Some new sets in Ideal topological spaces

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ABSTRACT In this paper, some related generalized sets of τ * namely R*-I closed sets, W-R*-I closed sets, I-R* closed sets in Ideal topological space are introduced. The relationships between these sets are investigated and some of the properties are also studied.

KEYWORDS IR*-closed,R*-I closed,Weakly R*-I closed

1. INTRODUCTION
The notion of generalized closed sets in Ideal topological spaces was studied by Dontchev et. al [4] in 1999. Further closed sets like Irg I rw were further developed by Navaneethakrishnan [10] and A.Vadivel [12] in 2009 and 2013 respectively. The main aim of this paper is to introduce some new related closed sets in the same space and study the relationships between them.

2. PRELIMINARIES
An ideal on a topological space (X, τ ) is a non empty collection of subsets of X which satisfies the following properties
(i) A ∈ I and B ⊆ A implies B ∈ I
(ii) A ∈ I and B ∈ I implies A ∪ B ∈ I.
An ideal topological space is a topological space with an ideal I on X and is denoted by (X, τ , I). For a subset A ⊂ X, A* (τ , I) = {x ∈ X: A ∩ U ∉ I for every U ∈ τ (X, x)} is called the local function of A with respect to I and τ . A Kuratowski’s closure operator cl*( ) for a topology τ *(I, τ ), called the *-topology, finer than τ is defined by cl*(A) = A ∪ A*(I, τ ). Moreover (G, τ , I) represents the relative topology on G denoted by τ and I g = {G ∩ I, j ∈ I} is an ideal topological space for (X, τ , I) and G ⊂ X.

Definition 2.1
A subset A of a topological space (X, τ ) is called
1. Regular open[10] if int(cl(A)) = A
2. Regular semi open [4] if there is a regular open set U such that U ⊂ A ⊂ cl(U). Also X\A is regular semi open.

Definition 2.2[7] The intersection of all regular closed subset of (X, τ ) containing A is called the regular closure of A and is denoted by rcl(A).

Definition 2.3 [7] A subset A of a space (X, τ ) is called R*-closed if rcl (A) ⊂ U whenever A ⊂ U and U is regular semiopen in (X, τ ). We denote the set of all R*-closed sets in (X, τ ) by R*-C(X).

Definition 2.4
A subset A of a space (X, τ , I) is called
1. *-closed [8] if A* ⊂ A
2. I-R closed [1] if A= cl*(int (A))
3. Regular-I closed [9] if A = (int(A))*
5. Pre -*closed[6] if cl*(int(A)) ⊂ A

3. R*-I-CLOSED SETS

Definition 3.1
The intersection of all regular –I closed sets containing A is called the regular-I-closure and is denoted by r*I cl (A).

Definition 3.2
A subset A of an ideal space (X, τ , I) is said to be R*-I closed if r*I cl (A) ⊂ U whenever A ⊂ U and U is regular semi open.

Definition 3.3
A subset A of a space (X, τ , I) is called R*-I open if X\A is R*-I closed.

Theorem 3.4: The union of two R*-I closed sets is R*-I closed.
Proof: Assume A and B are R*-I closed sets in (X, τ , I). Let U be a regular semi open in X such that A ∪ B ⊂ U. Then A ⊂ U and B ⊂ U. Since A and B are R*-I closed sets, r*I cl (A) ⊂ U and r*I cl (B) ⊂ U respectively, hence r*I cl (A ∪ B) ⊂ U. Therefore A ∪ B is R*-I closed.

Remark 3.5: The finite intersection of two R*-I closed need not R*-I closed.

Example 3.6: Let X = {a, b, c} τ = {X, φ , {a}, {c}, {a, c}} I = { φ , {a}}

A= {a,c} and B= {b,c} are R*-I closed sets, while A ∩ B= {c} is not an R*-I closed set. Remark 3.7: Every regular-I closed set is R*-I closed while the converse is not true.

Example 3.8: Let X = {a, b, c, d} τ = {X, φ , {a}, {b}, {a, b}, {b, c}, {a, b, c}}

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I = \{ \varnothing, \{a\}\}, \text{regular-I-closed sets} = \{ X, \varnothing, \{b, c, d\}\} and R^*-
I-closed sets are
\{ X, \varnothing, \{a, b\}, \{a, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}

Remark 3.9: Every regular-I closed set is I-R closed but not necessarily.

Example 3.10: In the above example 3.8, I-R closed sets are \{ X, \varnothing, \{a, b\}, \{b, d\}\}. The set \{a\} is not regular-I-closed.

Theorem 3.11: Let \( (X, \tau, I) \) be an ideal space and \( A \subseteq X \).
If \( A \) is R*I closed, then \( r^*_I \text{cl}(A) \setminus A \) does not contain any
nonempty regular semi open set.

Proof: Suppose \( A \) is R*I closed set in \( (X, \tau, I) \). Also let \( F \) be a regular semi closed set contained in \( r^*_I \text{cl}(A) \setminus A \). It
implies \( F \subseteq r^*_I \text{cl}(A) \setminus A \cap X \setminus A \). Since \( F \subseteq X \setminus A \), we have
\( A \subseteq X \setminus F \) which is a regular semi open set. Therefore \( r^*_I \text{cl}(A) \setminus X \setminus F \) and so \( F \subseteq X \setminus r^*_I \text{cl}(A) \). By hypothesis we have
\( F \subseteq r^*_I \text{cl}(A) \) and so \( F = \varnothing \). Hence \( r^*_I \text{cl}(A) \setminus A \) contains no
nonempty regular semi open set.

Theorem 3.12: Let \( A \) be a R*I closed set in an ideal
space \( X \) such that \( A \subseteq B \subseteq r^*_I \text{cl}(A) \), then \( B \) is also an R*I-
closed set.

Proof: Let \( U \) be a regular semi open set of \( X \), such that \( B \subseteq U \).
Then \( A \subseteq B \subseteq U \). Since \( A \) is R*I closed set, \( r^*_I \text{cl}(A) \subseteq U \).
Since \( A \subseteq B \subseteq r^*_I \text{cl}(A) \subseteq U \), it implies \( r^*_I \text{cl}(B) \subseteq r^*_I \text{cl}(A) \). Hence \( r^*_I \text{cl}(B) \subseteq r^*_I \text{cl}(A) \subseteq U \). Hence \( B \) is an
R*I-closed set.

4. I-R*-CLOSED SETS

Definition 4.1
The intersection of all I-R closed sets containing \( A \) is
called the I-R closure and is denoted by \( r^*_I \text{cl}(A) \).

Definition 4.2
A subset \( A \) of an ideal space \( (X, \tau, I) \) is said to be I-R*
closed if \( r^*_I \text{cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is regular
semi open.

Definition 4.3
A subset \( A \) is called I-R*-open if \( X \setminus A \) is I-
R*-closed.

Result 4.4
The finite union of two I-R*-closed sets is I-R*
closed set.

Proof: Let \( A \) and \( B \) be two I-R*-closed sets in \( X \). Let \( U \) be
regular semi open in \( X \). We have \( r^*_I \text{cl}(A) \subseteq U \), whenever
\( A \subseteq U \) and \( U \) is regular semi open and \( r^*_I \text{cl}(B) \subseteq U \), whenever
\( B \subseteq U \) and \( U \) is regular semi open. Let \( A \cup B \subseteq U \).
Hence \( r^*_I \text{cl}(A \cup B) \subseteq U \) whenever \( A \cup B \subseteq U \)
and \( U \) is regular semi open. Therefore \( A \cup B \subseteq \text{I-R*}
closed set.

Remark 4.5: The intersection of two I-R*-closed sets need not be I-R* closed set.

Example 4.6: Let \( X = \{ a, b, c, d \} \) \( \tau = \{ X, \varnothing, \{a\}, \{b, d\}\} \)
\( I = \{ \varnothing, \{a\}\} \). Then if
\( A = \{ b, d \}, B = \{ a, c, d \}, A \cap B = \{ d \} \) which is not I-R*-closed.

Theorem 4.7: In a topological space \( X \), if \( X \) and \( \varnothing \) are the
only regular semi open sets, then every subset of \( X \) is I-
R*-closed set.

Proof: Let \( X \) be a topological space and \( \{ X, \varnothing \} \) be the
regular semi open sets. Also let \( A \subseteq X \). Suppose \( A \neq \varnothing \), then \( X \) is the only the only regular semi open set containing \( A \) and so \( r^*_I \text{cl}(A) \subseteq X \). Hence \( A \) is I-
R* closed.

Remark 4.8: The converse of the above theorem need
not be true as shown in the following example.

Example 4.9: Let \( X = \{ a, b, c, d \} \) \( \tau = \{ X, \varnothing, \{a\}, \{c\}, \{b, c\}\} \)
\( I = \{ \varnothing, \{a\}\} \). Then all subsets are I-R*-closed and the
regular semi open set is
\( \{ X, \varnothing, \{a\}, \{b, c\} \} \).

Remark 4.10: Finite intersection of I-R* open sets is I-R*
closed set.

Theorem 4.11: Let \( (X, \tau, I) \) be an ideal space and \( A \subseteq X \).
If \( A \) is I-R* closed, then
\( r^*_I \text{cl}(A) \setminus A \) does not contain any nonempty regular semi open set.

Proof: Suppose \( A \) is I-R* closed set in \( (X, \tau, I) \). Also let \( F \) be a regular semi closed set contained in \( r^*_I \text{cl}(A) \). It
implies \( F \subseteq r^*_I \text{cl}(A) \setminus A \subseteq \text{X} \setminus F \) and so \( F \subseteq X \setminus r^*_I \text{cl}(A) \). By hypothesis we have
\( F \subseteq r^*_I \text{cl}(A) \) and so \( F = \varnothing \). Hence \( r^*_I \text{cl}(A) \setminus A \) contains no
nonempty regular semi open set.

Remark 4.12: The converse of the above theorem need
not be true from the following example.

Example 4.13: Let \( X = \{ a, b, c, d \} \) \( \tau = \{ X, \varnothing, \{a\}, \{c\}, \{a, c\}, \{a, b, c, \} \}\)
\( I = \{ \varnothing, \{a\}\} \). Then RSO(\( X \)) = \( \{ X, \varnothing, \{a\}, \{c\}, \{a, b, d\}\} \)
\( \{a, c, \} \{b, d\}, \{c\}\} \). Let \( A = \{ c, r^*_I \text{cl}(A) \setminus A = \{ b, c, d\}\} \setminus \{c\} = \{ b, d\} \). But \( A = \{ c\} \) is not I-R*closed set.

Theorem 4.14: Let \( (X, \tau, I) \) be an ideal topological space
and \( A \subseteq X \) be an I-R* closed set. Then \( A \cup \{ X \setminus (r^*_I \text{cl}(A)) \} \)
is a I-R* closed set in \( (X, \tau, I) \).
Proof: Let A be I-R* closed set in (X, τ, I). Suppose that U is a regular semi-open set such that A ∪ (X \ (r_τ^{**} cl(A))) ⊂ U. We have

\[ X \setminus U \subset X \setminus A \cup (X \setminus (r_τ^{**} cl(A))) = (X \setminus A) \cap (r_τ^{**} cl(A)) = r_τ^{**} cl(A) \setminus A \]

Since X \ U is regular semi-open set and A is a I-R* closed set, it follows from theorem 4.11 that X \ U = ∅. Hence X = U. Thus X is the only regular semi-open set containing A∪(X \ (r_τ^{**} cl(A))). Consequently, A ∪ (X \ (r_τ^{**} cl(A))) is I-R* closed set in (X, τ, I).

**Theorem 4.15:** Let (X, τ, I) be an ideal topological space and A ⊂ X be a I-R* closed set. Then r_τ^{**} cl(A) \ A is a I-R* open set in (X, τ, I).

**Proof:**

Since X \ (r_τ^{**} cl(A) \ A) = X \ (r_τ^{**} cl(A) \ A)^c = X \ [r_τ^{**} cl(A)]^c \cup [X \setminus A] = A \cup [X \setminus (r_τ^{**} cl(A))^c] = A \cup [X \setminus r_τ^{**} cl(A)]

By the previous theorem, A ∪ (X \ (r_τ^{**} cl(A) \ A) is I-R* closed set = X \ (r_τ^{**} cl(A) \ A) is I-R* closed set ⇒ r_τ^{**} cl(A) \ A is I-R* open set in (X, τ, I).

**Example 4.16:** Let X = \{a, b, c, d\}, τ = \{X, ϕ, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, c, d\}\} and I = \{ϕ, \{a\}\}. Then I-R* closed sets = \{X, ϕ, \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, c, d\}\}, I-R* open sets = \{X, ϕ, \{a\}, \{c\}, \{d\}, \{c, d\}\} and r_τ^{**} cl(A) \ A ∈ I-R* open sets.

**Theorem 4.17:** Let (X, τ, I) be an ideal topological space. The following properties are equivalent: (i) Each subset of (X, τ, I) is a I-R* closed set (ii) A is pre_τ^* closed set for each regular semi open set A in X.

**Proof:** (1) => (2) Suppose that each subset of (X, τ, I) is a I-R* closed set. Let A be a regular semi-open set. Since A is I-R* closed set, we have cl^*(int A) ⊂ A. Thus A is pre_τ^* closed set.

(2) => (1) Let A be a subset of (X, τ, I) and U be a regular semi-open set such that A ⊂ U. By (2), we have r_τ^{**} cl(A) ⊂ r_τ^{**} cl(U) ⊂ U. Thus A is I-R* closed sets in (X, τ, I).

**Theorem 4.18:** Let (X, τ, I) be an ideal topological space. If A is a I-R* closed set and A ⊂ U ⊂ r_τ^{**} cl(A) then U is a I-R* closed set.

**Proof:** Let U ⊂ K and K be a regular semi-open set in X. Since A ⊂ K and A be a I-R* closed set, then r_τ^{**} cl(A) ⊂ K. Since U ⊂ r_τ^{**} cl(A), then r_τ^{**} cl(U) ⊂ r_τ^{**} cl(A) ⊂ K. Thus, r_τ^{**} cl(U) ⊂ K and hence U is a I-R* closed set.

**Lemma 4.19:** [6] Let A be an open subset of a topological space (X, τ).

(i) If U is regular semi-open set in X, then so is U ∩ A in the subspace (A, τ_A).

(ii) If B ⊂ A is regular semi-open in (A, τ_A) then there exists a regular semi-open set U in (X, τ) such that B = U ∩ A.

**Theorem 4.20:** Let (X, τ, I) be an ideal topological space and U ⊂ A ⊂ X. If A is an open set in X and U is a I-R* closed set in A, then U is I-R* closed set in X.

**Proof:** Let K be a regular semi-open set in X and U ⊂ K. We have U ⊂ K ∩ A. By lemma 4.19, K ∩ A is a regular semi-open set in A. Since U is an I-R* closed set in A, then r_τ^{**} cl(A) \ U ⊂ K ∩ A. Also we have, r_τ^{**} cl(U) ⊂ r_τ^{**} cl(A) \ U ⊂ K ∩ A ⊂ K ⇒ r_τ^{**} cl(U) ⊂ K whenever U ⊂ K and K is a regular semi-open set, thus, U is I-R* closed set in (X, τ, I).

**Theorem 4.21:** Let (X, τ, I) be an ideal topological space and U ⊂ A ⊂ X. If A is a regular semi-open set in X and U is an I-R* closed set in X, then U is I-R* closed set in A.

**Proof:** Let U ⊂ K and K be a regular semi-open set in A. By lemma 4.21 there exist a regular semi-open set L in X such that K = L ∩ A. Since U is a I-R* closed set in X, then r_τ^{**} cl(U) ⊂ K. Also we have r_τ^{**} cl(A) \ U = r_τ^{**} cl(U) \ A ⊂ K ∩ A = K.

Thus r_τ^{**} cl(A) \ U ⊂ K. Hence U is I-R* closed set in A.

5. WEAKLY R*-I-CLOSED SETS

**Definition 5.1:** A subset A of an ideal space (X, τ, I) is said to be W-R*-I closed if (intA)^* ⊂ U whenever A ⊂ U and U is regular semi open set in X.

**Definition 5.2:** A subset A of an ideal space (X, τ, I) is said to be W-R*-I open set if X/A is W-R*-I closed set.

**Theorem 5.3:** Let (X, τ, I) be an ideal topological space and A ⊂ X. Then the following properties are equivalent.
1. A is W-R*-I closed set
2. cl*(int(A)) \subset U whenever A \subset U and U is regular semi open in X.

Proof: If A \subset U, then let A be a W-R*-I closed set in (X, \tau, I). Suppose that A \subset U and U is regular semi open in X. We know (int(A))^* \subset U and that int(A) \subset A \subset U. Hence we have (int(A))^* \cup int(A) \subset U which implies cl*(int(A)) \subset U.

2. \Rightarrow 1 Let cl*(int(A)) \subset U whenever A \subset U and U is regular semi open in X. It implies (int(A))^* \cup int(A) \subset U.

That is (int(A))^* \subset U whenever A \subset U and U is regular semi open. Hence A is W-R*-I closed set.

Theorem 5.4:
Let (X, \tau, I) be an ideal space and A \subset X. If A is open, regular semi open and W-R*-I closed, then A is \tau*-closed.
Proof: Let A be open, regular semi open and W-R*-I closed in (X, \tau, I). Since A is open. Hence cl*(A) = cl*(int(A)) \subset A. Thus A \subset A and hence A is \tau*-closed.

Theorem 5.5 Let (X, \tau, I) be an ideal space and A \subset X. If A is W-R*-I closed, then (int(A))^* \setminus A contains no nonempty regular semi open set.
Proof: Let A be W-R*-I closed and suppose that F is regular semi open set such that F \subset (int(A))^* \setminus A. Since A is W-R*-I closed, \forall F is regular open and A \subset \forall F, then (int(A))^* \subset \forall F. We have \forall F \subset (int(A))^* \cap (int(A))^* = \varnothing. Thus (int(A))^* \setminus A contains no nonempty regular semi open set.

Theorem 5.6:
Let (X, \tau, I) be an ideal space and A \subset X. If A is W-R*-I closed set, then cl*(int(A)) \setminus A contains no nonempty regular semi closed set.
Proof: Suppose U is a regular semi closed set such that U \subset cl*(int(A)) \setminus A. But cl*(int(A)) \setminus A = (int(A))^* \setminus (int(A)). The result follows from theorem 5.5.

Remark 5.7: The converse of the above theorem is not true in general as shown in the following example.

Example 5.8: Let X = \{a, b, c, d\}, \tau = (X, \varnothing, \{a\}, \{a, b, c\}), I=\{\varnothing, \{a\}\}. Let A = \{a\}, then cl*(int(A)) \setminus A does not contain any non empty regular semi open set but A is not W-R*-I closed set.

Theorem 5.9:
Let A be a W-R*-I closed set in an ideal space X such that A \subset \subset cl*(int(A)), then B is also a W-R*-I closed set.
Proof: Let U be a regular semi open set of X, such that B \subset U. Then A \subset B \subset U. Since A is W-R*-I closed set, cl*(int(A)) \subset U. Since A \subset B \subset cl*(int(A)) \subset U, it implies cl*(int(B)) \subset cl*(int(A)) \subset U. Hence B is a W-R*-I closed set.

Corollary 5.10: Let (X, \tau, I) be an ideal topological space. If G is a W-R*-I closed set and an open set, then cl*(G) is a W-R*-I closed set.
Proof: Let G be open and W-R*-I closed in (X, \tau, I). We have G \subset cl*(G) \subset cl*(int(G)). Hence by theorem 5.9, cl*(G) is a W-R*-I closed set.

Remark 5.11: (1) The intersection of two W-R*-I closed sets in an ideal topological space need not be a W-R*-I closed set.
(2) The union of two W-R*-I closed sets in an ideal topological space need not be a W-R*-I closed set.

Example 5.12: Let X= \{a, b, c, d\}, \tau = (X, \varnothing, \{a\}, \{c, d\}, \{a, c, d\})
I=\{\varnothing, \{a\}\}. A=\{a, c, d\} and B=\{b, c, d\} are W-R*-I closed sets but A \cup B=\{c, d\} is not an W-R*-I closed set.

Example 5.13: Let X= \{a, b, c, d\}, \tau = (X, \varnothing, \{a\}, \{c, d\}, \{a, c, d\})
I=\{\varnothing, \{a\}\}. \{c\} and \{d\} are W-R*-I closed sets but A \cup B=\{c, d\} is not an W-R*-I closed set.

Theorem 14: Let (X, \tau, I) be an ideal space and A \subset X. If A is nowhere dense in X, then A is a W-R*-I closed set.
Proof: Let A be a nowhere dense set in X. Since int(A) \subset cl*(int(A))= \varnothing, then int(A)= \varnothing. Hence cl*(int(A)) = \varnothing. Thus A is a W-R*-I closed set.

Remark 5.15: The converse of the theorem need not be true as shown in the following example.

Example 5.16: Let X= \{a, b, c\}, \tau = (X, \varnothing, \{a\}, \{b\}, \{a, b\})
I=\{\varnothing, \{a\}\}. Let A=\{b, c\}. Then A is W-R*-I closed set but it is not a nowhere dense set.

Theorem 5.17: Let (X, \tau, I) be an ideal space and H \subset G \subset X. If G is an open set in X and H is a W-R*-I closed set in G, then H is a W-R*-I closed set in X.
Proof: Let K be a regular semi open set in X and H \subset K. We have H \subset K \cap G. By Lemma 4.19 K \cap G is a regular semi open set in G. Since H is a W-R*-I closed set in G, cl*(Intc (H)) \subset K \cap G. Also, cl*(Int (H)) \subset cl*(Intc (H)) \subset K \cap G \subset K. Hence cl*(Int (H)) \subset K. Thus H is a W-R*-I closed set in X.

Theorem 5.18: Let (X, \tau, I) be an ideal space and H \subset G \subset X. If G is a regular semi open set in X and H is a W-R*-I closed set in G, then H is a W-R*-I closed set in G.
Proof: Let H \subset K and K be a regular semi open set in G. By Lemma 4.16 there exist a regular semi open set L in X such that K= L \cap G. Since H is W-R*-I closed set in G, cl*(Intc (H)) \subset K. Also we have cl*(Intc (H)) = cl*(Int (H)) \subset G \subset
Let $(X, \tau, I)$ be an ideal space and $G \subseteq X$. If $G$ is a $W$-R*-I closed set, then $\text{Int}^*(G)$ is a $W$-R*-I closed set.

**Theorem 5.19:** Let $(X, \tau, I)$ be an ideal space and $G \subseteq X$. If $G$ is a $W$-R*-I closed set, the following properties are equivalent:
1. $G$ is pre-$\tau$-closed,
2. $\text{Cl}^*(\text{int}(G)) \setminus G$ is regular semi closed,
3. $(\text{Int}(G))^* \setminus G$ is regular semi closed.

**Proof:**
1. Let $G$ be pre-$\tau$-closed. We have $\text{Cl}^*(\text{int}(G)) \subseteq G$. Then $\text{Cl}^*(\text{int}(G)) \setminus G = \varnothing$. Therefore $\text{Cl}^*(\text{int}(G)) \setminus G$ is regular semi closed.
2. Let $\text{Cl}^*(\text{int}(G)) \setminus G$ be regular semi closed. Since $G$ is a $W$-R*-I closed set, then by theorem 5.6, $\text{Cl}^*(\text{int}(G)) \setminus G = \varnothing$. Hence we have $\text{Cl}^*(\text{int}(G)) \subseteq G$. Thus $G$ is pre-$\tau$-closed.

2 $\iff$ 3: It follows easily since $\text{Cl}^*(\text{int}(G)) \setminus G = (\text{Int}(G))^*$.

**Theorem 5.20:** Let $(X, \tau, I)$ be an ideal space and $G \subseteq X$. Then $G$ is a $W$-R*-I open set if and only if $H \subseteq \text{int}^*\text{cl}(G)$ whenever $H \subseteq G$ and $H$ is regular semi closed set.

**Proof:**
Let $H$ be regular semi closed set in $X$ and $H \subseteq G$. It follows that $X \setminus H$ is regular semi open and $X \setminus G \subseteq X \setminus H$. Since $X \setminus G$ is a $W$-R*-I closed set, $\text{Cl}^*(\text{int}(X \setminus G)) \subseteq X \setminus H$. We have $X \setminus \text{int}^*\text{cl}(G) \subseteq X \setminus H$. Thus $H \subseteq \text{int}^*\text{cl}(G)$.

Conversely, let $K$ be a regular semi open set in $X$ and $X \setminus K \subseteq X$. Since $X \setminus K$ is a regular semi closed set such that $X \setminus K \subseteq G$, then $X \setminus K \subseteq \text{int}^*\text{cl}(G)$. We have $X \setminus \text{int}^*\text{cl}(G) \subseteq \text{Cl}^*(\text{int}(X \setminus G)) \subseteq X \setminus K$. Thus $X \setminus G$ is a $W$-R*-I closed set. Hence $G$ is $W$-R*-I open set in $(X, \tau, I)$.

**Theorem 5.21:** Let $(X, \tau, I)$ be an ideal space and $G \subseteq X$. If $G$ is a $W$-R*-I closed set, then $\text{Cl}^*(\text{Int}(G)) \setminus G$ is a $W$-R*-I open set in $(X, \tau, I)$.

**Proof:**
Let $G$ be a $W$-R*-I closed set in $(X, \tau, I)$. Suppose $H$ is a regular semi closed set such that $H \subseteq \text{Cl}^*(\text{Int}(G)) \setminus G$. Since $G$ is a $W$-R*-I closed set, it follows from theorem 5.6 that $H = \varnothing$. Thus, we have $H \subseteq \text{Cl}^*(\text{Cl}(\text{Int}(G)) \setminus G)$. It follows from theorem 5.20 that $\text{Cl}^*(\text{Int}(G)) \setminus G$ is a $W$-R*-I open set in $(X, \tau, I)$.

**Theorem 5.22:** Let $(X, \tau, I)$ be an ideal topological space. If $G$ is a $W$-R*-I open set in $(X, \tau, I)$ and $\text{Int}^*(\text{cl}(G)) \setminus G \subseteq H \subseteq G$, then $H$ is a $W$-R*-I open set.

**Proof:**
Let $G$ be a $W$-R*-I open set in $(X, \tau, I)$ and $\text{Int}^*(\text{cl}(G)) \setminus H \subseteq G$. Also let $K$ be regular semi closed. Since $G$ is a $W$-R*-I open set, from theorem 5.20 $K \subseteq \text{Int}^*(\text{cl}(G)) \subseteq \text{Int}^*(\text{cl}(H))$. Hence by theorem 5.20 $H$ is $W$-R*-I open set.

**Example 5.26:** Let $X = \{a, b, c, d\}$
$\tau = \{X, \varnothing, \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, c, d\}\}$
$\{\varnothing, \{a\}\}$

$R^*$-I closed sets are
$\{X, \varnothing, \{a, b\}, \{a, c\}, \{a, d\}, \{a, c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{a, b, c, d\}\}$

$R^*$-closed sets are
$\{X, \varnothing, \{a\}, \{a, c\}, \{a, d\}, \{a, c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{a, b, c, d\}\}$

$W$-R*-I closed sets are
$\{X, \varnothing, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{a, b, c, d\}\}$. 

**Corollary 5.23:** Let $(X, \tau, I)$ be an ideal topological space and $G \subseteq X$. If $G$ is a $W$-R*-I open set in $(X, \tau, I)$ and closed set, then $\text{Int}^*(G)$ is a $W$-R*-I open set.

**Proof:**
Let $G$ be a $W$-R*-I open set and closed set in $(X, \tau, I)$. Then $\text{Int}^*(\text{Cl}(G)) \subseteq \text{Int}^*(G) \subseteq G$. Thus by theorem 5.22, $\text{Int}^*(G)$ is a $W$-R*-I open set in $(X, \tau, I)$.
R*-closed sets are
\{X, \emptyset, \{d\}, \{a\}, \{a,d\}, \{a,b\}, \{a,b,d\}, \{a,c\}, \{a,b,c\}, \{b\}, \{b,d\}, \{c\}, \{a,c,d\}, \{b,c,d\}\}.

W-R*-I closed sets do not imply neither I-R* closed sets nor R*-I closed sets. R*-I closed set does not imply R*-I closed. W-R*-I closed sets does not imply R* closed sets.

I-R* closed sets and R*-I closed sets are independent with R* closed sets.

Example 5.27: Let X = \{a,b,c,d\} \tau =\{X, \emptyset, \{b\}, \{d\}\}
I = \{\emptyset, \{a\}\}.
W-R*-I closed sets are
\{X, \emptyset, \{a\}, \{a,c\}, \{b\}, \{c\}, \{a,b\}, \{a,b,d\}, \{a,c\}, \{a,b,c\}, \{b\}, \{b,c\}\}.

R*-closed sets are
\{X, \emptyset, \{a\}, \{a,c\}, \{b\}, \{c\}, \{a,b\}, \{a,b,c\}, \{a,b,d\}, \{a,c\}, \{a,b,c\}, \{b\}, \{b,c\}\}
W-R*-I closed sets does not imply R*-closed sets.

References
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