Abstract - This paper describes the design and implementation of ANFIS based Model Reference Adaptive PID controller for a nonlinear Conical Tank Level System (CTLS). The control structure is established on a CTLS. The mathematical model of CTLS is developed and an ANFIS based Model Reference Adaptive PID Controller is proposed for this level system. The result of proposed controller is compared with MRAC-PID and conventional PID to analyze the performance in terms of integral square error and Integral absolute error. The results proved that the efficiency of proposed controller.

Key Words: Conical tank, PID controller, ANFIS, MRAC.

1. INTRODUCTION

In most of the process industries, control of chemical process system is challenging problems due to their nonlinear dynamic behavior. In particular, one of the non linear systems like conical tank is extensively used in process industries, petrochemical industries, food process industries and wastewater treatment industries. Conical tank is highly non linear system because of cross section with respect to level. Conventional Controllers are normally used in process industries as they are simple and familiar to the field operator but they give poor performance because of tuning about one operating point. The variations in the process parameters can be trolled by persistent tuning of the controller parameters using adaptive intelligent techniques like adaptive based control strategy.

One of the most frequently used adaptive control technique is Model Reference Adaptive Control systems (MRAC). This control system [1-4] has received considerable attention, and many new approaches have been applied to practical process. But conventional MRAC will not give satisfactory response for non linear system because of its adaptation procedure. So a soft computing technique is introduced in the MRAC technique in order to overcome these problems. A control strategy that enhances a controller with a self-learning capability for achieving prescribed control objectives. In this sense, an Adaptive-Fuzzy Inference System (ANFIS) based MRAC PID architecture is employed [5-6] so that a MRAC structure is built for achieving a desired input/output mapping. The learning method used allows the tuning of parameters both of the membership functions and the consequents in a Sugeno-type inference system. In this paper the Conical tank level system has been considered as a typical representative of inherently nonlinear system, thus it is an ideal choice for testing the modeling capability of the ANFIS based MRAC algorithm.

The main contributions of this paper are the performance of the ANFIS based MRAC control strategy on the model of the conical tank level system through simulation studies. In section 2 the process description of conical tank is given. The MRAC is discussed in section 3. The design and structure of ANFIS control strategy is detailed in section 4. Simulation results are analyzed in section 5. Finally, section 6 is summing up of the entire work.

2. PROCESS DYNAMICS

Fig 1 shows the schematic Conical tank level system. Here $F_i$ is the inlet flow rate to the tank, $F_o$ be the outlet flow rate from the tank, $F_L$ be the disturbance applied to the tank.

![Fig 1: Conical Tank Level System](image_url)

- $F_i$ - Inlet flow rate to the tank (m³/min)
- $F_o$ - Outlet flow rate from the tank (m³/min)
- $F_L$ - Load applied to the tank (m³/min)
- $H$ - Height of the conical tank (m)
- $h$ - Height of the liquid level in the tank at anytime 't' (m)
- $R$ - Top radius of the conical tank (m)
- $r$ - Radius of the conical vessel at a particular level of height h(m)
A - Area of the conical tank (m²)

The nominal operating level \( h \) is given by

\[
F_{\text{in}} - F_{\text{out}} = A(h) \frac{dh}{dt} \tag{1}
\]

\[
\tan \theta = \frac{r}{H} = \frac{L}{h} \tag{2}
\]

At any level \( h \)

\[
\tan \theta = \frac{L}{h} \tag{3}
\]

Equating (2) and (3)

\[
R = \frac{r}{H} = \frac{L}{h} \tag{4}
\]

Cross sectional area of the tank at any level \( h \) is

\[
A(h) = \pi r^2 \tag{5}
\]

Substitute (4) in (5)

\[
A(h) = \frac{\pi R^2 h^2}{h^2} \tag{6}
\]

Also

\[
F_{\text{out}} = b \sqrt{h} \tag{7}
\]

Substituting (7) in (1)

\[
F_{\text{in}} - b \sqrt{h} = A(h) \frac{dh}{dt} \tag{8}
\]

From equation (8)

\[
F_{\text{in}} - \frac{bh}{2h} = A(h) \frac{dh}{dt} \tag{9}
\]

Where

\[
U = b \sqrt{h} = \text{Nominal value of outflow rate}
\]

Hence the transfer function of the above system is

\[
\frac{h(t)}{F_{\text{in}}(t)} = \frac{k}{\pi s + 1} \tag{11}
\]

where,

\[
h \text{ and } U \text{ are nominal values of PV and MV from equation (11)}
\]

Time constant of the level process \( \tau = \frac{1}{2k_b} \tag{12} \)

The gain constant of the level process \( k = \frac{2h}{U} \tag{13} \)

3. MODEL REFERENCE ADAPTIVE PID CONTROL

When the plant parameters and the disturbance are varying slowly or slower than the dynamic behavior of the plant, then a MRAC control can be used. The model reference adaptive control scheme is shown in figure 2. The adjustment mechanism uses the adjustable parameter known as control parameter \( \theta \) to adjust the controller parameters. The tracking error and the adaptation law for the controller parameters were determined by MIT rule.

Fig- 2: Structure of Model Reference Adaptive Controller

MIT (Massachusetts Institute of Technology) Rule is that the time rate of change of \( \theta \) is proportional to negative gradient of the cost function \( J \), that is:

\[
\frac{d\theta}{dt} = -\gamma \frac{\partial J}{\partial \theta} \tag{12}
\]

The adaptation error \( \epsilon = y_p(t) - y_m(t) \). The components of \( \frac{\partial \epsilon}{\partial \theta} \) are the sensitivity derivatives of the error with respect to the adjustable parameter vector \( \theta \). The parameter \( \gamma \) is known as the adaptation gain. The MIT rule is a gradient scheme that aims to minimize the squared model error \( \epsilon^2 \) from cost function.

\[
J(\theta) = \frac{1}{2} \epsilon^2(\theta) \tag{14}
\]

Model reference Adaptive Control: The goal of this section is to develop parameter adaptation laws for a PID control algorithm using MIT rule.

The reference model for the MRAC generates the desired trajectory \( y_m \) which the level \( y_p \) has to follow. Standard second order differential equation was chosen as the reference model given by

\[
h_p[\dot{a}] = \frac{k}{2} + a_0 + \frac{1}{2} \tag{15}
\]

Consider also the adaptive law of MRAC structure taken as the following form

\[
u(t) = \left(K_p e(t) + K_i \int e(t) dt - K_d e^*(t) y_p\right) \tag{16}
\]

Where; \( e(t) = u_c - 1 \), \( K_p \) is proportional gain, \( K_i \) is integral gain, \( K_d \) is derivative gain and \( u_c \) is a unit step input. In the laplace domain, equation (18) can be transformed to

\[
U = \left(K_p \dot{E} + \frac{K_i}{s} E - sK_d Y_p\right) \tag{17}
\]

It is possible to show that applying this control law to the system gives the following closed loop transfer function:
Apply MIT gradient rules for determining the value of PID controller parameters \( \{K_p^*, K_i^*, K_d^*\} \). The tracking error equation satisfies:

\[
\epsilon = \frac{G_p K_p (s + K_i^*)}{G_m K_m (s + K_i^*)} Y_m
\]

The exact formulas that are derived using the MIT rule cannot be used. Instead some approximations are required. An approximation made which valid when parameters are closed to ideal value is as follows:

\[
\frac{dK_i}{dt} = \frac{\epsilon}{\eta K_i}
\]

Then the approximate parameter adaptation laws are as follows

\[
K_p^* = \frac{-\epsilon}{s} e \left( \frac{s}{a_0 s^2 + a_M s + a_H} \right) e
\]

\[
K_i^* = \frac{-\epsilon}{s} e \left( \frac{1}{a_0 s^2 + a_M s + a_H} \right) e
\]

\[
K_d^* = \frac{-\epsilon}{s} e \left( \frac{s^2}{a_0 s^2 + a_M s + a_H} \right) e
\]

Above equations show the change in PID controller parameters with respect to time. By assuming the reference model has 5% maximum overshoot, settling time of 30 seconds and rise time of about 1 seconds, the second order transfer function of the Model Reference as follows

\[
G(s) = \frac{0.5}{s^2 + 0.2666s + 0.3831}
\]

4. ANFIS MODEL REFERENCE ADAPTIVE PID CONTROL

ANFIS stands for Adaptive Neural Fuzzy Inference System. Using a given input/output data set, ANFIS constructs a Fuzzy Inference System (FIS) whose membership function parameters are adjusted using back propagation algorithm in combination with a least squares technique. This allows fuzzy system to learn from the data. The Takagi-Sugeno ANFIS architecture is shown in Figure 3. The circular nodes represent nodes that are fixed whereas the square nodes are nodes that have parameters to be learnt.

5. RESULTS AND DISCUSSION

Performances of proposed controller are analyzed using step input at various level in the CTLS. Initially the tank is maintained at 30% operating level, after that, a step size of 20% of level is applied to control loop with ANFIS based MRAC PID control strategy. In the same way, test runs of MRAC PID and conventional PID control values are carried out and their responses are presented in Figure 5. It is found that in ANFIS based MRAC PID makes the system to settle with minimum overshoot at all.

To validate the performance proposed controller, the same procedure is repeated for 70% and 90% level and given in the same figure. From the results, the performances are analyzed in terms of ISE and IAE are tabulated in Table 1. From Fig 5, it is also clear that proposed controller tracks the set point quickly without any oscillations. The magnified view of output response is presented in Figure 6. The results prove that ANFIS based MRAC-PID controller is appropriate.
for non linear process as it has least error values than the other controller strategies.

![Comparison of performances of PID, MRAC-PID and ANFIS based MRAC-PID for conical tank level process.](image)

**Fig-5: Comparison of performances of PID, MRAC-PID and ANFIS based MRAC-PID for conical tank level process.**

![Magnified view of output response.](image)

**Fig-6: Magnified view of output response.**

**Table.1. Performance Indices at different Operating range.**

<table>
<thead>
<tr>
<th>Controller</th>
<th>ISE OP (30%-50%)</th>
<th>ISE OP (50%-70%)</th>
<th>ISE OP (70%-90%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PID</td>
<td>1171</td>
<td>2456</td>
<td>4290</td>
</tr>
<tr>
<td>MRAC-PID</td>
<td>552.7</td>
<td>866.7</td>
<td>1083</td>
</tr>
<tr>
<td>ANFIS MRAC-PID</td>
<td>497.7</td>
<td>737.6</td>
<td>964.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Controller</th>
<th>IAE OP (30%-50%)</th>
<th>IAE OP (50%-70%)</th>
<th>IAE OP (70%-90%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PID</td>
<td>166.9</td>
<td>354.1</td>
<td>594.9</td>
</tr>
<tr>
<td>MRAC-PID</td>
<td>59.81</td>
<td>102.2</td>
<td>128.1</td>
</tr>
<tr>
<td>ANFIS MRAC-PID</td>
<td>53.15</td>
<td>85.38</td>
<td>114.3</td>
</tr>
</tbody>
</table>

**6. CONCLUSION**

In this paper, ANFIS based MRAC-PID control strategy has been developed and implemented for a conical tank level system. This method is suitable for process control applications with a large delay, where a conventional PID controller yield a poor performance. The simulation results are furnished to illustrate the efficiency of proposed controller with those of MRAC-PID and PID control approaches. The performance indices are also proved that the proposed controller gives a superior performance than the existing control strategies.

**REFERENCES**