

# NUMERICAL ANALYSIS OF THERMAL CONVECTION OF A FLUID WITH INCLINED AXIS OF ROTATION USING GALERKIN METHOD

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**Abstract** - Numerical analysis using Galerkin technique is carried out for finding the effect of axis of rotation and limiting condition of temperature gradient being oblique to gravity on the stability of fluids saturating porous medium. Various cases of fluids having large and small Prandtl number have been studied which tend to destabilize the system by stationary and oscillatory modes.. Brinkman and Darcy models are used for large and small permeabilities respectively. Numerical calculations are made to indicate the effect of inclination of axis of rotation on setting up of thermo convective instability.

**Key Words:** Thermal instability, inclined axis, Brinkman model, Darcy model, Porous medium, Galerkin technique.

## 1. INTRODUCTION:

Thermoconvective instability in the case of rotating fluids is applicable in geo-thermal analysis, where the effect of rotation is justified as the earth rotates with constant angular velocity about the vertical axis. It is also of great importance to hydrologists and soil scientists.

Generally, many researchers have taken gravity and rotation axis along the vertical direction. But in the actual study of geo-physical system, the axis of rotation is inclined to the direction of gravity at an angle of  $23.5^\circ$ . Hence the effect of obliqueness has to be considered in the stability analysis. Knowledge of temperature gradient constituting thermal convection in mineral fluids is very much essential for the extraction of energy from geo-thermal systems. More over, even petroleum fluids in the

form of crude oil are also embedded under shale rock-beds. Hence, for the extraction of crude from mine bed, the estimation of temperature gradient for onset of convective currents is a must. The mechanism of convection cannot be completely understood if the inclination of axis of rotation to the direction of gravity is not considered. Lapwood (1948) and Wooding(1960) have determined critical Rayleigh number necessary to set up convection current in fluids embedded in a porous medium.

The linear stability of convective flow in a porous medium, which is induced by an inclined temperature gradient in a shallow horizontal layer was first studied by Weber(1974) and then by Nield(1991). Nield et.al.(1993) and Manole et.al(1994) have investigated the linear stability problems for convection in a porous medium induced by inclined thermal and solutal gradients with/without horizontal mass flow rate. Nield (1994) used higher order Galerkin approximation and found that beyond a certain value the effect of horizontal Rayleigh number is changed from stabilizing to destabilizing. Kaloni and Zongchun Qiao (1997, 2001) discussed the non-linear stability of convection in a horizontal porous layer subjected to an inclined temperature gradient by using the energy method. The compound matrix method is used to solve the associated Eigen value problem. Later they also considered a variable gravity field. They had found that the preferred mode and that a decrease in gravity variation has a stabilizing effect on the system.

Chandrasekhar (1961) has determined the critical Rayleigh number for setting up of convection currents in a non-porous rotating system using normal mode techniques. Rudraiah and Prabhamani (1973) extended the normal mode analysis of Chandrasekhar (1961) to

porous medium taking the axis of rotation along the gravity axis.

Several models had been analysed for rapidly rotating spherical systems with internal heat sources. Roberts (1968) and Busse (1970) analysed the onset of convection in rapidly rotating fluid spheres using linear stability analysis. They considered Boussinesq fluids with uniform distribution of heat sources such that unstable radial temperature gradients were established. Heard(1972) studied similar results. Hathaway et.al (1979) carried out a linear stability analysis of fluid layers under uniform rotation, which possesses both vertical and horizontal temperature gradients. Sun and Schubert (1995) carried out numerical stimulations of thermal convection in rapidly rotating spherical fluid shell at high Taylor number and Rayleigh number with a non-linear three-dimensional time dependant spectral-transform code. Zhang and Gubbins(1996) examined thermal convection in a rotating spherical shell with central gravity and a spatially non-uniformly heated outer surface at two values of Prandtl number by numerical calculation. Shin-ichi Takehiro et.al.(2002)studied linear stability of thermal convection in rotating systems with fixed heat flux boundaries. I.K.Choi et.al(2006) analysed Endwall heat transfer and fluid flow around an inclined short cylinder. Barletta (2007) analysed Parallel and non-Parallel laminar mixed convection flow in an inclined tube. Cha'o-kuang Chen and Ming-Che Lin (2009) analysed Weakly nonlinear hydrodynamic stability of the thin Newtonian fluid flowing on a rotating circular disk.

In the present work, Thermal convection of a fluid with inclined axis of rotation analysed using Galerkin technique.

## 2. MATHEMATICAL FORMULATION

An infinitely spread quiescent layer of incompressible Boussinesq fluid saturating a porous medium is confined between two stress free thermally conducting boundaries at  $z = \frac{-1}{2}$  and  $z = \frac{1}{2}$ . The axis of rotation is inclined to the direction of gravity at an angle  $\phi$ . Gravity defined the vertical direction and the rotation vector is tilted at an angle  $(90^\circ - \phi)$  from the vertical. The angular velocity vector  $\Omega$  has components  $(\Omega \cos \phi, \Omega \sin \phi, 0)$ .

Using the Brinkman Model, the governing equations are :

Continuity Equation:

$$\nabla \cdot \mathbf{q} = 0 \tag{1}$$

Momentum Equation:

$$\frac{\partial \mathbf{q}}{\partial t} + (q \cdot \nabla) \mathbf{q} + 2(\Omega \times \mathbf{q}) = - \left( \frac{1}{\rho_0} \right) \nabla p + \left( \frac{\rho}{\rho_0} \right) \mathbf{g} - \left( \frac{\nu}{k_0} \right) \mathbf{q} + \nu \nabla^2 \mathbf{q} \tag{2}$$

Energy equation:

$$\frac{\partial T}{\partial t} + (\mathbf{q} \cdot \nabla) T = K_1 \nabla^2 T + \nu \Phi' \tag{3}$$

where  $\Phi'$  is non linear viscous dissipation function.

Equation of the state:

$$\rho = \rho_0 [1 - \alpha(T - T_0)] \tag{4}$$

where  $\rho_0$  is the density at temperature  $T_0$ ,  $\mathbf{g} = (0, 0, -g)$  is the acceleration due to gravity,  $\mathbf{q}$  is the velocity,  $t$  is the time,  $T$  is the temperature,  $p$  is the pressure,  $\nu = \frac{\eta}{\rho}$  is the kinematic viscosity,  $\kappa = \frac{K_1}{\rho_0 C}$  is the thermometric conductivity,  $\Omega$  is the angular velocity and  $k_0$  is the permeability parameter.

Basic state is assumed to be a uniformly moving state, where  $\mathbf{q} = (U(z), 0, 0)$  and the variation in  $x, y, t$  are assumed to be zero.

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = \frac{\partial}{\partial t} = 0, \quad \frac{\partial}{\partial z} \neq 0, \quad \text{when } \rho = \rho_c \text{ and } T = T_c$$

This, when substituted in the governing equations, yields:

$$U(z) = \frac{g \alpha T_y \left( z - \frac{1}{2} \right)}{2 \Omega \sin \phi}$$

$$\text{where } T_y = \frac{\partial T}{\partial y}$$

Linear theory is used for analyzing the stability of the system. Using normal mode technique, the dynamic variables are written in the form:

$$f(x, y, z, t) = f(z) \exp[ikx + ily + \sigma t]$$

where  $k^2 + l^2 = a^2$  is square of the wave number and  $\sigma$  is the complex growth rate.

The real part of  $\sigma$  represents the growth rate of disturbance and the imaginary part of  $\sigma$  describes the propagation features of the mode.  $W(z)$  is the amplitude function of the vertical velocity and  $Z(z)$  is the amplitude function for vertical vorticity. The  $z$  component of vorticity vector is written as

$$\zeta = \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) = Z(z) \exp[ikx + ily + \sigma t]$$

A small perturbation is superimposed on the basic state. The perturbed state quantities are:

$$\mathbf{q} = (U + u, v, w); \quad p_c = (p + \pi') \quad \text{and} \quad T_c = T + \theta$$

Assuming the velocity perturbation to be small, the governing equations become:

$$\left[ \frac{\partial}{\partial t} + \frac{v}{k_0} - \nu(D^2 - a^2) \right] (D^2 - a^2)W = -2\Omega \sin \phi DZ + g\alpha \nabla_1^2 \theta \tag{5}$$

$$\left[ \frac{\partial}{\partial t} + \frac{v}{k_0} - \nu(D^2 - a^2) \right] DZ = 2\Omega \sin \phi D^2 W \tag{6}$$

$$\left[ \frac{\partial}{\partial T} + U \frac{\partial}{\partial x} \right] \theta = \kappa \nabla^2 \theta + w\beta \tag{7}$$

where  $D = \frac{\partial}{\partial z}$  and  $\beta = -\frac{\partial T}{\partial z}$  is the adverse temperature gradient.

The final equations obtained from equation (5) to equation (7) for  $W, Z$  and  $\theta$  are:

$$\frac{\partial}{\partial t} (\nabla^2 W) + U (\nabla^2 W_x) - W_x U_{zz} + 2\Omega \cos \phi Z_y + 2\Omega \sin \phi DZ = g\alpha \nabla^2 \theta - \left( \frac{\nu}{k_0} \right) \nabla^2 W + \nu (D^2 - a^2)^2 W \tag{8}$$

$$\left[ \frac{\partial}{\partial t} + \frac{v}{k_0} - \nu(D^2 - a^2) \right] DZ = \left[ -U_z \frac{\partial}{\partial x} - U \frac{\partial^2}{\partial x \partial z} \right] DZ + \frac{\partial}{\partial z} (W_y U_z) + 2\Omega \sin \phi W_{zz} + 2\Omega \cos \phi W_y \tag{9}$$

$$\left[ \frac{\partial}{\partial t} + U \frac{v}{k_0} \right] \theta = \kappa \nabla^2 \theta + w\beta \tag{10}$$

where suffices  $x, y, z$  of dynamical variables represent partial differentiation with respect to  $x, y, z$  respectively.

The dynamical variables are non-dimensionalised as follows :

$$t^* = \frac{\nu}{d^2} t; \quad W^* = \frac{d}{\nu} W; \quad \theta^* = \frac{\kappa \alpha \sqrt{R_a}}{\beta \nu d} \theta; \quad z^* = \frac{z}{d};$$

$$U^* = U \frac{d}{\nu}; \quad a^* = \frac{a}{d};$$

$$k_0 = \frac{k_0}{d^2} \tag{11}$$

For applying linear theory in the limiting condition of variation of temperature along the  $y$  direction, the thermal gradient along the  $Y$  direction,  $T_y$  is taken to be zero.

This leads to reduction of the basic state velocity along  $x$  direction to zero.

The non-dimensional form of the governing equations are:

$$\left[ \frac{\partial}{\partial t^*} + \frac{1}{k_0^*} \right] (D^2 - a^{*2}) W^* = -T_a^{\frac{1}{2}} \sin \phi DZ^* - a^* (R_a)^{\frac{1}{2}} \theta^* \tag{12}$$

$$P_r \left[ \frac{\partial}{\partial t^*} \theta^* \right] = (D^2 - a^{*2}) \theta^* + a^* R_a^{\frac{1}{2}} W^* \tag{13}$$

$$\left[ \frac{\partial}{\partial t^*} + \frac{1}{k_0^*} \right] DZ^* = T_a^{\frac{1}{2}} \sin \phi D^2 W^* \tag{14}$$

The non-dimensional parameters used are:

$$\text{Rayleigh number } R_a = \frac{g\alpha\beta d^4}{\kappa\nu}$$

$$\text{Taylor number } T_a = \frac{4\Omega^2 d^4}{\nu^2}$$

$$\text{Prandtl number } P_r = \frac{\nu}{k}$$

### 3. NUMERICAL SOLUTION USING GALERKIN METHOD

The boundary conditions are Finlayson(1970)

$$W = D^2W = \theta = 0 \text{ at } z = \pm \frac{1}{2}$$

Using Galerkin technique the power series expansion for the variables are taken as

$$w(z, t) = Aw_1(z)e^{i\sigma t}$$

$$\theta(z, t) = B\theta_1(z)e^{i\sigma t}$$

$$\zeta(z, t) = CZ_1(z)e^{i\sigma t}$$

Using these, we get

$$\left[ \sigma + \frac{1}{k_0} \right] \left( w_1 D^2 w_1 - a^2 w_1^2 \right) A + T_a^{\frac{1}{2}} \sin \phi w_1 DZC + a(R_a)^{\frac{1}{2}} w_1 \theta B = 0 \tag{15}$$

$$R_a^{\frac{1}{2}} w_1 \theta_1 A + \left[ (P_r \sigma + a^2) \theta_1^2 - \theta_1 D^2 \theta_1 \right] B = 0 \tag{16}$$

$$\left[ \sigma + \frac{1}{k_0} \right] (DZ)^2 C - T_a^{\frac{1}{2}} \sin \phi DZD^2 w_1 A = 0 \tag{17}$$

Taking

$$w(z) = \frac{z^4}{2} - \frac{3z^2}{4} + \frac{5}{32}$$

$$D^2 w(z) = 6z^2 - \frac{3}{2}$$

$$\theta(z) = z^4 + z^2 - \frac{5}{16}$$

$$D\theta(z) = 4z^3 + 2z$$

$$DZ = 6z^2 - \frac{3}{2}$$

so as to satisfy the boundary conditions we get the equations

$$\left[ \sigma + \frac{1}{k_0} \right] \left( (-0.1214286) - a^2(0.01230158) \right) A + T_a^{\frac{1}{2}} \sin \phi (-0.1214286) C + a(R_a)^{\frac{1}{2}} (-0.025992063) B = 0 \tag{18}$$

$$R_a^{\frac{1}{2}} (0.025992063) A + \left[ (P_r \sigma + a^2) (0.05515873) + (0.569047619) \right] B = 0 \tag{19}$$

$$\left[ \sigma + \frac{1}{k_0} \right] (1.2) C - T_a^{\frac{1}{2}} \sin \phi (1.2) A = 0 \tag{20}$$

for the existence of non-trivial solutions for the above equations the determinant of the coefficients of A, B and C in (18), (19), and (20) is equated to Zero. Stationary stability is analyzed, using the expression

$$R_a = \frac{x_1 x_5 x_7 - x_3 x_5 x_6}{x_2 x_4 x_7}$$

where

$$x_1 = -\frac{1}{k_0} [0.1214286 + a^2(0.01230158)]$$

$$x_2 = -a(0.025992063)$$

$$x_3 = -(T_a)^{\frac{1}{2}} \sin \phi (0.1214286)$$

$$x_4 = a(0.025992063)$$

$$x_5 = a^2(0.05515873) + (0.569047619)$$

$$x_6 = -(T_a)^{\frac{1}{2}} \sin \phi (1.2)$$

$$x_7 = \frac{1.2}{k_0}$$

#### 4. RESULT AND DISCUSSION

An infinitely spread fluid layer heated from below saturating a porous medium is studied. The porosity of the porous medium is varied over wide range from 0.0001 to 0.1. The axis of rotation is assumed to be inclined at an angle of  $23\frac{1}{2}^\circ$  with respect to gravity axis, experimental

and theoretical curiosity led to study the effect of various inclinations ranging from  $15^\circ$  to  $90^\circ$  and rotations prescribed by Taylor number varying from 10 to  $10^8$ . It is found that the system cannot destabilize through oscillatory mode for a fluid having larger Prandtl number greater than 1, because  $\sigma^2$  tends to be negative. An interesting phenomenon of interaction of rotation, permeability and inclination is seen. For small rotation ( $T_a < 10^4$ ), the destabilizing effect due to increase in permeability against increase in inclination is observed. However, for large rotation ( $T_a \geq 10^4$  but  $T_a < 10^6$ ) from Table 1 and Table 2, the destabilizing effect due to increase in permeability is compensated by increase in

inclination. From Figure 1 and 2, the marginal instability in the fluid for Darcy and Brinkman models are observed for  $T_a = 10^6$ . Similar to Hathaway et.al (1979), for any permeability, the increase in rotation exhibits increase in Rayleigh number as well as the wave number thereby showing stabilizing nature of the system.

### 5. CONCLUSION

For small rotation, even though rotation tends to stabilize, the increase in permeability will favour the convection to a greater extent. Inclination also has its own effect of stabilization. When the rotation is increased, stabilizing effect due to rotation and inclination is more which tends to reduce the easiness for the flow and nullify the destabilizing effect. For still larger rotation, the stabilizing effect of rotation is so large that for none of the inclination, the destabilizing effect is not observed.

**Table -1: Stationary instability for variation of Permeability parameter  $k_0$  for different angles of inclination  $\phi$  with  $a_c = 3.25, T_a = 10^6$  (Darcy Model).**

$\phi$	$K_0$	$R_a$	$N_c$	
0	0.001	40567.64844	40567648.43800	
15		41872.60156	41872601.56200	
30		45439.87891	45439878.90600	
45		50319.33594	50319335.93800	
60		55211.30469	55211304.68800	
75		58812.80469	58812804.68800	
90		60164.5625	60164562.50000	
0		0.0001	405676.5	405676500.00000
15			405807	405807000.00000
30	406163.7188		406163718.75000	
45	406651.6563		406651656.25000	
60	407140.875		407140875.00000	
75	407501.0313		407501031.25000	
90	407636.1875		407636187.50000	

**Table -2: Stationary instability for variation of Permeability parameter  $k_0$  for different angles of inclination  $\phi$  with  $a_c = 3.25, T_a = 10^6$  (Brinkman Model).**

$\phi$	$K_0$	$R_a$	$N_c$	
0	0.01	4056.765137	4056765.137	
15		17106.29102	17106291.02	
30		52779.08594	52779085.94	
45		101573.625	101573625	
60		150493.3281	150493328.1	
75		186508.3125	186508312.5	
90		200025.9063	200025906.3	
0		0.02	2028.382568	2028382.568
15			28127.43359	28127433.59
30	99473.02344		99473023.44	
45	197062.0938		197062093.8	
60	294901.5		294901500	
75	366931.5		366931500	
90	393966.6875		393966687.5	
0	0.03		1352.255005	1352255.005
15			40500.83203	40500832.03
30		147519.2188	147519218.8	
45		293902.8125	293902812.5	
60		440661.9375	440661937.5	
75		548706.9375	548706937.5	
90		589259.6875	589259687.5	
0		0.04	1014.191284	1014191.284
15			53212.29297	53212292.97
30	195903.4688		195903468.8	

45	391081.625	391081625
60	586760.4375	586760437.5
75	730820.375	730820375
90	784890.75	784890750

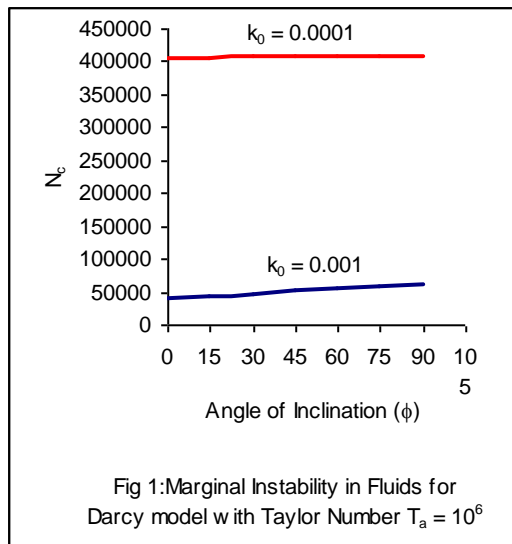


Fig 1: Marginal Instability in Fluids for Darcy model with Taylor Number  $T_a = 10^6$

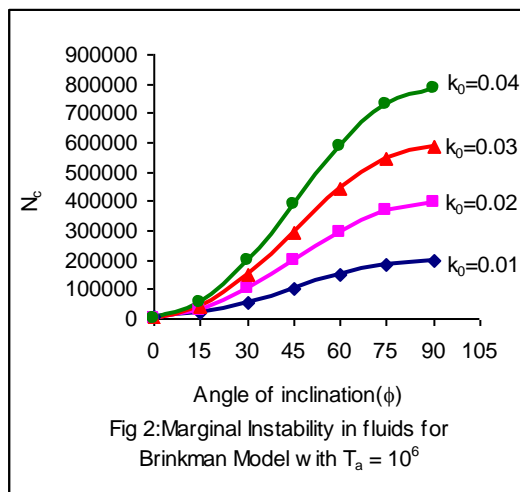


Fig 2: Marginal Instability in fluids for Brinkman Model with  $T_a = 10^6$

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### Nomenclature

- $a$  - dimensionless wave number
- $d$  - thickness of the fluid
- $g$  - gravitational acceleration
- $k$  - dimensional wave number
- $k_1$  - thermal conductivity
- $k_0$  - permeability of the porous medium
- $k_x, k_y$  - wave number in the x and y directions
- $N$  - magnetic Rayleigh number
- $p$  - hydrodynamic pressure
- $Pr$  - Prandtl number
- $q$  - velocity of the fluid (u,v,w)
- $R$  - Rayleigh number
- $t$  - time
- $T$  - temperature
- $T_a$  - average temperature of the lower and upper surfaces
- $T_0$  - constant temperature at the lower surface of the layer
- $T_1$  - constant temperature at the upper surface of the layer
- $\nabla$  - vector differential operator
- $\alpha$  - coefficient of volume expansion
- $\beta$  - adverse basic temperature gradient
- $\rho$  - density of the fluid

- $\rho_0$  - reference density at  $T = T_a$
- $\nu$  - kinematic viscosity,  $\frac{\mu_1}{\rho_0}$
- $\phi$  - magnetic scalar potential
- $\Omega$  - angular velocity vector
- $\sigma$  - complex growth rate
- $\theta$  - perturbed temperature
- $\zeta$  - vorticity vector

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