Design of Alpha Stirling Engine in Conjunction with Solar Concentrator

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Abstract – This paper provides an design approach and thermodynamic analysis of an solar stirling model. Out of the three Configurations viz. Alpha, Beta and Gamma an Alpha configuration was Chosen because of its ease of fabrication. Parabolic solar concentrator is used to provide Heat to the Engine. The main objective was to determine how much work can be extracted from a 49cc Stirling Engine. Further the Effect of regenerator effectiveness on Stirling cycle efficiency, Variation of expansion volume, Compression volume and total volume is also calculated with respect to crank angle variation.

Key Words: Solar Stirling model, Alpha Configuration, Stirling Engine, Parabolic Solar Concentrator, regenerator effectiveness, crank angle, expansion volume, compression volume.

1.1 Configuration of Stirling Engine

There are mainly three configurations of stirling engine namely, Alpha, Beta and Gamma.

- In the alpha-configuration a displacer is not used. Two pistons, called the hot and cold pistons, are used on either side of the heater, regenerator, and cooler. These pistons move uniformly in the same direction to provide constant-volume heating or cooling processes of the working fluid. When all the working fluid has been transferred into one cylinder, one piston will be fixed and the other piston moves to expand or compress the working fluid. The expansion work is done by the hot piston while the compression work is done by the cold piston.

- In the beta-configuration, a displacer and a power piston are incorporated in the same cylinder. The displacer moves working fluid between the hot space and the cold space of the cylinder through the heater, regenerator, and cooler. The power piston, located at the cold space of the cylinder, compresses the working fluid when the working fluid is in the cold space and expands the working fluid when the working fluid is moved into the hot space.

- The gamma-configuration uses separated cylinders for the displacer and the power pistons, with the power cylinder connected to the displacer cylinder. The displacer moves working fluid between the hot space and the cold space of the displacer cylinder through the heater, regenerator, and cooler. In this configuration, the power piston both compresses and expands the working fluid. The gamma-configuration with double-acting piston arrangement has theoretically the highest possible mechanical efficiency. This configuration also shows good self-pressurization.

1.INTRODUCTION

Stirling engine is a mechanical device working theoretically on the Stirling cycle, in which compressible fluids, such as air, hydrogen, helium, nitrogen or even vapors, are used as working fluids. The Stirling engine offers possibility for having high efficiency engine with less exhaust emissions in comparison with the internal combustion engine. The earlier Stirling engines were huge and inefficient. However, over a period of time, a number of new Stirling engine models have been developed to improve the deficiencies.

The modern Stirling engine is more efficient than the early engines and can use any high temperature heat source. The Stirling engine is an external combustion engine. Therefore, most sources of heat can power it, including combustion of any combustible material, field waste, rice husk or the like, biomass methane and solar energy. In principle, the Stirling engine is simple in design and construction, and can be operated easily.

Direct solar-powered Stirling engines may be of great interest to countries where solar energy is available in unlimited quantity. To use direct solar energy, a solar concentrator and absorber must be integrated with the engine system.
1.2 Principle of Operation

The Stirling hot air engine is a simple type of engine that uses a compressible fluid as the working fluid. Because the working fluid is in a closed system, there are no problems with contamination and working fluid costs. Heat transfer to the working fluid is very important. High mass flow is needed for good heat transfer. The working fluid should be of low viscosity to reduce pumping losses. Using higher pressure or lower viscosity, or combinations thereof, could reduce the high mass flow required. The Stirling engine could theoretically be a very efficient engine in upgrading from heat to mechanical work with the Carnot efficiency. The thermal limit of the operation of the Stirling engine depends on the material used for construction. Engine efficiency ranges from about 30 to 40% resulting from a typical temperature range of 923–1073 K, and a normal operating speed range from 2000 to 4000 rpm.

1.3. Stirling Cycle

The idealized Stirling cycle consists of four thermodynamic processes acting on the network of the fluid;

i. 3-4: Isothermal expansion: the expansion space and associated heat exchanger are maintained at a constant high temperature so that the gas undergoes isothermal expansion absorbing heat from the heat source.

ii. 4-1: Constant volume heat removal: the gas is passed through a regenerator where it cools transferring heat to the regenerator for use in the next cycle.

iii. 1-2: Isothermal compression: the compression space and associated heat exchanger are maintained at a constant low temperature so that the gas undergoes isothermal compression, rejecting heat to the cold sink.

iv. 2-3: Constant volume heat addition: the gas passes back through the regenerator where it recovers much of the heat transferred in (2) above, heating up on its way to the expansion space.

2. Design Methodology.

- We decided to manufacture a Stirling engine having 49cc displacement capacity and accordingly the L/D ratio was calculated for the cylinders and the piston.
- After finalizing the dimensions of each and every component of the engine, a CAD model of a alpha configuration of Stirling Engine was developed on CREO Software.
- Parabolic Solar Concentrator was manufactured by utilizing a Old Cable Dish. The dish was further given a mirror finish by adhering several small mirrors over its Concave surface.

2.1 Calculation.

i. Calculation of solar radiation incident on Earth.

The Solar Radiation Intercepted by earth surface was Calculated By Following Equation:

\[
\cos(\theta_z) = \sin\phi (\sin\delta \cos\beta + \cos\delta \cos\gamma \sin\phi) + \cos\phi (\cos\delta \cos\gamma \cos\beta - \sin\delta \sin\gamma \sin\omega \sin\beta) \\
\]

(1)

Where,
- \( \theta_z \) - zenith angle
- \( \delta \) - declination angle
- \( \beta \) - slope angle
- \( \gamma \) - surface azimuth angle
- \( \omega \) - hour angle
- \( \phi \) - latitude angle

According to Empirical relation derived zenith angle \( \theta_z \) is given by

- \( \cos(\theta_z)=\sin\phi (\sin\delta \cos\beta + \cos\delta \cos\gamma \cos\phi) + \cos\phi (\cos\delta \cos\gamma \cos\beta - \sin\delta \sin\gamma \sin\omega \sin\phi) \)

(1)
Now according to ASHRAE Model
\[ I_{bn} = A \exp\left(-B / \cos \Theta z\right) \]  
\[ I_d = C I_{bn} \]  
Where,
- \( I_{bn} \): Beam radiation in direction of rays
- \( I_d \): Diffused radiation
- \( I_g \): Hourly global radiation
- \( I_b \): Hourly beam radiation
- \( I_b = I_{bn} \cos \Theta z \)  
- \( I_g = I_b + I_d \)

Using Eqn. (1), (2), (3), (4), (5) and taking suitable value of constant A, B and C from ASHRAE databook, The global Radiation was found to be:
\[ I_g = 1030.79 \, \text{W/m}^2 \]

### ii. Calculation of Parabolic Solar Concentrator
- The effective surface area of Absorber is given by
  \[ A_{abs} = \frac{\pi \times D_p^2}{4} + \pi D_p L \]
- Area of parabolic Dish is given by
  \[ A_p = \pi \times r_p^2 \]
- Concentration Ratio, \( C = \frac{A_p}{A_{abs}} \)
- Now, \( C = \frac{1}{\sin^2 \theta} \)
- Rim Angle, \( \Theta_{rim} = 90 - \Theta \)
- Also Focal Length, \( f = \frac{D_p^2}{16h} \)

Using the above equations the Rim Angle was found to be 76.26°, Concentration Ratio was Calculated as 17.32 and Focal Length of the parabolic dish was found to be 0.37m.

### iii. Calculation of Temperature achieved by the absorber
The temperature achieved by the absorber was calculated using following equation.
\[ m_a \times C_{pa} \times (T_{abs} - T_{ambient}) = \eta \times I_g \times A_p \]
Since the efficiency of parabolic solar concentrator is between 0.3 to 0.6, Therefore taking \( \eta = 0.3 \).

Where \( m_a \) is the mass of air inside engine. Using the Above equation the temperature achieved by the absorber was found to be
\[ T_{abs} = 540.20^\circ \text{C} \]

### iv. Thermodynamic Analysis of Stirling Cycle With Regeneration.
- Total Heat added considering regeneration is given by the equation
\[ Q_a = m C_p \left(T_3 - T_1\right)(1-e) + (k-1)T_2 \ln V_{inh} + V_{sec} + k T_3 V_{sec} + V_{sec} \]

In Above Equation \( m \) is mass of air inside the cylinder, \( C_p \) is specific heat of air at constant volume, \( T_3 \) is the temperature of expansion cylinder, \( T_1 \) is the temperature of compression cylinder , \( e \) is regenerator effectiveness, \( k \) is ratio of specific heat, \( K \) is ratio of Dead volumes with there respective temperatures and \( V_{inh} \) and \( V_{sec} \) are Swept volume of expansion and Compression Cylinder.

- Total Heat Rejected Considering Regeneration is given by the Equation-
\[ Q_{out} = m C_p \left(T_3 - T_1\right)(1-e) + (k-1)T_2 \ln V_{inh} + V_{sec} + k T_3 V_{sec} + V_{sec} \]

- Net work Done is Calculated from the equation
\[ W_{net} = \sum Q = Q_{in} + Q_{out} \]

- Thermal Efficiency is Given by-
\[ \eta_{th} = \frac{W_{net}}{Q_{in}} \]

Using the Above Four equations the net work output without regeneration i.e \( e=0 \), was found to be 10.36 J and the efficiency of the engine was found to be 12.36%.

**Table-1:** Cycle Efficiency For Different Regenerator Effectiveness.

<table>
<thead>
<tr>
<th>Regenerator Effectiveness (( e ))</th>
<th>( Q_{in} )</th>
<th>( Q_{out} )</th>
<th>Stirling Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>60.21</td>
<td>48.76</td>
<td>0.19</td>
</tr>
<tr>
<td>0.2</td>
<td>55.32</td>
<td>44.14</td>
<td>0.202</td>
</tr>
<tr>
<td>0.3</td>
<td>50.72</td>
<td>39.52</td>
<td>0.220</td>
</tr>
<tr>
<td>0.4</td>
<td>46.13</td>
<td>34.89</td>
<td>0.2436</td>
</tr>
<tr>
<td>0.5</td>
<td>41.53</td>
<td>30.27</td>
<td>0.2711</td>
</tr>
</tbody>
</table>
Stirling Cycle Simulation Using Schmidt Analysis

The equations used for cycle Simulation are:

I. Expansion Space Volume Variation, $V_E$

$$V_E = \frac{V_{swk}}{2} (1 + \cos \Theta)$$

II. Compression Space Volume Variation, $V_C$

$$V_C = \frac{V_{swk}}{2} (1 + \cos(\Theta - \alpha))$$

III. Engine Cycle Pressure, $p$

$$p = mR \left( \frac{V_{swk}}{T_1} + \frac{V_{swk}}{T_5} \right)^{-1}$$

IV. Variation Of Volume and pressure with Crank angle

Using equation (I.), (II.) and (III.) the variation of various volumes and engine cycle pressure $p$ is calculated and the results are tabulated below.

In Above Equations $\Theta$ represents Crank Angle, $\alpha$ represents phase angle viz. 90° in case alpha Stirling engine.

Table 2: Variation of pressure and volume w.r.t. Crank angle.

<table>
<thead>
<tr>
<th>$\Theta$</th>
<th>$V_E$</th>
<th>$V_C$</th>
<th>$V_T$</th>
<th>$p$ (bar)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>49</td>
<td>24.5</td>
<td>73.5</td>
<td>2.53</td>
</tr>
<tr>
<td>30</td>
<td>45.71</td>
<td>36.75</td>
<td>82.46</td>
<td>2.01</td>
</tr>
<tr>
<td>60</td>
<td>36.75</td>
<td>45.71</td>
<td>82.46</td>
<td>1.82</td>
</tr>
<tr>
<td>90</td>
<td>24.5</td>
<td>49</td>
<td>73.5</td>
<td>1.74</td>
</tr>
<tr>
<td>120</td>
<td>12.25</td>
<td>45.71</td>
<td>57.96</td>
<td>2.14</td>
</tr>
</tbody>
</table>

3. CONCLUSIONS

I. Effect of regenerator effectiveness on thermal efficiency.

In regenerative cycle, the heat transferred to regenerator matrix by working fluid and the same heat is recovered back and hence the heat transfer process at regenerator governs the regeneration effectiveness. The engine cycle simulation has shown that the engine attains maximum thermal efficiency of 61% with regeneration effectiveness unity. The regeneration effectiveness does not affect maximum cycle pressure, net cycle work, and power output.

Chart 1: Regenerator Effectiveness Vs. Stirling Cycle Efficiency.
II. Regenerator Effectiveness Vs. Heat Rejected

It is clear from the calculation that as the regenerator effectiveness increases the heat rejected decreases. The graph of Regenerator Effectiveness vs Heat rejected is shown below.

![Chart -2: Regenerator Effectiveness Vs. Heat Rejected](image)

III. Regenerator Effectiveness Vs. Heat Supplied

As the regenerator absorbs heat when the fluid passes from hot cylinder to cold cylinder and supply some amount of heat on the return process, the heat required to be supplied decreases. The graph of Regenerator Effectiveness Vs. Heat supplied is shown below.

![Chart -3: Regenerator Effectiveness Vs. Heat Supplied](image)

IV. Crank Angle Vs. Volume Variation

Volume variation of expansion Cylinder, compression cylinder and the total volume has been plotted with respect to Crank Angle. It has been found that the volume shows sinusoidal variation with respect to crank angle. The graph of Crank angle Vs. Volume variation is shown below.

![Chart -4: Crank Angle Vs. Volume Variation](image)

V. Actual Pressure - Volume Diagram

There is always some variation in the ideal pressure-volume diagram Vs. actual pressure-volume diagram. The actual pressure-volume diagram is shown below:

![Chart -5: Actual p-V Diagram](image)

REFERENCES


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