = \tilde{x}_1$, the signal detection for the first user. The received signal of the second user is given as

$$y_2 = l_{21} x_1^{TH} + l_{22} x_2^{TH} + b_2$$

$$= l_{21}\tilde{x}_1 + l_{22} x_2^{TH} + b_2 \quad (26)$$

Using Equation (23) Equation (24) can be expressed as

$$y_2 = l_{21}\tilde{x}_1 + l_{22}\left(\tilde{x}_2 - \frac{l_{21}}{l_{22}}\tilde{x}_1 + 2A.m_2 + j2A.n_2\right) + b_2$$

$$= l_{22}(\tilde{x}_2 + 2A.m_2 + j2A.n_2) + b_2 \quad (27)$$

Defining $\tilde{y}_2$ as a scaled version of y_2, that is,

$$\tilde{y}_2 = \frac{y_2}{l_{22}} = \tilde{x}_2 + 2A.m_2 + j2A.n_2 + \frac{b_2}{l_{22}} \quad (28)$$

The second-user signal $\tilde{x}_2$ can be detected with the modular operation

$$\tilde{x}_2 = mod_A(\tilde{y}_2) \quad (29)$$

The received signal of the third user is given a

$$y_3 = l_{31} x_1^{TH} + l_{32} x_2^{TH} + l_{33} x_3^{TH} + b_3$$

$$= l_{31} x_1^{TH} + l_{32} x_2^{TH} + l_{33}(\tilde{x}_3 - \frac{l_{31}}{l_{33}} x_1^{TH} + \frac{l_{32}}{l_{33}} x_2^{TH} + 2A.m_3 + j2A.n_3) + b_3$$

$$= l_{33}(\tilde{x}_3 + 2A.m_3 + j2A.n_3) + b_3 \quad (30)$$

The third-user signal $\tilde{x}_3$ can be detected as,

$$\tilde{x}_3 = mod_A(\tilde{y}_3) \quad (31)$$

Where

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$$\tilde{y}_3 = \frac{y_3}{l_{33}} = x_3 + 2A.m_3 + j2A.n_3 + \frac{b_3}{l_{33}}$$

3. SIMULATION RESULT

Fig.1 shows that the BER performance of dirty paper coding (DPC), method for N_B=4, in which four users with the highest channel norm values are selected out of K = 10.

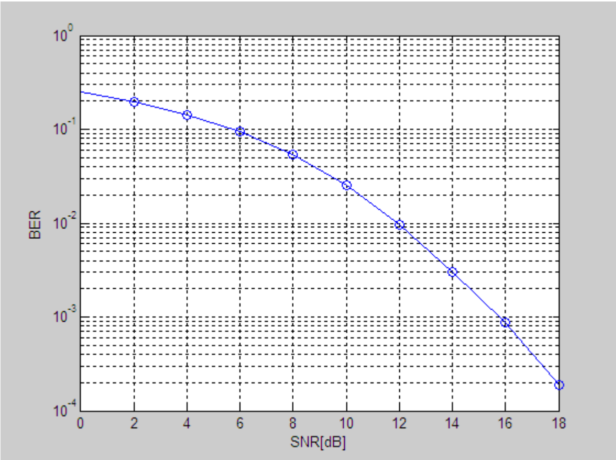

Fig-1: BER performance of Dirty paper Coding method

Fig.2 shows that the BER performance of Tomlinson-Harashima Precoding method for N_B=4, in which four users with the highest channel norm values are selected out of K = 10.

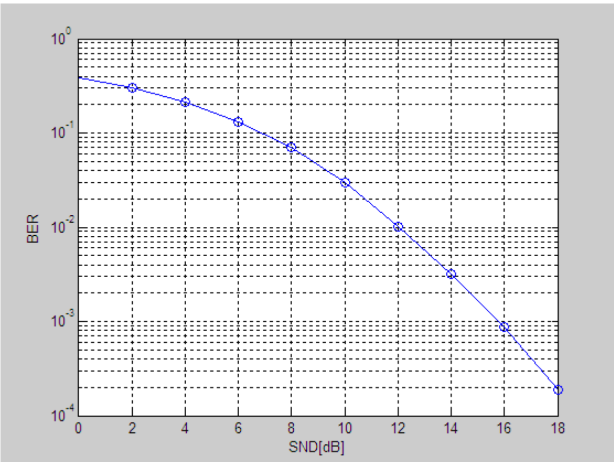

Fig-2: BER performance of Tomlinson-Harashima Precoding method

4. CONCLUSION

The above mentioned broadcast channel techniques are used to remove the interference in the communication system. Dirty Paper Coding is a technique of precoding the data effect the interference can be canceled to some interference that is known to the transmitter. Tomlinson-Harashima Precoding (THP) in Decision Feedback Equalization (DFE) is to cancel the post-cursor ISI in the transmitter, where the past transmits symbols are known without possibility of errors. The presented results show that the Dirty Paper Coding and Tomlinson-Harashima Precoding BER performance.

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