Abstract: The optimal energy-efficient design of uplink (UL) for multi-user (MU) multiple-input multiple-output (MIMO) system in a single cell environment is addressed in this paper. Throughput per Joule is the unit for energy efficiency. Radio frequency (RF) transmission power and device electronic circuit power are taken into consideration. The energy efficiency (EE) capacity for UL MU-MIMO is defined and the power allocation for achieving this capacity is studied. Only those user antennas with good spatial channels should be operated, so that it improves the overall network EE. Mobile devices may contain better circuit management capability to turn off circuit operations, when some antennas are not working so that power consumption is optimized. Where users may have improved circuit management capability and turn off part of the circuit operations when some antennas are not used to reduce the circuit power consumption. The problem is non-concave and multiple local maximums may exist. Furthermore, algorithms that converge to the global optimum are developed. The gain in network energy efficiency is demonstrated through simulation results.

Index Terms: Energy efficiency, Multi-user MIMO, Power allocation, SDMA.

I. INTRODUCTION

As the researchers are designing higher capacity wireless links to meet increasing demand from multimedia applications, device power consumption is also increasing. Slow improvement of battery technologies [1] has led to an exponentially increasing gap. Therefore, recent research has focused on energy-efficient (EE) wireless communication techniques [3]. In [4], optimal EE orthogonal frequency-division multiple access (OFDMA) is designed to balance the circuit power consumption as well as the transmission power consumption on all OFDM sub channels.

In a multi-user (MU) scenario, MU multiple-input multiple-output (MIMO) systems can provide a substantial gain in networks by allowing multiple users to communicate in the same frequency and time slots [3]. On the other hand, MIMO has been a key technology for wireless systems. As increasing bandwidth allocation for a user always improves energy efficiency, all sub channels cannot be allocated exclusively to one user in a MU system [5]. MU-MIMO takes the advantage of both high capacity achieved with MIMO processing and the benefits of space-division multiple access (SDMA) and has been accepted by major wireless standards like IEEE 802.16m [5] and 3GPP Long Term Evolution (LTE) [5]. A low-complexity EE and reconfigurable reduced dimension maximum likelihood MIMO detector is proposed in [2]. In addition to energy saving, energy-efficient communications have the benefit of reducing interference to other co-channel users as well as lessening environmental impacts [1]. However there is very limited research studying EE MU-MIMO and its optimal power allocation. This motivates the work in this paper.

The EE design of UL for MU-MIMO systems in a single cell environment is addressed in this paper. The proposed scheme balances the energy consumption of circuit operations and radio frequency (RF) transmissions of all users to achieve the maximum network EE. Assume that all users consume a fixed amount of circuit power in addition to the RF power and demonstrate the existence of a unique globally optimal power allocation that achieves the energy efficiency capacity. And also a one-dimensional iterative algorithm to obtain the optimum is defined. Furthermore energy-efficiency in UL MU-MIMO is studied with improved circuit management to show that some antennas, even with good channel states should be turned off as lot of circuit power is utilized when they are turned on. Low-complexity EE power allocation algorithms are developed based on theoretical analysis. Through simulation results the gain in the network energy efficiency is demonstrated.
II. ENERGY-EFFICIENT MU-MIMO

The EE MU-MIMO has been introduced. Throughout the paper, matrices are shown with capital bold-face letters, vectors with lowercase boldface, and scalars with either upper or lowercase letters without boldface.

Consider a MU-MIMO system, as illustrated in Fig. 1, where one access point (AP) is serving K users that desire best-effort data service.

User i has $k_i$ antennas and $i=1 \leq N$, the received signal at the AP is given by

$$y = H \cdot Q \cdot P \cdot x + n = \sum_{i=1}^{K} H_i \cdot Q_i \cdot P_i \cdot x_i + n$$  \hspace{1cm} (1)

Where $y = [y_1, y_2, ..., y_N]^T$, $x_i = [x_{i1}, x_{i2}, ..., x_{ik_i}]^T$ consists of transmitted signals of User i and $E[x_i] = 1$, where E is the expectation. Here $[\cdot]^T$ is the transpose of a vector. $P_i = \text{diag} \left( \sqrt{P_{i1}}, \sqrt{P_{i2}}, ..., \sqrt{P_{ik_i}} \right)$ is the power allocation matrix of User i. $Q_i$ is the precoding matrix of User i. $\sigma_i^2 I_{N_i}$, where $I_{N_i}$ is the identity matrix of size $N_i$.

$x = [x_1, x_2, ..., x_K]^T$, $P = \text{diag} \{P_1, P_2, ..., P_K\}$, $Q = \text{diag} \{Q_1, Q_2, ..., Q_K\}$, and $H = [H_1, H_2, ..., H_K]$.

With a linear detector, the decision vector for the transmitted symbols is

$$\hat{x} = w \cdot y = w \cdot H \cdot Q \cdot P \cdot x + w \cdot n.$$  \hspace{1cm} (2)

Using singular value decomposition (SVD), the received signal at the AP is

$$H_i = U_i \left[ \begin{array}{c} \Lambda_i \\ 0 \end{array} \right] V_i^H = \left[ \begin{array}{c} \hat{U}_i \hat{U}_i \end{array} \right] \left[ \begin{array}{c} \Lambda_i \\ 0 \end{array} \right] V_i^H = \hat{U}_i \Lambda_i \hat{V}_i^H$$  \hspace{1cm} (3)

Where $U_i$ and $V_i$ are $N \times N$ and $k_i \times k_i$ unitary matrices and $[\cdot]^H$ is the Hermitian transpose. $U_i$ consists of the first $k_i$ columns of $U_i$.

$\Lambda_i = \text{diag} \{\lambda_{i1}, \lambda_{i2}, ..., \lambda_{iki}\}$ where $\lambda_{ii} \geq 0$.

With local channel knowledge $H_i$, User i sets the precoding matrix $Q_i = V_i$. Define

$$U = \left[ \hat{U}_1, \hat{U}_2, ..., \hat{U}_K \right]$$

and $\Lambda = \text{diag} \{\Lambda_1, \Lambda_2, ..., \Lambda_K\}$.

It is easy to see the decision vector at the AP is

$$\hat{x} = w \cdot U \cdot \Lambda \cdot P \cdot x + w \cdot n.$$  \hspace{1cm} (4)

There are many ways of designing the linear receiver $w$. $w = (U^H U)^{-1} U^H$, \hspace{1cm} (5)

Note that the restriction on $\sum_{i=1}^{K} k_i \leq N$ is needed for the existence of the ZF receiver. The decision vector is

$$\hat{x} = \Lambda \cdot P \cdot x + \hat{n},$$  \hspace{1cm} (6)

where $\hat{n} = (U^H U)^{-1} U^H n$, which is also Gaussian distributed with a zero mean and a covariance matrix

$$E[\hat{n}^H \hat{n}] = \sigma^2 \left( (U^H U)^{-1} \right)^H,$$  \hspace{1cm} (7)

with all elements in the diagonal being $\sigma^2$.

From (6), the transmissions of different users are uncoupled. The AP can detect each symbol independently and the achieved signal-to-noise ratio (SNR) of all the symbols for User i is

$$\eta_i = \left[ \frac{P_{i1} \lambda_{i1}}{\sigma^2}, \frac{P_{i2} \lambda_{i2}}{\sigma^2}, ..., \frac{P_{iki} \lambda_{iki}}{\sigma^2} \right]^T$$  \hspace{1cm} (8)

Define $B$ as the system bandwidth. The achievable data rate is determined by

$$R_i = \sum_{k=1}^{k_i} r_{ik}$$  \hspace{1cm} (9)

$$r_{ik} = B \log_2 \left( 1 + \frac{\eta_{ik}}{\Gamma} \right)$$  \hspace{1cm} (10)
where \( \eta_k = \frac{P_k \lambda_k^2}{\sigma^2} \) and \( \Gamma \) is the SNR gap that defines the gap between the channel capacity and a practical coding and modulation scheme, and other implementation factors.

\[
\Gamma = 10^{(9.8+\gamma_m-\gamma_c)/10},
\]

where \( \gamma_m \) is the system design margin and \( \gamma_c \) is the coding gain in dB. Define the overall transmission power of User i to be \( P_{\text{r}} \) such that

\[
P_{\text{r}} = \sum_{k=1}^{i} P_k
\]

where \( \zeta \in [0, 1] \) is the power amplifier efficiency. The overall power consumption of User i will then be

\[
P_\text{P} = P_{\text{r}} + P_{\text{t}}
\]

The optimal transmission power allocation to maximize

\[
\sum_i R_i \Delta t \over \Delta e
\]

which is equivalent to maximizing

\[
U(P) = \sum_i R_i \over \alpha \sum_i P_i + \beta P_{\text{t}}
\]

U is the total number of bits sent per Joule of energy consumption. U is called the energy efficiency of MU-MIMO. The energy efficiency capacity of MU-MIMO is defined as

\[
U^* = \max_P \sum_i R_i \over \alpha \sum_i (P_{\text{r}} + P_{\text{t}}) + \beta P_{\text{t}}
\]

and the optimal energy-efficient power allocation achieving the energy efficiency capacity is

\[
P^* = \arg \max_P \sum_i R_i \over \alpha \sum_i (P_{\text{r}} + P_{\text{t}}) + \beta P_{\text{t}}
\]

III. PRINCIPLES OF ENERGY-EFFICIENT MU-MIMO POWER ALLOCATION

In the following, unique globally optimal power allocation always exists and gives the necessary and sufficient conditions for a power allocation scheme to achieve the energy efficiency capacity. It can be proved that U has the following properties.

Lemma 1. U is strictly quasi-concave in P.

For a strictly quasi-concave function, if a local maximum exists, it is also globally optimal. Hence, a unique globally optimal power allocation always exists.

Theorem 1. There exists a unique globally optimal energy-efficient power allocation \( P^* \) that achieves the energy efficiency capacity, where \( P_k^* \) is given by

\[
P_k^* = \begin{cases} \frac{B_\gamma \lambda_k^2}{\alpha U^* \ln 2} \Gamma \sigma^2 \over \lambda_k, & \text{if } B_\gamma \lambda_k^2 > U^*, \\ 0, & \text{otherwise}, \end{cases}
\]

\[
U^* = U(P^*)
\]

Theorem 1 says that the kth antenna of User i should be used only when the corresponding spatial channel, characterized by \( \lambda_k \), is sufficiently good such that using it improves the overall network energy efficiency. Based on Theorem, we have the following basic properties of power allocation.

Proposition 1. The energy efficiency capacity decreases strictly, while the optimal allocated power on each spatial channel, if nonzero, increases strictly with the circuit power of any user.

The main intuition behind Proposition 1 is that as circuit power increases, higher power should be allocated to achieve higher data rate such that each information bit can be transmitted faster and less circuit energy is consumed. Similarly Proposition 2 follows.

Proposition 2. When receiving power is considered (\( \beta > 0 \)), the energy efficiency capacity decreases strictly, while optimal allocated power on each spatial channel, if nonzero, increases strictly with the receiving power.

IV. A ONE-DIMENSIONAL LOW-COMPLEXITY ALGORITHM

Theorem 1 provides the necessary and sufficient condition for a power allocation to be the unique and globally optimum one. Therefore, an iterative method to search for the optimal \( P^* \) based on the analysis of the optimal power allocation in Theorem 1 was developed.

Define \( p_k(\mu) = \left[ \mu - \frac{\Gamma \sigma^2}{\lambda_k^2} \right] \), where \( \lfloor x \rfloor^+ = \max (x, 0) \), and the corresponding power allocation matrix to be \( P(\mu) \) clearly when

\[
\mu = B_\gamma \sigma^2 \over \alpha U^* \ln 2
\]

Define

\[
f(\mu) = U(P(\mu))
\]

Table-I

Energy-Efficient MU-MIMO Power Allocation

Algorithm Energy-Efficient MU-MIMO Power Allocation
1. \( \mu_1 \leftarrow \min_{i,j} \frac{\Gamma \sigma_i^2}{\lambda_{ik}^2} \)

(* \( \mu^* \) is above \( \min_{i,j} \frac{\Gamma \sigma_i^2}{\lambda_{ik}^2} \); otherwise \( p_{ik} = 0 \) for all \( i, k^* \))

2. \( \alpha \leftarrow \text{a Value above} \ i, \text{i.e.} \ 10; \mu_2 \leftarrow \mu_1 - \alpha \)

3. While \( f'(\mu_2) > 0 \)

4. do \( \mu_1 \leftarrow \mu_2; \mu_2 \leftarrow \mu_2 - 3\alpha \)

5. While no convergence

(* search the optimum interactively*)

6. do \( \mu \leftarrow \frac{\mu_2 - \mu_1}{2} \)

7. if \( f'(\mu) > 0 \)

8. then \( \mu_1 \leftarrow \mu \);

9. else \( \mu_2 \leftarrow \mu \);

10. return \( \mu \) and \( p_{ik} \leftarrow \left[ \mu - \frac{\Gamma \sigma_i^2}{\lambda_{ik}^2} \right]^{+} \)

And it is easy to see that the optimal \( \mu^* \) that maximizes \( f(\mu) \) equals \( \frac{B_C}{\alpha U^* \ln 2} \). Therefore find \( \mu^* \) such that

\[ \mu^* = \arg \max_{\mu} f(\mu) \]  

(21)

when \( f(\mu) > 0 \), \( f(\mu) \) is strictly quasi-concave in \( \mu \). Hence a unique globally optimal \( \mu^* \) exists such that for any \( \mu < \mu^*, f(\mu) > 0 \), and for any \( \mu > \mu^*, f(\mu) < 0 \), \( \leq (\mu)|\mu^* \) If \( f(0) = 0, \mu^* \) is found. If \( f'(\mu)|_{\hat{\mu}} = 0 \), then \( \mu_1 < \mu^* < \hat{\mu} \) and replace \( \mu_2 \) with \( \hat{\mu} \); otherwise, replace \( \mu_1 \) with \( \mu \). This iteration continues until \( \mu_2 - \mu_1 \) is sufficiently small to meet the convergence requirement.

**Proposition 3.** EMMPA converges to the globally optimal \( \mu^* \). Any \( \mu \), which satisfies \( |\mu - \mu^*| \leq \varepsilon \) can be found within

\[ \left[ \log_2 \left( \frac{(\alpha - 1)|\mu^*|}{\varepsilon} - 1 \right) \right] \] iterations. The EMMPA algorithm should be implemented at the AP. Each user needs to report its circuit power to the AP before the communications. This is a one-time report and the signaling overhead is negligible.

**V. ENERGY-EFFICIENT MU-MIMO WITH IMPROVED CIRCUIT MANAGEMENT**

According to Theorem 1, the power allocated on some antennas may be zero. User \( i \) can turn off these antennas to reduce circuit energy consumption. With the improved circuit management, circuit power is a function of the set of antennas that are turned on. In the following, for notation simplicity, Assign the circuit power of User \( i \) to \( P_{C_i}(k_i^0) \)

where \( k_i^0 \) is the number of antennas that have positive power allocation. \( P_{C_i}(k_i^0) \) is increasing in \( k_i^0 \).

\[ P_{C_i}(k_i^0) = k_i^0 P_\alpha + I(k_i^0)P_\beta \]  

(22)

Where \( P_\alpha \) is the extra antenna-related circuit power consumption and \( P_\beta \) is the power consumption of circuit components. The indicator function \( I(A) \) is defined as

\[ I(A) = \begin{cases} 1 & \text{if } A > 0 \\ 0 & \text{otherwise} \end{cases} \]  

(23)

The energy efficiency capacity is given by

\[ U^* = \max_P U(P) = \max_P \frac{\sum_i R_i}{\alpha \sum_i (P_{T_i} + P_{C_i}(k_i^0)) + \beta P_t} \]  

(24)

and the optimal energy-efficient power allocation achieving the energy efficiency capacity is

\[ P^* = \arg \max_P U(P) = \arg \max_P \frac{\sum_i R_i}{\alpha \sum_i (P_{T_i} + P_{C_i}(k_i^0)) + \beta P_t} \]  

(25)

**A. Principles of Energy-Efficient Power Allocation**

With improved circuit management, the energy efficiency function is no longer continuous or quasi-concave in \( P \). Observe Antenna \( j \) of User \( i \) and define it to be Antenna \((i, j)\). Antenna \((i, j)\) may have two states, on or off. If it is on, the energy efficiency is

\[ U^*(p_{ij}(k_j^0)(p_{ij})) = \frac{R_{ij}^0 + \log_2 \left( 1 + \frac{p_{ij}^2 R_{ij}^0}{\Gamma \sigma_i^2} \right)}{P_{ij}^0 + p_{ij}^0 \alpha + P_{C_i}(k_i^0)} \]  

(26)

The partial derivative of \( U \) with respect to \( p_{ij} \) is
\[ \frac{\partial \hat{U}}{\partial p_{ij}} = \frac{f(p_{ij})}{1 + \frac{P_{ij}^2}{\sigma^2}} \left( \frac{\alpha}{\gamma} p_{ij}^\gamma + \frac{P_{ij}^0 + P_{C_i}^0(k_i^0)^2}{\gamma} \right)^{-\gamma} \]  

(27)

**Theorem 2.** With improved circuit management, the optimal energy-efficient power allocation \( P^* \) achieving the energy efficiency capacity satisfies, for antennas that are turned on,

\[ p_{ij}^* = \frac{B_S}{\alpha U^* \ln 2} - \frac{\gamma^2}{\lambda_{ij}^2} \]  

(28)

and these antennas have channel conditions

\[ \frac{\lambda_{ij}^2}{\Gamma^2} > \frac{R_i^0 \alpha \ln 2}{\left( p_{ij}^0 + P_{C_i}^0(k_i^0) \right) B_S}. \]  

(29)

where \( k_i^0 \) is the number of antennas of User \( i \) when Antenna \((i, j)\) is turned on. Correspondingly, the energy efficiency capacity is

\[ U^* = \hat{U}(P^*) \]  

(30)

According to Theorem 2, whether or not Antenna \((i, j)\) should be turned on is determined by multiple factors. Characterizes the channel condition of Antenna \((i, j)\) and determines the effective receiver SNR once the power is allocated. If it is above the threshold \( \frac{R_i^0 \alpha \ln 2}{\left( p_{ij}^0 + P_{C_i}^0(k_i^0) \right) B_S} \), Antenna \((i, j)\) should be used since using it improves the overall network energy efficiency.

**Proposition 4.** With improved circuit management, the energy efficiency capacity decreases strictly and the optimal allocated power on each spatial channel, if nonzero, increases strictly with the circuit power of any antenna that is on. If receiving power is considered \( (\beta > 0) \), the energy efficiency capacity decreases strictly while the optimal allocated power on each spatial channel, if nonzero, increases strictly with the receiving power.

**Table II**

**Algorithm** Exhaustive search power allocation

1. \( U_{\text{max}} \leftarrow 0; \) \( P^* \leftarrow 0; \)
2. for all antenna configurations
3. Calculate the circuit power for each user;
4. Use EMMPA to find the optimal \( \mu \) power allocation \( P \), and the EE \( U \);
5. If \( U > U_{\text{max}} \) and all antennas turned on have positive power allocation
6. \( c \leftarrow \) current antenna configuration;
7. \( \mu^* = \mu,U_{\text{max}} \leftarrow U, \) and \( P^* \leftarrow P; \)
8. return \( c, \mu^*, P^*, \) and \( U_{\text{max}} \).

**B. Algorithm Development**

Different from Theorem 1, Theorem 2 only gives the necessary conditions of globally optimal energy efficient power allocation. An example is given in Fig. 2, assume one user with two antennas is communicating to the AP, i.e., a MIMO system. The circuit power of the user is assumed to be \( P_{C_i} = k_i^0 \). Observing Fig. 2, \( U \) has three local maximums, each of which satisfies Theorem 2. When both antennas are turned on, there is a unique power allocation that maximizes \( U \). When the state of one antenna switches from on to off, the energy efficiency \( U \) increases abruptly because of the reduction of circuit power.

1) **An Exhaustive Search Algorithm (ESPA):** Define the antenna configuration to be a binary vector of length \( \Sigma_k k_n \), in which 1 indicates the corresponding antenna is on and 0 otherwise. EMMPA can be used to determine the corresponding optimal power allocation for that antenna configuration. One simple approach is that, exhaustively search all antenna configurations and use EMMPA to determine the maximum energy efficiency achieved for each configuration. The complexity of ESPA grows exponentially with the total number of antennas of all users and based on Proposition 3. It can be easily shown that the convergence rate is characterized by Proposition 5.

![Fig. 2. An example of non-quasi-concave energy efficiency function \( \hat{U} \)](image-url)
Proposition 5. ESPA converges to the globally optimal power allocation. The optimal antenna configuration, as well as the power allocation for antennas turned on, can be found within at most \( \log_2 \left( \frac{(\alpha-1)\mu^*}{\epsilon} - 1 \right) \) \( \sum ki - 1 \) iterations.

2) A Quadratic-Complexity Algorithm: For a small number of users and antennas, ESPA is effective in finding the globally optimal solution. Further a low-complexity algorithm has been developed. This algorithm consists of two steps. In the first step, assume all antennas are turned on and the circuit power, of all users can be determined. If any antenna is turned off, the circuit power of the corresponding user is reduced. This indicates that \( S^{(1)} \) belongs to the set of antennas that should be turned off in the globally optimal antenna configuration. \( S^{(1)} \) can be determined by EMMPA. In the round 2, turn off all antennas in \( S^{(1)} \) and calculate the circuit power, \( P_{Ci}(k_i^0) \) of all users. Then use EMMPA again to determine \( S^{(2)} \). Similarly \( S^{(2)} \) also belongs.

Table -III

Iterative EMMPA

Algorithm Iterative EMMPA

1. Let \( S^{(0)} \) be an empty set.
2. Assume all antennas are turned on and calculate the circuit power, \( P_{Ci}(k_i^0) \) of all users.
3. Use EMMPA to determine \( S^{(1)} \) and the corresponding optimal \( \mu^* \) and \( P^* \), \( m \leftarrow 1 \).
4. while \( S^{(m)} \) differs from \( S^{(m-1)} \)
5. do Turn off all antennas in \( S^{(m)} \) and calculate the circuit power, \( P_{Ci}(k_i^0) \) of all users.
6. \( m \leftarrow m + 1 \).
7. Use EMMPA to determine \( S^{(m)} \) and the corresponding optimal \( \mu^* \) and \( P^* \).
8. return \( S^{(m)}, \mu^*, \) and \( P^* \).

Table -IV

Improved EMMPA

Algorithm Improved EMMPA

1. Use iterative EMMPA to determine \( S^{(0)} \) and \( U^{(0)} \).
2. \( m \leftarrow 0 \) and \( U_{max} \leftarrow 0 \).
3. repeat
4. for Antenna (i, j) in \( S^{(m)} \)
5. do Turn on only antennas in \( S^{(m)} \) excluding (i,j) and calculate circuit power of all users.
6. Use EMMPA to determine \( U^{(m)}_{i,j} \) and the corresponding \( \mu \) and \( P \).
7. if \( U^{(m)}_{i,j} > U_{max} \)
8. then \( U_{max} \leftarrow U^{(m)}_{i,j} \) and \((k, l) \leftarrow (i, j), \mu^* \leftarrow \mu, P^* \leftarrow P.\)
9. if \( U_{max} > U^{(m)} \)
10. then \( S^{(m+1)} \leftarrow S^{(m)} \) excluding \((k, l)\)
11. \( U^{(m+1)} \leftarrow U_{max} \).
12. \( m \leftarrow m + 1 \);
13. until \( U_{max} \leq U^{(m-1)} \).
14. return \( S^{(m-1)}, \mu^*, \) and \( P^* \).

no antennas in \( S^{(0)} \) should be turned off.

\[
\max_{(i, j) \in S^{(0)}} U^{(0)}_{i,j}
\]  

(31)
Then a higher energy efficiency \( U_k^{(0)} \) is achieved. In the second round, let \( U_k^{(1)} = U_k^{(0)} \). Define the set of remaining antennas that are still on to be \( \mathcal{S}^{(1)} \). The above selection process can be repeated until in round \( m \), no antennas should be turned off.

**Proposition 7.** The output of the improved EMMPA is obtained within at most

\[
\log_2 \left( \frac{\alpha - 1}{\mu^*} \right) \left( \sum_{k=1}^{K} k_i \right),
\]

iterations.

Similar to EMMPA, the algorithms proposed in this section should also be implemented at the AP.

To determine their optimal power allocations.

**VI. SIMULATION RESULTS FOR ENERGY-EFFICIENT MU-MIMO**

Simulation results for a single-cell cellular network to demonstrate the performance of energy-efficient MU-MIMO was provided. System parameters are listed in Table V.

**A. Performance of Energy-Efficient MU-MIMO without Improved Circuit Management**

Consider the case that no circuit management is used and each user consumes a fixed amount, \( P_A = 100 \text{ mW} \), of circuit power. Fig. 3 gives the average energy efficiency capacity when there are two users in the network and each user has 1, 2, 3, or 4 antennas. The number of AP antennas is varied to observe its impact on energy efficiency capacity. On the other hand, Fig. 4 compares the average energy efficiency capacity when the AP has 64 antennas. Without circuit management, more users and more antennas always help improve the energy efficiency capacity of MU-MIMO. Fig. 5 compares the energy efficiency of EMMPA and that of the fixed power allocation (FPA). With the fixed power allocation, each user employs a fixed amount of transmission power, given by the value in the x axis. As shown in Fig. 5, significant gain in energy efficiency can be observed by using EMMPA.

**B. Performance of Energy-Efficient MU-MIMO with Improved Circuit Management**

In the following, consider energy-efficient MU-MIMO with improved circuit management and assume

\[
P_{C, I}(k_i^0) = P_A k_i^0 + P_B \mathcal{J}(k_i^0) \text{ mW for all users.}
\]

Verify the global optimality of improved EMMPA. Fig. 6 gives the normalized energy efficiency of improved and iterative EMMPA when the AP has 16 antennas and each user has 4 antennas. From Fig. 6, improved EMMPA performs exactly the same as ESPA. Therefore, improved EMMPA is also globally optimal.

Fig. 7 compares the average computing time of improved EMMPA, iterative EMMPA, and ESPA with the same simulation setting as that in Fig. 6. When \( K \) increases, the computing time of ESPA grows exponentially, more than ten times when \( K \) is increased by one. Fig. 8 gives the average energy efficiency capacity when the AP has 16 antennas while each user has two antennas. More users always help improve the network energy efficiency because of increased multi-user diversity. Fig. 9 illustrates the average energy efficiency capacity when the AP has 16 antennas and two users are accessing the AP. When \( P_A \) is small, more antennas always improve the energy efficiency capacity.
network energy efficiency. When $P_a$ is large, more antennas do not help improve the network energy efficiency because most antennas should be turned off in this case to reduce circuit power consumption.

Fig. 10 compares the average energy efficiency of improved EMMPA, iterative EMMPA, and FPA in the same two scenarios as in Fig. 5. Iterative EMMPA achieves very close performance to that of improved EMMPA.

Fig. 5. Comparison between EMMPA and Fixed Power Allocation (Scenario 1: $N = 8, K = 4, k_i = 2$; Scenario 2: $N = 8, K = 1, k_i = 4$).

Fig. 6. Sub optimality gap of improved EMMPA and iterative EMMPA ($N = 16$ and $k_i = 4$).

Fig. 7. Complexity comparison of improved EMMPA, iterative EMMPA, and ESPA ($N = 16$ and $k_i = 4$).

Fig. 8. Relationship between energy efficiency capacity and number of users ($N = 16$ and $k_i = 2$).

Fig. 9. Relationship between energy efficiency capacity and user antennas ($N = 16$ and $K = 2$).
Fig. 10. Comparison between improved EMMPA, iterative EMMPA, and FPA (Scenario 1: N = 8, K = 4, ki = 2; Scenario 2: N = 8, K = 1, ki = 4).

Table-V
Simulation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Carrier frequency</td>
<td>1.5 GHz</td>
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<tr>
<td>System bandwidth</td>
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</tr>
<tr>
<td>Thermal noise power, N₀</td>
<td>-141 dBW/MHz</td>
</tr>
<tr>
<td>User antenna height</td>
<td>1.6 m</td>
</tr>
<tr>
<td>BS antenna height</td>
<td>40 m</td>
</tr>
<tr>
<td>Environment</td>
<td>Macro cell in urban area</td>
</tr>
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</table>

VII. CONCLUSION

The optimal energy-efficient MU-MIMO was investigated in the present paper. Both electronic circuit and RF transmission power consumptions have been considered. MU-MIMO system was analyzed based on distributed SVD decomposition of the channels of all users and derived the achieved SNR conditions for all users. Then the concept of energy efficiency capacity for MU-MIMO is defined. Following that the existence of a uniquely globally optimal power allocation that could achieve this energy efficiency capacity has been demonstrated. The optimal power allocation is shown to be a dynamic water-filling approach where the water level is determined by the energy efficiency capacity. A one-dimensional low-complexity algorithm has been developed to obtain the globally optimal power allocation and this algorithm converges to the global optimum at an exponential speed. Energy-efficient MU-MIMO was further studied with improved circuit management to facilitate users to turn off electronic circuit operations when some antennas are not used. Certain antennas are not to be used even though they have good channel states, because using them consumes a lot of circuit power. Furthermore to determine the set of antennas that should be kept on and the respective power allocation globally optimal algorithms were developed. Comprehensive simulation results have been provided to demonstrate the algorithm performance and the significant gain in energy efficiency for a cellular network.
REFERENCES


BIOGRAPHIES

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