Analysis of Natural Frequencies for Cantilever Beam with I- and T- Section Using Ansys

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Abstract - ***Modal analysis is a process to determine the vibration characteristics (natural frequencies and mode shapes) of a structure or a machine component while it is being designed. It has become a major alternative to provide a helpful contribution in understanding control of many vibration phenomena which encountered in practice. In this work we compared the natural frequency for different material having same I and T cross- sectional beam. The cantilever beam is designed and analyzed in ANSYS. The cantilever beam which is fixed at one end is vibrated to obtain the natural frequency, mode shapes and deflection with different loads.***

Key Words: *I-Section, T-Section, Mode Shapes, Natural Frequency*

1. INTRODUCTION

In engineering field vibration behavior of an element plays a key role without which it is incomplete. Resonance is a key aspect in dynamic analysis, which is the frequency of any system matches with the natural frequency of the system which may lead to catastrophes or system failure. Modal analysis has become a major alternative to provide a helpful contribution in understanding control of many vibration phenomena which encountered in practice [1].
A new finite element model for laminated composite beams. The model includes sufficient degrees of freedom to allow the cross-sections of each lamina to deform into a shape which includes up through cubic terms in thickness co-ordinate. The element consequently admits shear deformation up through quadratic terms for each lamina but not interfacial slip or delamination [2]. Higher order shear deformation theory is used for the analysis of composite beams. Nine nodes are parametric elements are used in the analysis. Natural frequencies of composite beam are compared for different stacking sequences, different (l/h) ratios and different boundary conditions. They had shown that natural frequency decreases with an increase in ply angle and a decrease in (l/h) ratio [3].The symbolic computation technique to analyze the free vibration of generally layered composite beam on the basis of a first-order shear deformation theory. The model used considering the effect of poisson effect, coupled extensional, bending and torsional deformations as well as rotary inertia [4]. It has investigated the free vibration of axially laminated composite Timoshenko beams using dynamic stiffness matrix method. This is accomplished by developing an exact dynamic stiffness matrix of a composite beam with the effects of axial force, shear deformation and rotatory inertia taken into account. The effects of axial force, shear deformation and rotator inertia on the natural frequencies are demonstrated. The theory developed has applications to composite wings and helicopter blades [5].
A finite element model to investigate the natural frequencies and mode shapes of the laminated composite beams. The FE model needed all lamina had the same lateral displacement at a typical cross-section, but allowed each lamina to rotate to a different amount from the other. The transverse shear deformations were included [6]. The effects of the location and depth of the cracks, and the volume fraction and orientation of the fibers on the natural frequencies and mode shapes of the beam with transverse non-propagating open cracks, were explored. The results of the study leaded to conclusions that, presented method was adequate for the vibration analysis of cracked cantilever composite beams, and by using the drop in the natural frequencies and the change in the mode shapes, the presence and nature of cracks in a structure can be detected [7]. They had done free vibration analysis of a cross-ply laminated composite beam on Pasternak Foundation. The model is designed in such a way that it can be used for single-stepped cross section. For the first time to-date, the same analysis was conducted for a single-stepped LCB on Pasternak foundation. Stiffness and mass matrices of a cross-ply LCB on Pasternak foundation using the energy method are computed [9]. The cracks can be present in structures due to their limited fatigue strengths or due to the manufacturing processes. These cracks open for a part of the cycle and close when the vibration reverses its direction. These cracks will grow over time, as the load reversals continue, and may reach a point where they pose a threat to the integrity of the structure. As a result, all such structures must be carefully maintained and more generally, SHM denotes a reliable system with the ability

to detect and interpret adverse "change" in a structure due to damage or normal operation. [10]. The finite beam element was formulated using the composite element method with a one-member–one-element configuration with cracks where the interaction effect between cracks in the same element was automatically included. The accuracy and convergence speed of the proposed model in computation were compared with existing models and experimental results. [11].The effects of crack depth and location, fiber orientation, and fiber volume fraction on the flexibility and consequently on natural frequency and mode shapes for cracked fiber-reinforced composite beams are investigated [12].

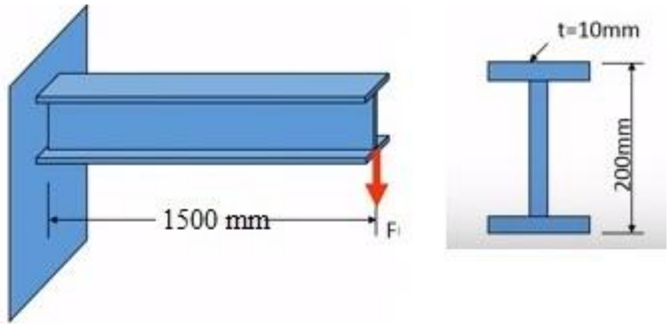
Vibration is a mechanical oscillation about a reference position. Any system has certain characteristics to be fulfilled before it will vibrate. To put in simple words, every system has a stable position in which all forces are equivalent and when this equilibrium is disturbed, the system will try to regain its stable position To remain stable, structure exhibits vibration at different magnitude when excited, the degree of vibration varies from point to point (node to node), due to the variation of dynamic responses of the structure and the external forces applied Therefore, vibration may also be described as the physical manifestation of the interchange between kinetic and potential energy [13].

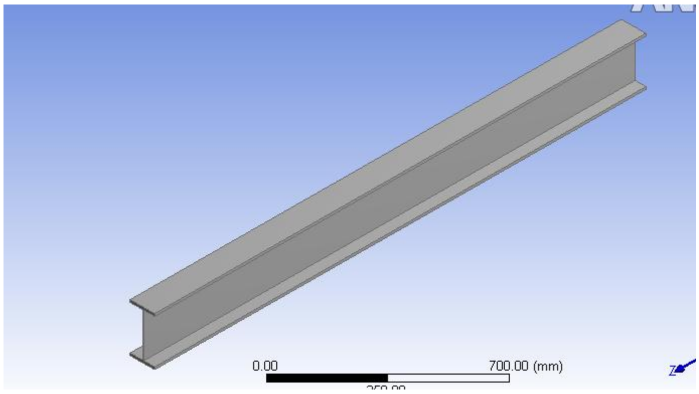
The mechanical properties of aluminum and fiber (Nylon and Glass fiber reinforcement plastic) are measured a universal testing machine. The three-dimensional finite element models of composite beam with and without cracks are constructed and then computational modal analysis on ANSYS-14 is then performed to generate natural frequencies and mode shapes [14]

They considered geometric non-linearity and have solved the expression using Variation Iteration Method (VIM). Also the different nonlinear frequencies have been considered for different shapes of modes [15].

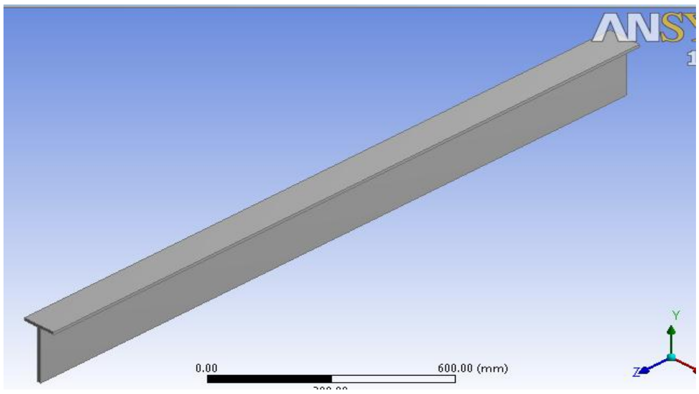
2. MATHEMATICAL MODEL

In the present case I and T section length of 1500 mm long and having the cross-sectional area of as shown in figure 1 is considered. One end of the beam is fixed and the system is subjected to vibration. The material which is assumed is structural steel, cast iron and stainless steel.





(a)



(b)

Figure 1:- Mathematical Model of (a) I-Section, (b) T-Section

Beam Specifications:

Table 1:- Dimesion and Properties of a Cantilever beam

Dimension/ Properties	Structural Steel	Grey Cast Iron	Stainless Steel
Length (mm)	1500	1500	1500
Width (mm)	100	100	100
Thickness (mm)	10	10	10
Height (mm)	200	200	200
Young Modulus (MPa)	2x10^5	1.1x10^5	1.93x10^5
Density(Kg/m³)	7850	7200	7750

The dimension and properties of a cantilever beam with I and T cross section has one end fixed and other end free as shown in table-1, and load applied to free end.

3. GOVERNING EQUATION [14]

3.1 Modal Analysis

3.1.1 Damping Matrices

Damping may be introduced into a transient, harmonic, or damped modal analysis as well as a response spectrum. The type of damping allowed depends on the analysis as described in the subsequent sections.

3.1.2 Transient Analysis and Damped Modal Analysis

The damping matrix, [C], may be used in transient and damped modal analyses as well as substructure generation.

In its most general form, the damping matrix is composed of the following components.

$$[C]=\alpha[M]+(\beta+\frac{2}{\Omega}g)[K]+\sum_{i=1}^{N_{ma}}\alpha_i^m[M_i]+\sum_{j=1}^{N_m}[(\beta_j^m+\frac{2}{\Omega}g_j+\frac{1}{\Omega}g_j^E)[K_j]]+\sum_{k=1}^{N_e}[C_k]+\sum_{m=1}^{N_v}\frac{1}{\Omega}[C_m]+\sum_{l=1}^{N_g}[G_l] \quad (1)$$

where: [C] = structure damping matrix, α = mass matrix multiplier, [M] = structure mass matrix, β = stiffness matrix multiplier, [K] = structure stiffness matrix, N_{ma} = number of materials, α_i^m = mass matrix multiplier for material i, $[M_i]$ = portion of structure mass matrix based on material i, N_{mb} = number of materials, $[K_j]$ = portion of structure stiffness matrix based on material j, N_e = number of elements with specified damping, $[C_k]$ = element damping matrix, Ng = number of elements with Coriolis or gyroscopic damping, $[G_l]$ = element Coriolis or gyroscopic damping matrix, β_j^m = stiffness matrix multiplier for material.

3.1.3 Harmonic Analysis

The damping matrix ([C]) used in harmonic analyses is composed of the following components.

$$[C]=\alpha[M]+(\beta+\frac{2}{\Omega}g)[K]+\sum_{i=1}^{N_{ma}}\alpha_i^m[M_i]+\sum_{j=1}^{N_m}[(\beta_j^m+\frac{2}{\Omega}g_j+\frac{1}{\Omega}g_j^E)[K_j]]+\sum_{k=1}^{N_e}[C_k]+\sum_{m=1}^{N_v}\frac{1}{\Omega}[C_m]+\sum_{l=1}^{N_g}[G_l] \quad (2)$$

The input exciting frequency, Ω, is defined in the range between Ω_B and Ω_E via

$\Omega_B = 2\pi f_B$, $\Omega_E = 2\pi f_E$, f_B = beginning frequency, f_E = end frequency

Substituting equation (2) into the harmonic response equation of motion and rearranging terms yields

$$[[K]+i\{2g[K]+\sum(2g_j+g_j^E)[K_j]+\sum[C_m]\}+i\Omega[\alpha[M]+\sum\alpha_i^m[M_i]+\beta[K]+\sum\beta_j^m[K_j]+\sum[C_k]+\sum[G_j]]](u_1+iu_2)=F_1-\Omega^2[M] \quad (3)$$

The complex stiffness matrix in the first row of the equation consists of the normal stiffness matrix augmented by the structural damping terms given by g, g_i, g_i^E, and $[C_m]$ which produce an imaginary contribution. Structural damping is independent of the forcing frequency, Ω, and produces a damping force proportional to displacement (or strain). The terms g, g_i, and g_i^E are damping ratios (i.e., the ratio between actual damping and critical damping, not to be confused with modal damping). The second row consists of the usual viscous damping terms and is linearly dependent on the forcing frequency, Ω, and produces forces proportional to velocity.

3.1.4 Mode-Superposition Analysis

The damping matrix is not explicitly computed, but rather the damping is defined directly in terms of a damping ratio ξd. The damping ratio is the ratio between actual damping and critical damping. The damping ratio ζ_i^d for mode i is the combination of

$$\zeta_i^d=\zeta+\zeta_i^m+\frac{\alpha}{2\omega_i}+\frac{\beta}{2}\omega_i \quad (4)$$

ξ = constant modal damping ratio, ζ_i^m = modal damping ratio for mode shape i (see below), ω_i = circular natural frequency associated with mode shape i = $2\pi f_i$, f_i = natural frequency associated with mode shape i, α = mass matrix multiplier

The modal damping ratio ζ_i^m can be defined for each mode directly (undamped modal analyses only).

Alternatively, for the case where multiple materials are present whose damping ratios are different, an effective mode-dependent damping ratio ζ_i^m can be defined in the modal analysis if material-dependent damping is defined and the element results are calculated. This effective damping ratio is computed from the ratio of the strain energy in each material in each mode using,

$$\zeta_i^m = \frac{\sum_{j=1}^{N_m} \beta^m{}_j E_j{}^s}{\sum_{j=1}^{N_m} E_j{}^s}$$

4. Result and Analysis

4.1 Natural frequency and deflection on various loads

In order to analyze the natural frequency and deflection of I and T cross sectional cantilever beam, first we calculate the mechanical properties of structural steel, cast iron and stainless steel. These mechanical properties are fed into ANSYS-14 to calculate the deflection and natural frequency for cantilever beam.

Table 2:- Deflection and Natural frequency of Structural Steel for T- Section

T-Section Steel		
Load (KN)	Deflection(mm)	Natural frequency
10	4.73	19.028
20	9.46	54.141
30	14.19	79.988
40	18.92	97.096
50	23.65	171.64
60	28.38	222.82

Table 3:- Deflection and Natural frequency of Cast Iron for T-Section

T-Section Cast Iron		
Load (KN)	Deflection(mm)	Natural frequency
10	8.6	14.377
20	17.2	43.979
30	25.8	61.935
40	34.4	77.247
50	43	138.98
60	51.6	173.61

Table 4:- Deflection and Natural frequency of Stainless steel for T-section

T-Section Stainless Steel		
Load (KN)	Deflection(mm)	Natural frequency
10	4.9	18.78
20	9.8	53.52
30	14.7	79.082
40	19.6	95.923
50	24.5	169.76
60	29.4	219.98

Table 5:- Deflection and Natural frequency of Structural Steel for I- Section

I-Section Structural Steel		
Load(KN)	Deflection(mm)	Natural frequency
10	2.59	26.341
20	5.18	56.716
30	7.77	94.068
40	10.36	161.14
50	12.95	230.46
60	15.54	415.08

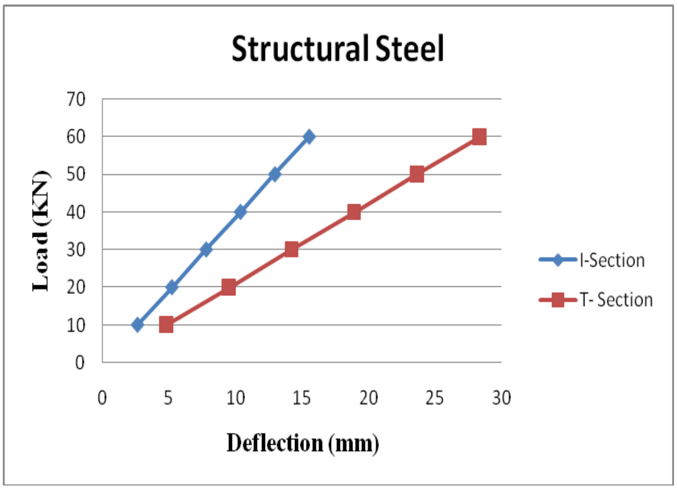
Table 6:- Deflection and Natural frequency of Cast Iron for I-Section

I-Section Cast Iron		
Load (KN)	Deflection(mm)	Natural frequency
10	4.751	20.372
20	9.5	44.045
30	14.25	72.857
40	19	124.75
50	23.75	178.88
60	28.5	321.16

Table 7:- Deflection and Natural frequency of Stainless steel for I-section

I-Section Stainless Steel		
Load (KN)	Deflection(mm)	Natural frequency
10	2.687	26.043
20	5.374	55.939
30	8.061	92.994
40	10.74	159.32
50	13.43	227.59
60	16.12	410.54

As the load on the cantilever beam increases, the deflection also increases. The deflection is minimum at 10N of I-section for structural steel whereas deflection is maximum at 50 KN of T-section for Cast iron as shown in table 2-7.



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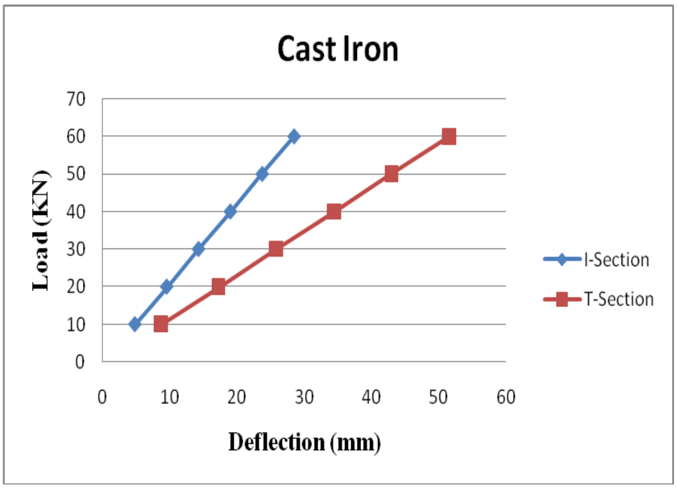

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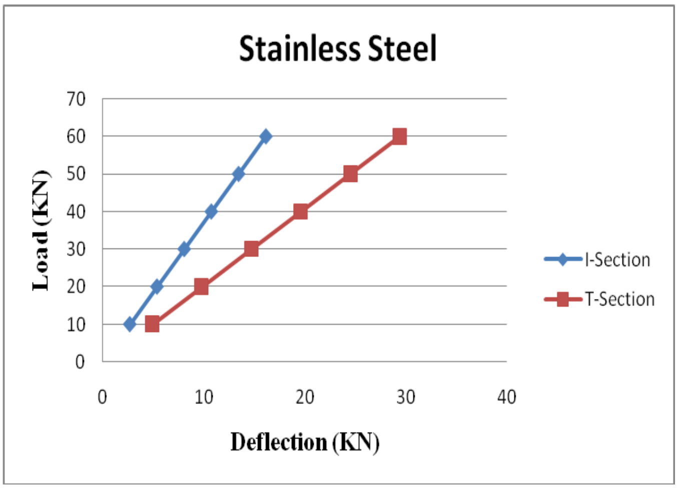

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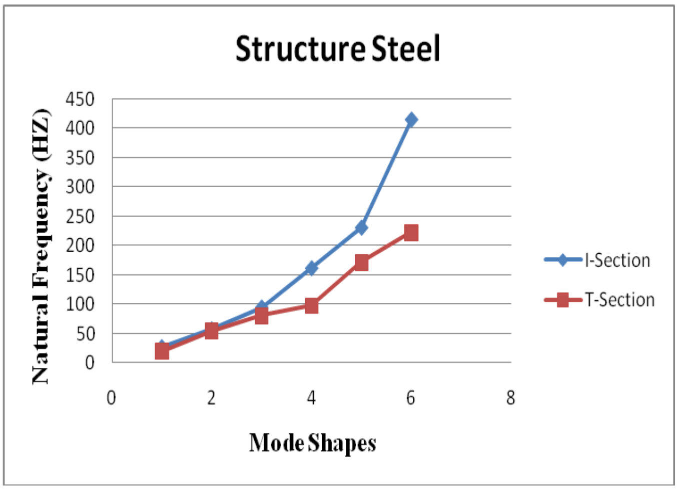
Figure 2:- Comparison between deflection for T and I section (a) Structural Steel (b) cast Iron, (c) Stainless Steel



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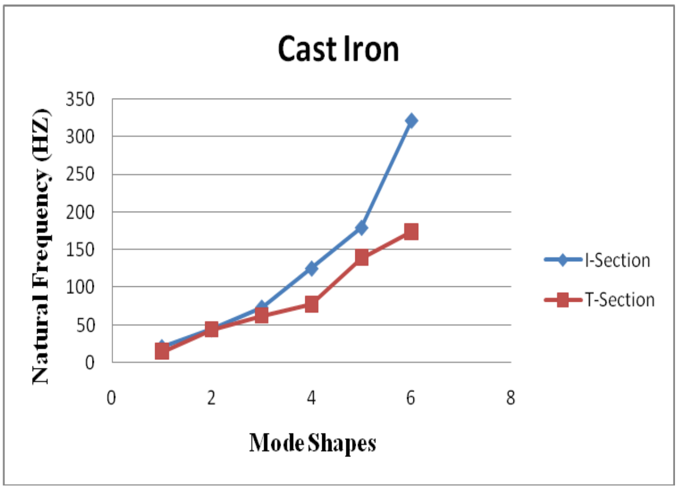

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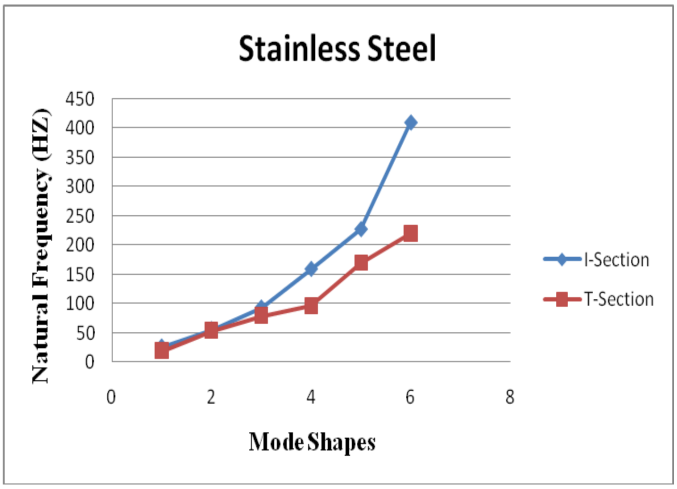

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Figure 3:- Comparison between natural frequency for T and I section (a) Structural Steel (b) cast Iron, (c) Stainless Steel

4.2 Natural frequency for different mode shapes for T-section

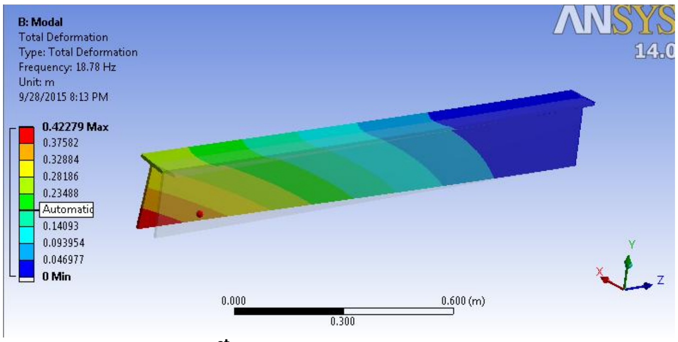

Figure 4:- 1st mode of vibration for T-section

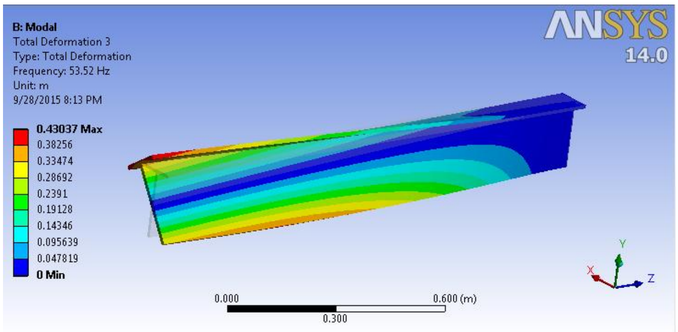

Figure 5:- 2nd mode of vibration for T-section

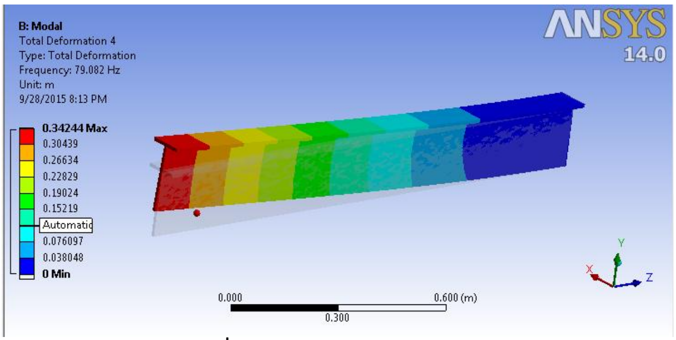

Figure 6:- 3rd mode of vibration for T-section

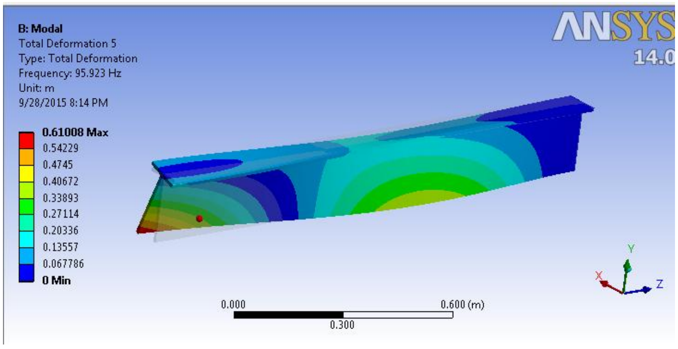

Figure 7:- 4th mode of vibration for T-section

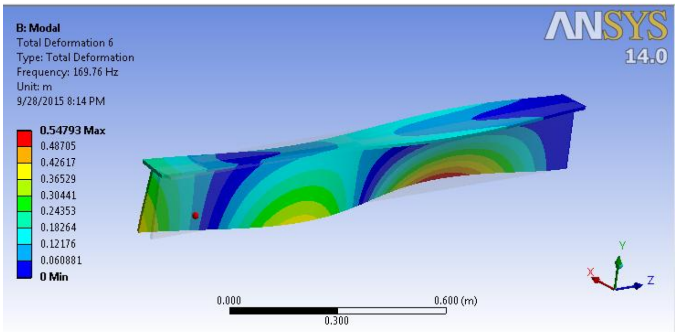

Figure 8:- 5th mode of vibration for T-section

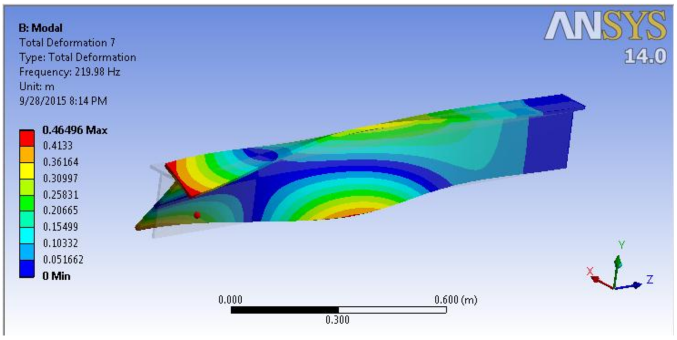

Figure 9:- 6th mode of vibration for T-section

The first mode of vibration for T- section is a bending mode in horizontal direction. In this mode shape, the frequency is 18.78 Hz. The beam is tending to bend about the root section's minimum moment of inertia. The second modes of vibration is also bending mode having natural frequency 53.52 Hz. The third mode of vibration is also bending mode in vertical direction. The frequency of the third mode shape is 79.08 Hz. The fourth mode of vibration is twisting about the root, t he frequency is affected by tip rotational moment of inertia. The frequency of the fifth mode is 95.92 Hz. The fifth mode of vibration is bending and twisting mode in horizontal with frequency 169.76 Hz. The sixth mode shape is twisting with natural frequency 219.98 Hz

4.3 Natural frequency for different mode shapes for I-section

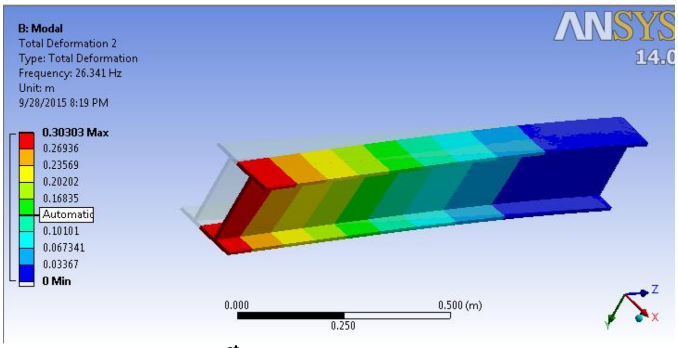

Figure 10:- 1st mode of vibration for I-section

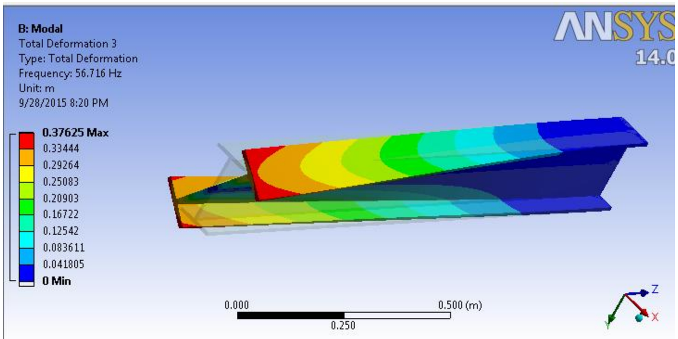

Figure 11:- 2nd mode of vibration for I-section

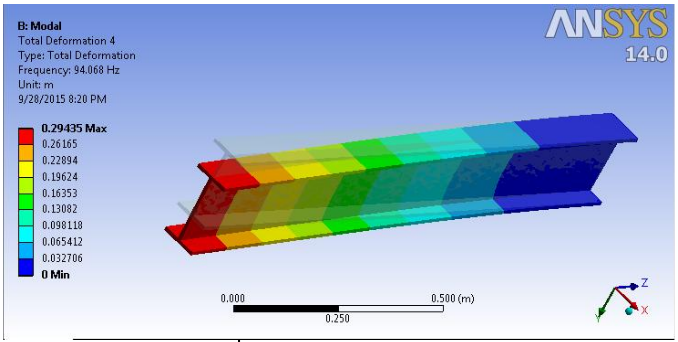

Figure 12:- 3rd mode of vibration for I-section

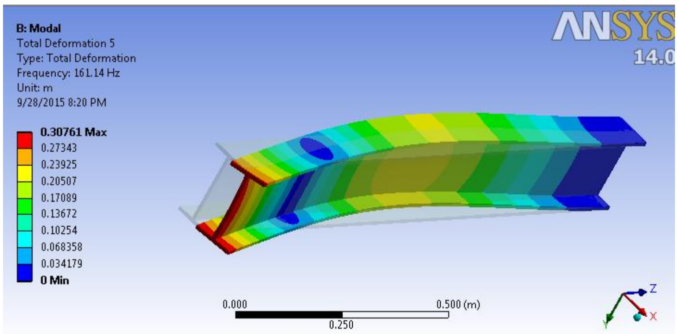

Figure 13:- 4th mode of vibration for I-section

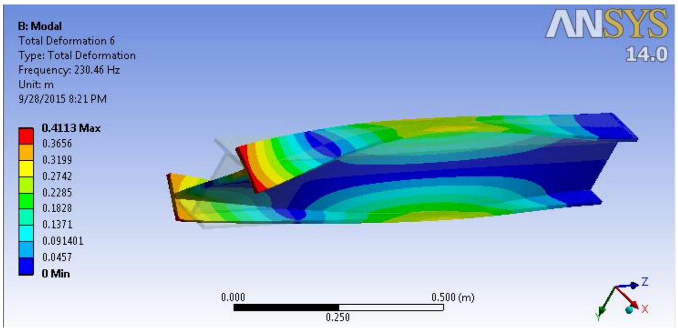

Figure 14:- 5th mode of vibration for I-section

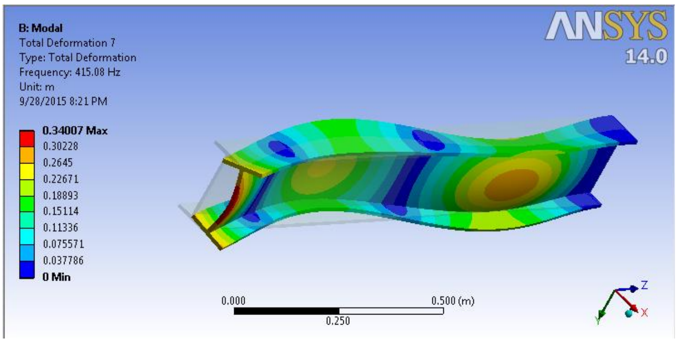

Figure 15:- 6th mode of vibration for I-section

The first mode of vibration for I- section is a bending mode in horizontal direction. In this mode shape, the frequency is 26.34 Hz. The beam is tending to bend about the root section's minimum moment of inertia. The second modes of vibration is also bending mode having natural frequency 56.71 Hz. The third mode of vibration is also bending mode in vertical direction. The frequency of the third mode shape is 94.06 Hz. The fourth mode of vibration is twisting about the root, t he frequency is affected by tip rotational moment of inertia. The frequency of the fifth mode is 161.14 Hz. The fifth mode of vibration is bending and twisting mode in horizontal with frequency 230.46 Hz. The sixth mode shape is twisting with natural frequency 415.08.

5. Conclusion:

The following conclusions are as given below.

- The deflection of I section is less than the T-section
- Maximum deflection is seen in T-section for cast iron i.e. 51.6 mm for 50 KN load.
- The Minimum deflection is seen in I-section structural steel i.e. 2.59 mm for 10 KN load.
- The minimum natural frequency is obtained in T-section for cast iron i.e. 14.3 Hz, whereas the maximum natural frequency is obtained in I section for structural steel i.e. 415.08 Hz

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