Digital Image Reconstruction through its projected views by using Discrete Fourier Slice Theorem

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Abstract - ***The Discrete Fourier Transform (DFT) give a way of solution for many problems commonly possessing missing or unmeasured frequency data. In this paper, DFT is used as a fast and robust method for recovering missing slices of the image is described. The Discrete Fourier Slice Theorem (DFST) is referred as the Discrete Radon Transform. The Discrete Radon Transform uses the projection slices are originated from the exact partitioning of DFT space. The DFST is a vital tool for image processing or signal processing. The DFST is used as a exact reconstruction of an image through its projected views. The proposed algorithm gives the computational complexity as the two-dimensional fast fourier transform and resistant to the significant levels of noise. A mapping is allowed to construct an aperiodic projections are transformed into a periodic image boundary conditions. Every remapped projection forms as a one dimensional slice of the two dimensional discrete fourier transform. The projection angles are chosen so that the set of remapped one dimensional slices exactly tile the two dimensional DFT space. This permits the exact reconstruction of image via inverse DFT. The improvement in noise suppression is essential in the reconstruction of images are to be resistant against when increases the computational complexity. This paper describes a significant application to fast and robust reconstruction of an image. It is derived from the sets of projection slices are obtained from a limited range of projection angles.***

Key Words: *Discrete Fourier Slice Theorem, Image Reconstruction, Image Watermarking, Mojette Transform, Filtered Back Projection*

1. INTRODUCTION

The DFT is used as a important tool in Digital Signal Processing. It allows the system to be analyzed in the frequency domain. It deals with a finite amount of data, it can be implemented in computers by using numerical algorithms. The DFT provides the solutions to the problems ranging from filtering and convolution, where it is used as a strategy to recover an object from its discrete projected views or projections [1].

This paper is structured as follows. In section II, presents literature survey. In section III, presents the proposed method. In section IV, presents the Discrete Fourier Slice Theorem. In section V, presents the Fast Mojette Reconstruction. In section VI, presents the experimental results of the proposed method shows the reconstruction of an image.

2. LITERATURE SURVEY

There are two methods to reconstruction of an image i.e. Analytical reconstruction and iterative reconstruction. These two Methods are depends on Filtered Back Projection (FBP). Presently, Analytical reconstruction method is used to reconstruction of an image. This method is mainly used in clinical CT scanners for achieving computational efficiency. These two methods are used to reduce noise in a image while maintaining high-contrast resolution. If it is used as excessively, image may tend to change the texture and resulted a low-contrast image. Therefore, a powerful and efficient technique is needed to reconstruct an image.

The DFST provides the way to reconstruct an image by using its projection slices with achieving computational complexity. It is a property of the DFT. The images are viewed through the Fourier transform in three-dimensional form. To recovering a reconstruction of an image, the inverse Fourier transform is used. The paper presents a method of DFST is used to recovering an image from its projected views or slices. This technique both fast and robust in the presence of noise. The projected views are referred in a Mojette Transform (MT) method[2].

3. PROPOSED METHOD

The proposed method allows the discrete projections to be exactly packed within the slices of the DFT, so that the inverse two dimensional DFT is used to reconstruct the image without any additive noise. (see Fig. 1).

This technique is used to reconstruction of an image and reduces the image noise to achieve a computational complexity.

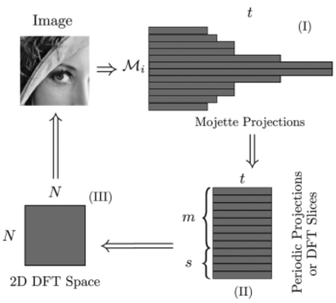

Fig -1: A schematic way with in the fast mojette transform. (I)The projections of a Mojette Transform (II) The Projections are viewed as periodic projections (III) The mapping is needed to inversion of an image on DFT space.

4. DISCRETE FOURIER SLICE THEOREM

The DFST is referred as a finite randon transform. The slices resulting from discrete fourier slice theorem are the one dimensional DFT of periodic projections taken as sums along the lines [3].

$$Y = mx + t \pmod N \qquad (1)$$

$$X = psy + t \pmod N \qquad (2)$$

The slopes of m and s, intercepts the values of t and x, y, m, s, t Є z for a image of size $N= p^n$. Here, p is a prime number which includes the powers of two. The lines of (1) and (2) are equivalently formed by the vectors [1, m] and [ps, 1], i.e. m pixels across and one pixel down or one pixel across and ps pixels down.(see Fig.2).

The lines of (1) and (2) are utilized for the set of slopes.

$$m= \{m: m<N, m Є N_0\} \qquad (3)$$

$$s= \{s: s<N/P, s Є N_0\} \qquad (4)$$

and the set of translates,

$$t=\{t: t<N, t Є N_0\} \qquad (5)$$

Here, N_0 represents whole numbers, i.e. all numbers including with zero. All the elements are fully tiled at least once. It means that, cover all possible coefficients in the NxN DFT space, i.e. a total of N+N/P slices and the DFT space is tiled completely.

Suppose the case n>1, N+N/P projections are required results in a certain amount of over sampling, which is easily and exactly corrected.

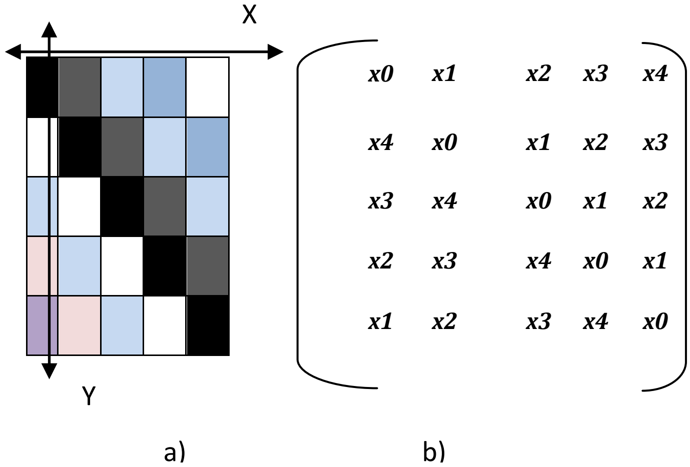

Fig -2: The lines are shows in the geometry of the Discrete Fourier Transform with a slope of a 5 × 5 image. (a) specifies the behavior of the lines in an image lattice and each pixel is translate. (b) specifies the equivalent circulant matrix for the lines.

In most practical cases, only aperiodic discrete projections exist in various signal and image processing applications ranging from data integrity, packet networks via an n-dimensional mojette transform, lossless networking, image compression, scalable multimedia distribution, image In most practical cases, only aperiodic projections exist in various signal and image processing applications ranging from data integrity, packet networks via n-dimensional MT, lossless networking, image compression, scalable multimedia distribution, image coding and digital image watermarking.

Recent methods are used to apply the MT on real data to reduce the number of projections to reconstruct an image is easier and effective. The methods including in reconstruction of an image are conjugate gradient method [4] and a geometric graph approach [5]. The DFST procedure is used to recover the missing slices of the DFT which is exactly tiled.

5. FAST MOJETTE RECONSTRUCTION

The N'xN' image of a object is to exactly tile an NxN DFT space, where N= kN' and k>1 or k=1.

Where k>1, means that a mechanism for reducing the effects of noise using of projections. The projection lines of the MT is non- periodic and parallel lines.

$$\Gamma_{t,\theta p,q} = \begin{cases} t = qy - px & \text{if } q/p >= 0 \\ t = px - qy & \text{of } q/p < 0 \end{cases} \qquad (6)$$

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Here, the projection is taken at angle $\theta_{pq} = \tan^{-1}(q/p)$ with p, q, t Є Z of an object with its convex support. Convex support is not having any singularities in the DFT space.
Consider the size of the image is $N=2^n$. Then, the total number of MT projections are M=N+N/2. These projections can satisfy exact reconstruction requirements. Each mojette projection is mapping with the unique periodic projections of DFT slice to tile its space.

5.1 Finite projection mapping

Suppose, case n=1 in $N= p^n$, the projection $M_{\theta pq}$ of the MT is mapping with the unique periodic projection as,

$$m=PQ^{-1} \text{ (mod N)} \qquad (7)$$

Here Q^{-1} represents the multiplicative inverse of q. suppose the case n>1, the projection $M_{\theta pq}$ maps with the m and s projections. The mojette set is ensuring with a one-to-one correspondence to the periodic projection set. It means the DFT space is filled completely.

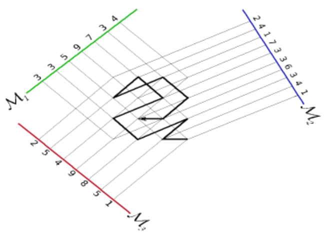
$$2s= P^{-1}Q \text{ (mod N)} \qquad (8)$$

y

x

2	4	0	4
1	3	2	3
1	0	3	1
3	2	4	1

a)



b)

Fig -3: A Mojette Transform example. (a) An image of size 4×4 using the three projections [1, 1], [1,−1] and [1,−2]. (b) Shows the thick lines within the grid show a possible reconstruction path using a corner-based reconstruction method.

5.2 Angle Set

A new mojette discrete angle set is finite, fully tilling and spans the range {0,π]. The mojette projection set has to be constructed so as to fill DFT space completely with as little redundancy may occur. The number of bins B in a mojette projection depends on the angle θ_{pq} as,

$$B= |P| (Q-1) + |Q| (P-1) + 1 \qquad (9)$$

of a rectangular PxQ image.

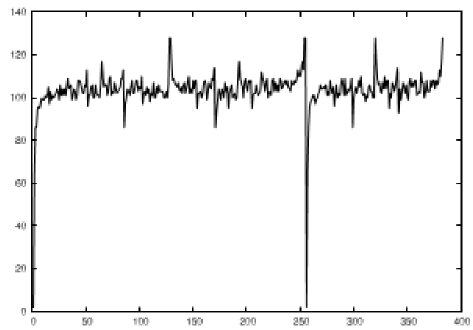
5.3 Finite Conversion

Once the projection set is known for a original image and DFT space, the mojette projections are converted to periodic projections. This can be done by equating equations in (6), as well as (1) and (2), with the mapping of equations (7) and (8).
A mojette translate t_M and a periodic projection translate t_R gets,

$$t_R = \begin{cases} Q^{-1}t_M \text{(mod N), if gcd(p,N)>1} \\ P^{-1}t_M \text{ (mod N), if gcd(q,N)>1} \end{cases} \qquad (10)$$

The conversion can be done before the inversion of the mojette projections, or as the projections are being acquired. Mojette projections are more desirable than the periodic projections and all have the same size.

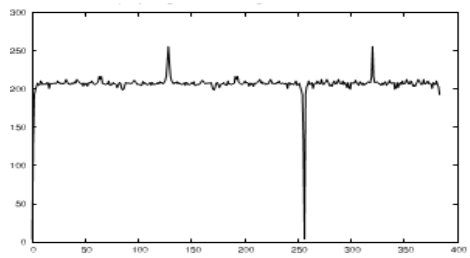
Count



Finite Angle

a) Multiplicity Histogram of the Finite angles for N=256 in one quadrant

Count



Finite Angle

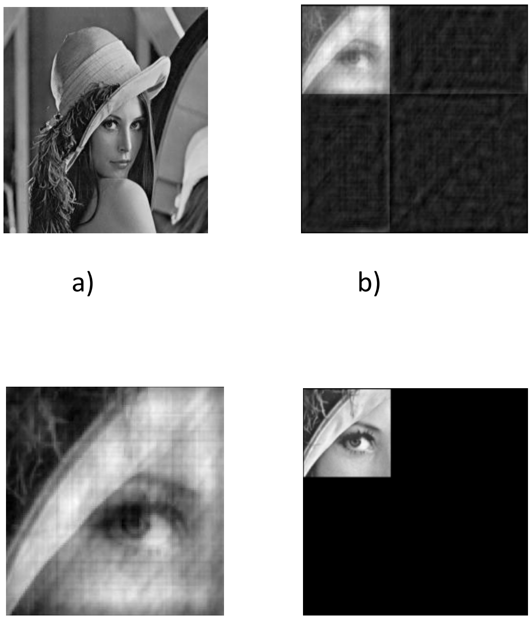
b) Multiplicity Histogram of the ration angle for N=256 in the Half-plane

Fig 4. The multiplicity of the mapping in (a) and (b) shows the finite angle and rational angle sets for N = 256. The value of N is between 0<m < N and here 0<s < N/2.

The mapping defines the number of octants in the half-plane used in the projection set. The projection set is limited, but DFT space covers completely for each finite angle [1,m]. This is shown in graphs in Fig. 4.

6. EXPERIMENTAL RESULTS

Numerical simulations of the proposed method were conducted of a 256x256 images. The Fig (a),(b),(c), and (d) shows an example of a simulated reconstruction of images using DFST algorithm.



a) b)

c) d)

Fig -5: The results of the DFST to reconstruction of an image (a) shows the original image.(b) shows resulting ghosts when (a) is reconstructed.(c) shows cropped version of (b).(d) shows that DFST can recover the reconstructed image exactly.

The reconstruction of images are specified with Gaussian noise present in the mojette projections with in a 256x256 DFT space. The image shows that reconstruction of an image is resistant at the levels of noise which is suitable precision for lossy image and video encoding techniques. The above figures represent the reconstruction of a images without loss of a content in the original image.This solution gives a computational complexity of O(n $logn$) in an exact reconstruction of an image.

7. CONCLUSION AND FUTURE WORK

Image Reconstruction can improve the quality of an image. Reconstruction of an image is used in various levels of application area. Image Reconstruction mainly helps in bio- medical industry. Image Reconstruction techniques are helpful in image analysis- for instance, to enhance ultra sound or MRI images.

The projection slices in image reconstruction suffers with the inherent difficulty. Certainly, the tremendous images in the usage get identical projections repeatedly. But, it is essential to identify the projections of a image. Based on the images used the application area design algorithms that are appropriate to work.

The algorithm that is presented in the paper is developed based on the digital assumption. The DFST is used to reconstruct the image effectively. It is not specified exactly to give correct results even though using the number of projections are small. When simulate the realistic data, the reconstruction of an image from a small number of projection views is may loss content in a image.

Further work on image reconstruction needs to be described in selecting geometries of an images and conducting tomographic experiments on real data to compare the effects of several noise types.

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