ANALYSIS OF NATURAL FREQUENCY OF PRESTRESSED STEEL BEAM WITH TRIANGULAR TENDON PROFILE

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Abstract – *Pre-stressing has become a very useful technique in the field of concrete and steel structure design. There are two types of pre-stressing in use. One is pre-stressing with bonded and second pre-stressing with unbonded tendons. Pre-stressed steel beams are lighter than traditional ones that have the same length and vertical load capacity; this aspect could make them economically advantageous and viable solution in many practical situations. In the present work Finite Element Approach is used to model and analyze the pre-stressed steel beam. Finite element software ANSYS 15.0 is used. Non-linear static analysis is performed to get the static response for unbonded pre-stressed beam. Triangular tendon profile is considered for analysis. Dynamic analysis is performed to study the effect of pre-stressing force and effect of eccentric tendon on the flexural natural frequency of the beam.*

***Key Words*:** **Pre-stressed steel beam, flexural natural frequency, pre-stressing force, eccentricity.**

1. INTRODUCTION

Pre-stressing has become a very useful technique in the field of concrete and steel structure design. Historically, pre-stressing was developed simultaneously for concrete and steel structures. One of the basic objectives of pre-stressing is to avoid compressive forces in slender tendons at any load configuration.
The technique of bridge strengthening using pre-stressing with external tendons has been studied as a possible means of strengthening single-span steel-concrete girders. Also the pre-stressed steel literature emphasizes on the issues like self- stressing force in tendons and buckling aspects.
Dynamic problems involving girder bridges include the problem of vibration, noise due to the flexural rigidity of main girders, and fatigue due to repetitive live loads.In the strengthening method, which pre-stresses a girder with external tendons, the girder is given an axial pre-stressing force, the value of which is determined according to the required degree of strengthening. In this section the study for analysis of pre-stressed beam is presented. In the dynamic analysis of pre-stressed beams, which are widely used in bridges, knowledge of the flexural natural frequencies is of vital importance. Many researchers have studied the influence of pre-stressing force on the flexural natural frequencies of the beams. Pre-stressing lowers the natural frequency of the girder; thus, using external pre-stressing tendons, the vibration characteristics,which include the natural frequency, of the existing structural system are affected.

2. LITERATURE REVIEW

Pre-stressing techniques for steel beams were developed many years ago, both for construction of new structures and for rehabilitation of existing structures. This technique has been adopted mainly for bridges and rarely for roof structures. In the available literature concerning pre-stressed structures, it was found that the literature on pre-stressed steel structures is less in comparison with pre-stressed concrete structures.

2.1EXPERIMENTAL AND ANALYTICAL WORK

Park et al. (2007) [1] studied the flexural behavior and strengthening effect of bridge using steel I- beam that had been externally pre-stressed with un-bonded tendons. For the purpose of analysis 11 steel beams were fabricated and tested in terms of tendon type, amount of tendon, pre-stressing force installation of deviator and embedment of a draped tendon. It was observed that the externally pre-stressing method created a stiffer steel beam when the proper amount of pre-stressing force was applied. The experimental results were compared with theoretical solution in order to verify whether the external post-tensioning method is useful for strengthening a steel bridge and whether it was applicable in the construction field.
Saiidi *et al*. (1994) [2] studied if vibration frequencies of pre-stressed member can be used to establish pre-stress losses. Both field and laboratory tests were carried out. For the purpose of field test a post-tensioned bridge called the Golden Valley Bridge was used. A post-tensioned beam was used for laboratory test. Since actual pre-stresses were known in bridge and beam therefore it was possible to correlate the measured frequencies and pre-stresses forces. It was observed that the measured frequencies for both bridge and beam showed that as pre-stressing force Increases the frequency decreases.
Jaiswal O.R. (2008) [2] studied the effect of pre-stressing force on first flexural natural frequency of beams. Finite element technique was used to model the beam tendon system and pre-stressing force is applied in form of initial tension in the tendon. Depending on bonded and unbounded nature of tendon, and eccentricity of tendon, he studied the effect of pre-stressing force on first natural frequency.This study concluded that if the eccentricity of tendon is small, then the first flexural natural frequency decreases with pre-stressing force.
Belletti and Gasperi (2010) [4] studied the behavior of simply supported I-shaped cross section steel beams pre-stressed by tendons. For analysis a pre-stressed steel beam having a medium span ranging from 35 to 45 m was used. This study focused on two parameters for the design: the number of deviators and the value of the pre-stressing force. This study has been carried out with non-

linear finite-element analyses that took into account both mechanical and geometrical non-linearities. ABAQUS code was used for this purpose. Four-node shell elements S4 were employed for beam and stiffening ribs, three-dimensional pipe elementsB31 for deviators having a radius of 70 mm and a thickness of 12 mm, and tube-tube contact elements ITT31 for modeling the finite-sliding interaction where tendons are inside the deviators. Fixed boundary conditions were imposed to simulate supports and lateral restraint resulting from the bracing.

3. METHODOLOGY

The pre-stressed beam used for modeling and analysis is a mono-symmetric I-section. Mono-symmetrical section is economical and hence it is used in designing of girders. The beam considered for analysis is pre-stressed by un-bonded tendon.

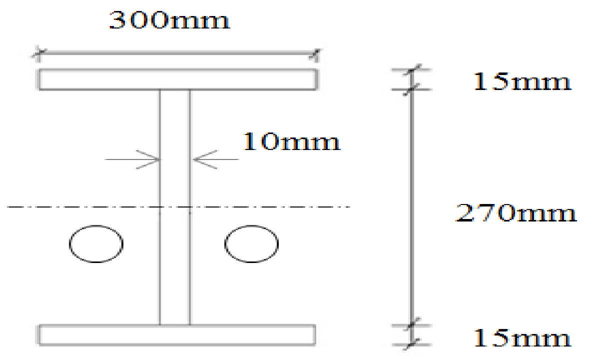

Fig -1: Cross-section of the beam.

Table -1: Dimensions and Properties of cross-section

Dimensions	Properties
Top flange – 300x15mm Web section – 10x270mm Bottom flange – 300x15mm Length of beam – 4300mm	Modulus of Elasticity - 2E05 N/mm2 Density of steel - 7850 kg/m3 Yield strength - 250 N/mm2 Poisson's ratio - 0.3

Table -2: Material properties of beam and Tendon

Type	Cross section area (m2)	Moment of Inertia (m3)	Modulus of Elasticity (GPa)
Steel beam	0.01198	2.41 x 10-4	200
Tendon	0.000491	1.917 x 10-6	200

3.1Static analysis of beam with triangular tendon profile

static analysis for triangular tendon profile is performed. While modeling the beam with triangular tendon profile, the eccentricities at the end of the beam are taken as zero (i.e. ends of the tendon are at C.G.) and the eccentricity is varied at the mid-span. Fig. 3.17 shows the model of the beam pre-stressed with triangular tendon profile with e = 50 mm. The analysis is done for the different values of pre-stressing force from 0kN to 3000kN.

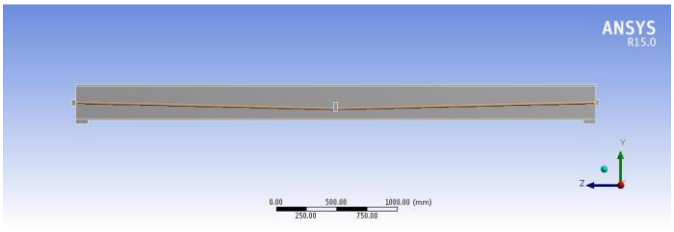

Fig -2: Model of steel I beam with triangular tendon profile in ANSYS

Table -3: Mid-span deflection of beam with triangular tendon profile for e = 50mm

Remote force (kN)	0	200	400	600	800
Pre-stressing force (kN)		Mid Span Deflection (mm)			
0	0	-4.034	-8.069	-12.104	-16.139
500	2.113	-1.989	-6.018	-10.052	-14.087
1000	4.226	0.122	-3.968	-8	-12.035
1500	6.338	2.142	-2.044	-5.953	-9.983
2000	8.452	4.591	0.245	-4.011	-7.936
2500	10.565	6.699	2.402	-1.841	-5.91
3000	12.677	8.81	4.97	0.368	-3.471

By changing the values of remote force, load deflection curves are plotted for different values of pre-stressing force as shown in figure 3 for e = 50 mm

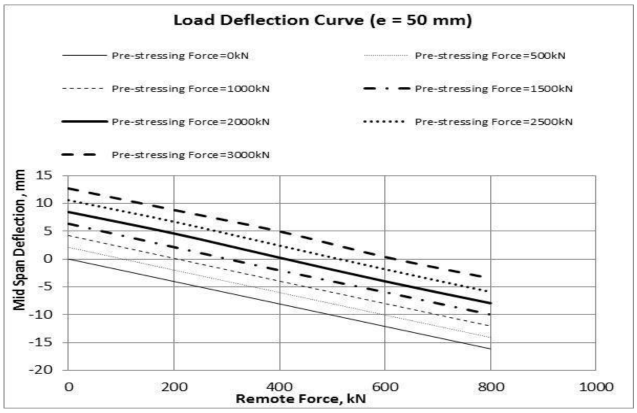

Fig -3: deflection Curve with trapezoidal tendon profile for e=50mm.

Table -4: Mid-span deflection of beam with triangular tendon profile for e = 100mm

Remote force (kN)	0	200	400	600	800
Pre-stressing force (kN)		Mid Span Deflection (mm)			
0	0	-4.036	-8.073	-12.109	-16.146
500	3.837	-0.271	-4.329	-8.365	-12.401
1000	7.677	3.801	-0.541	-4.633	-8.658
1500	11.512	7.6312	3.784	-0.458	-4.769

2000	15.349	11.466	7.602	3.776	-0.139
2500	19.186	15.301	11.43	7.582	3.003
3000	23.024	19.138	15.262	11.403	7.567

By changing the values of remote force, load deflection curves are plotted for different values of pre-stressing asshown infigure 4 for e = 100 mm.The mid-span deflection reduces with increase in the pre-stressing force.

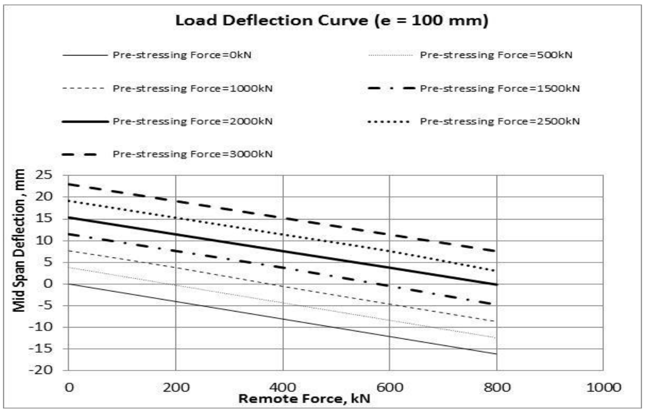

Fig -4: deflection Curve with trapezoidal tendon profile for e=100mm

From the above tables and graphs, it can be seen that the values obtained from ANSYS for mid span deflection decrease with increase in pre-stressing force for a particular case with tendon eccentricity value constant.

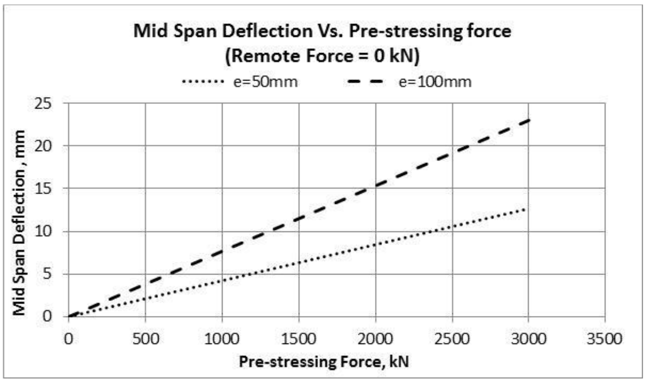

Fig -5: Load-deflection Curve with triangular tendon profile for 0kN remote force.

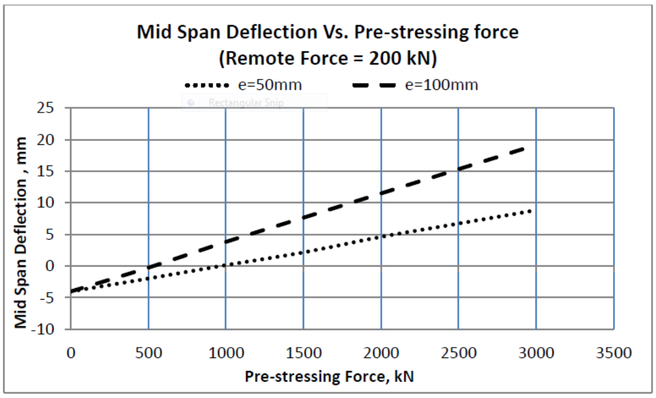

Fig -6: Load-deflection Curve with triangular tendon profile for 200kN remote force

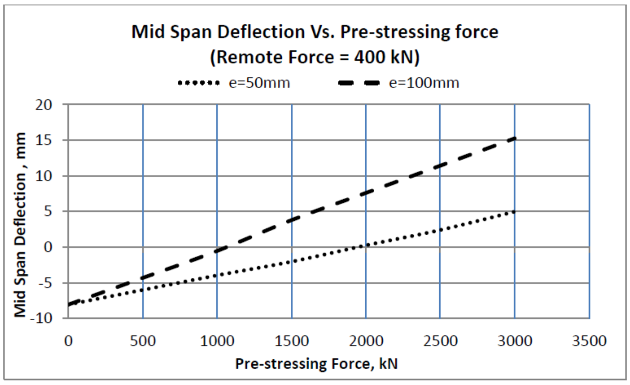

Fig -7: Load-deflection Curve with triangular tendon profile for 400kN remote force

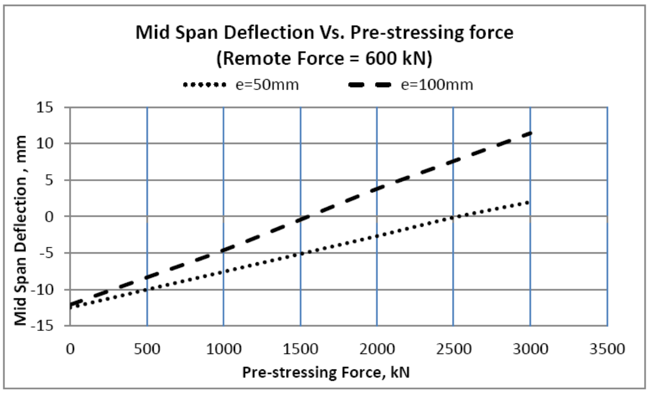

Fig -8: Load-deflection Curve with triangular tendon profile for 600kN remote force

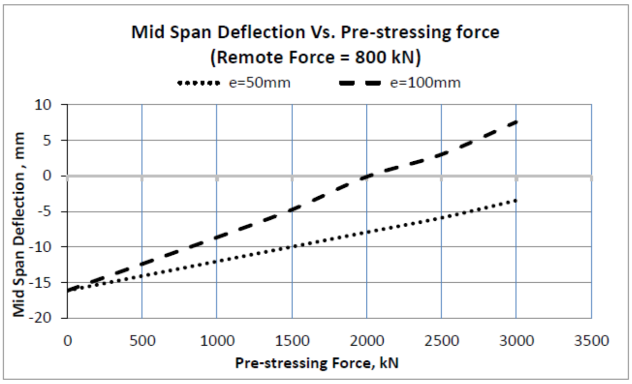

Fig -9: Load-deflection Curve with triangular tendon profile for 800kN remote force

keeping the pre-stressing force constant, mid-span deflection decreases with increase in eccentricity of tendon.

3.2Static analysis of beam with triangular tendon profileusing ANSYS 15.0

The deflection results obyained from ansys are compared to the analytical results by using the following equation for Triangular tendon profile.

$$\delta = \frac{P_f \, e \, L^2}{12 \, E \, I_{xx}}$$

4. Results

4.1 Comparing the static behavior in ANSYS with analytical results for the I beam with triangular tendon profile

Table -5: Analytical and ANSYS Mid-span deflection of beam with triangular tendon profile for e = 50mm

Pre-stressing Force (kN)	ANSYS DEFLECTION (mm)	ANALYTICAL DEFLECTION (mm)	% Error
0	0	0	-
500	2.113	2.027	-4.222
1000	4.226	4.055	-4.222
1500	6.338	6.082	-4.205
2000	8.452	8.110	-4.222
2500	10.565	10.137	-4.222
3000	12.677	12.164	-4.213

Table -6: Analytical and ANSYS Mid-span deflection of beam with triangular tendon profile for e = 100mm

Pre-stressing Force (kN)	ANSYS DEFLECTION (mm)	ANALYTICAL DEFLECTION (mm)	% Error
0	0	0	-
500	3.837	3.988	-3.945
1000	7.677	7.977	-3.904
1500	11.512	11.965	-3.936
2000	15.349	15.953	-3.938
2500	19.186	19.942	-3.939
3000	23.024	23.930	-3.936

As seen from table 5 and 6 the analytical and ANSYS results for the mid-span deflection of the I-beam with triangular tendon profile are in good agreement with each other and the % error is within ±5 %.

4.2 Dynamic analysis of pre-stressed beam with triangular tendon profile

The dynamic analysis aims to study the effect of pre-stressing force on natural frequency of beam.

Asta and Dezi (1996) have derived the expression for the flexural natural frequencies. In second discussion on Saiidi's paper, Deak (1996) opined that if the tendon is unbonded and attached to the beam only at its ends, then only the pre-stressing force could be treated as external axial force. In the third discussion, Jain and Goel (1996) pointed out that since the tendon becomes integral part of the system, tension in the tendon cannot be treated as an external force, and hence, the pre-stressing force will not affect the flexural natural frequencies of the beam.The considered beam being pre-stressed with un-bonded tendon, the equation used to calculate the natural frequency will be the same which is used for external axial load.

$$\omega_n^2 = \left(\frac{n\pi}{L}\right)^4 \frac{E_b I_b}{m} - \left(\frac{n\pi}{L}\right)^2 \frac{N}{m}$$

Where, n, Ib, L, Eb, and m respectively denote mode number, moment of inertia in m4, cross-sectional area in m2, span length in m, elastic modulus in N/m2, and mass per unit length in kg/m of the beam and N is pre-stressing force in N.

4.3Dynamic analysis of pre-stressed beam with triangular tendon profile in ANSYS

Dynamic Analysis is carried out for different values of eccentricity e = 50/100 mm, remote forces and pre-stressing force in ANSYS.

Table -7: Natural frequency of beam with triangular tendon profile for e = 50mm

Remote force (kN)	0	200	400	600	800
Pre-stressing force (kN)		Natural Frequency (Hz)			
0	60.899	60.882	60.865	60.847	60.83
500	60.638	60.648	60.568	60.55	60.638
1000	60.478	60.456	60.434	60.413	60.391
1500	60.421	60.399	60.377	60.355	60.333
2000	60.29	60.268	60.246	60.224	60.201
2500	60.127	60.105	60.083	60.061	60.038
3000	59.947	59.926	59.904	59.882	59.86

Table -8: Natural frequency of beam with triangular tendon profile for e = 100mm

Remote force (kN)	0	200	400	600	800
Pre-stressing force (kN)		Natural Frequency (Hz)			
0	60.84	60.824	60.807	60.792	60.752
500	60.579	60.584	60.513	60.498	60.578
1000	60.411	60.39	60.369	60.348	60.327
1500	60.35	60.329	60.308	60.287	60.266
2000	60.216	60.195	60.174	60.152	60.131
2500	60.05	60.029	60.008	59.986	59.965
3000	59.867	59.845	59.824	59.803	59.781

As seen from the table 7-8 , the natural frequency reduces with increase in the pre-stressing force.

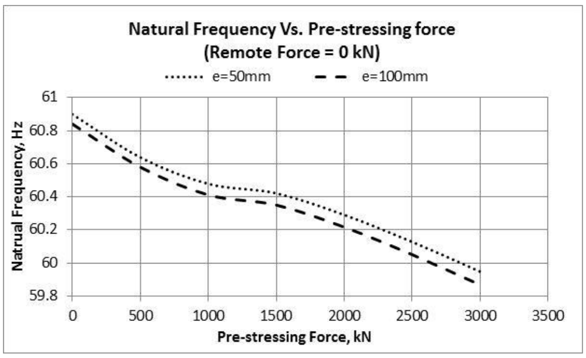

Fig -10 Natural frequency of I-beam with triangular tendon profile for 0kN remote force.

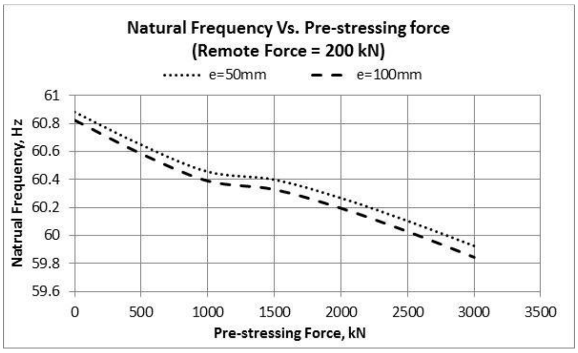

Fig -11 Natural frequency of I-beam with triangular tendon profile for 200kN remote force.

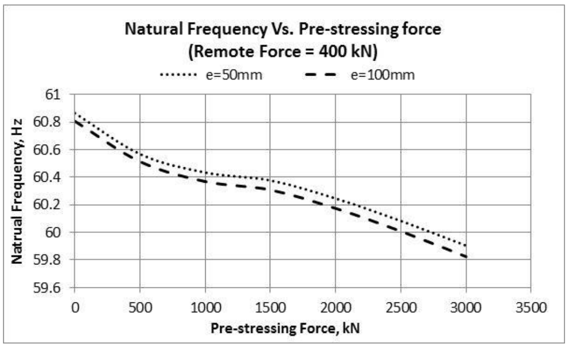

Fig -12 Natural frequency of I-beam with triangular tendon profile for 400kN remote force.

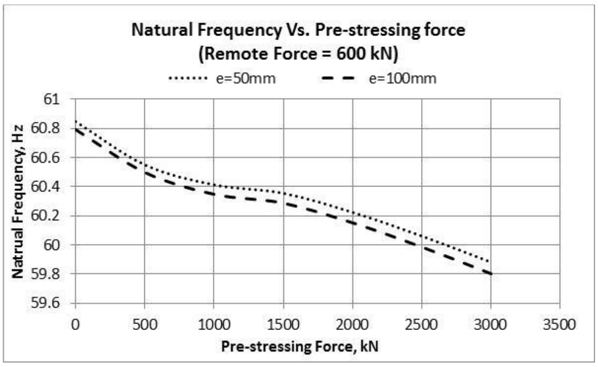

Fig -13 Natural frequency of I-beam with triangular tendon profile for 600kN remote force.

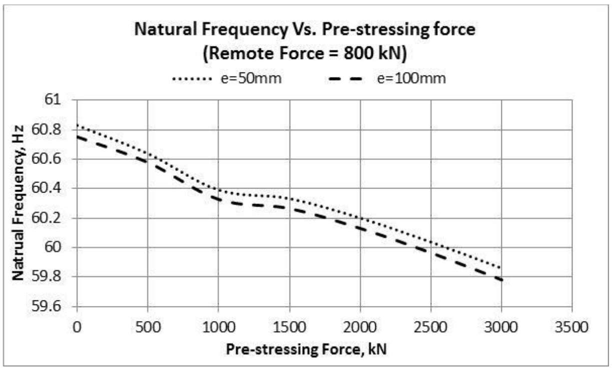

Fig -14 Natural frequency of I-beam with triangular tendon profile for 600kN remote force

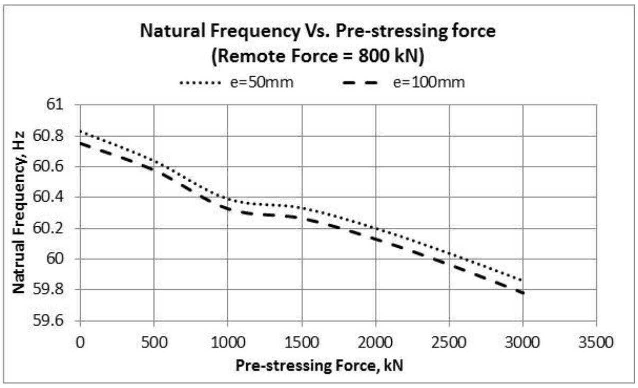

Fig -15 Natural frequency of I-beam with triangular tendon profile for 800kN remote force

As seen from the fig 10-15, the natural frequency reduces with increase in the pre-stressing force and eccentricity.

5. Discussion

The beam is pre-stressed with triangular tendon profile. The eccentricity is e=50 mm, e=100 mm.
It is seen that the results obtained from finite element analysis are in good agreement with the analytical results. It is observed that as the pre-stressing force increases, the natural frequency of the beam reduces. Also the effect of eccentricity of tendon on the natural frequency of the beam is observed, where the natural frequency of the beam decreases when the eccentricity of the tendon is increased.

6. Conclusion

For the beam pre-stressed externally with un-bonded tendons, it can be concluded that the increase in pre-stressing force significantly reduces the natural frequency and mid-span deflection of the beam.

Variation of eccentricity at which the pre-stressing force is applied also affects the natural frequency of the beam. The natural frequency reduces with increase in eccentricity of the tendon.
Results obtained from ANSYS are in good agreement with that obtained analytically and the error is within ±5%.

The externally un-bonded pre-stressing method is useful for strengthening steel I-beams because it provides an ease of application and economic feasibility.

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