A STUDY ON PROBABILITY FAILURE OF A COLUMN IN RC FRAME

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Abstract - The present method of designing Reinforced Concrete Columns is based on limit state philosophy which makes use of partial safety factors for material strengths and loads. However, the overall safety of a column as a member and the probability of failure of a column against a given load is not explicitly obtained. The design variables being random, it becomes much more important to assess the level of safety in the probabilistic design situation. Column being the vital most structural element, probability of failure of a column is linked to the overall safety of a structural system. With this in view, an attempt is made to assess the safety levels ensured by the design methodology of the present code of practice IS 456-2000. This requires the information on the probability distributions of the resistance part and action part in each limit state.

For a particular column statistics and probability distribution of axial load with uniaxial moments are generated by digital simulation on a R.C. frame using ETABS 2013. The generated data on axial load with uniaxial moment is subjected to statistical analysis, probability modeling using program supported on MATLAB. Resistance statistics are generated using the relevant equations form IS 456-2000. Several design situations corresponding to different material grades and dimensions are considered. Probability of failure is obtained by Monte Carlo Simulation which establishes the statistics of safety margin M = R – S.

It is attempted to see if the safety level is uniform over a range of design situation.

Key Words: Reliability Analysis, Probability of Failure, RC Frame, Monte-Carlo Simulation

1. INTRODUCTION

Reliability analysis is defined as the consistent evaluation of design risk using probability theory. The reliability is the probability of an item performing its intended function over a given period of time under the operating conditions encountered. The four significant elements are:

a) Probability
b) Intended function
c) Time
d) Operating conditions

1.1 Uncertainties

The reliability is a probability, which is the first element in the definition. The second point, intended function, signifies that the reliability is a performance characteristic. For a structure to reliable, it must perform certain function or functions satisfactorily for which it has been designed. Reliability is always related to time. In case of structure, it must perform the assigned function satisfactorily. The last point is operating conditions, this establishes the actions, stresses that will be imposed on the structure. These may be loads, temperature, corrosive atmosphere, shock, etc.

1.2 Probability of failure

The achievement of absolute safety or reliability is impossible, a probabilistic approach to a evaluation of safety becomes a sensible solution which are subjected to random variables. There is a need for a rational approach to the evaluation of the structural safety, taking into account these random variations. The study of variability comes under the domain of statistics and probability. Using the probabilistic approach, there is a possibility of obtaining uniform reliability (uniform performance in structure under different design situation). Hence probabilistic approach must me used. For convenience, the reliability, R, is defined in terms of the probability of failure, P₀, which is taken as R=1-P₀. When probability theory is used in the limit state design, the method is called probability-based limit state design.
2. DETERMINISTIC DESIGN DETAILS

For deterministic design single bay RC Portal frame fixed at the base is considered with a following data.

2.1 Symmetrical Loading on RC frame

- Height of column=4m
- Length of the beam=8m
- Column dimension 400*600mm
- Beam dimension 400*700mm
- Characteristic strength $f_{ck}=20 \text{N/mm}^2$, $f_y=415 \text{N/mm}^2$
- Live Load on the beam=20kN/m.

In the deterministic design of RC frame due to symmetrical loading, the column is designed by taking $M_2=101.29 \text{kN-m}$, $P_2=107.99 \text{kN}$ and by using interaction curves of SP-16 the percentage of the steel in the column was found to be 0.8 ($p_t=0.8\%$) and the area of the steel=1920mm$^2$.

2.2 Unsymmetrical Loading on RC frame

- Height of column=4m
- Length of the beam=8m
- Column dimension 400*600mm
- Beam dimension 400*700mm
- Characteristic strength $f_{ck}=20 \text{N/mm}^2$, $f_y=415 \text{N/mm}^2$
- Lateral Load at 2/3rd from the base=15kN
- Point Load at 3/4th of length=25kN.

In the deterministic design of RC frame due to unsymmetrical loading, the column is designed by taking $M=33.62$ and $P=32.47$, $M_4=47.74$ kN-m and $P_4=48.50$ kN and by using interaction curves of SP-16 the percentage of the steel in the column was found to be 0.8 ($p_t=0.8\%$) and the area of the steel=1920mm$^2$.

3. GENERATION OF LOAD STATISTICS AND RESISTANCE STATISTICS

In the process of codal assessment, reliability analysis of existing design of column as per the current codal provisions are carried out for limit state of collapse in compression and then the reliability levels of the present designs under different design situations are established. Columns subjected to axial load with uniaxial bending are considered.

3.1 Distribution of parameters

The basic design variables are identified as,

a) Geometrical dimensions
b) Material properties
c) Load

<table>
<thead>
<tr>
<th>Geometrical Properties</th>
<th>Material properties</th>
<th>Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>Breadth</td>
<td>3%</td>
<td>$f_{ck}$</td>
</tr>
<tr>
<td>Depth</td>
<td>5%</td>
<td>$f_y$</td>
</tr>
<tr>
<td>Length</td>
<td>3%</td>
<td></td>
</tr>
</tbody>
</table>

3.2 Monte Carlo simulation
The Monte Carlo simulation is a technique used to generate the random variables whose probability distribution is known.

a) Generate random numbers between 0 to 1. (v1 & v2)

b) The normal deviates is given by
\[ u_1 = [2\ln(1/v_1)]^{0.5} \cos(2\pi v_2) \]
\[ u_2 = [2\ln(1/v_1)]^{0.5} \sin(2\pi v_2) \]

c) Normal variates
\[ y_1 = \mu + \sigma u_1 \]
\[ y_2 = \mu + \sigma u_2 \]

100 data sets were randomly generated for each cross section of beam and column, height of the column, length of the beam, characteristic strength of concrete and steel, live loads and each data set varied randomly as a function of statistical models for the variables involved by using Monte-Carlo simulation in EXCEL.

3.3 Probability Modeling of Action

100 data sets of values were obtained by Monte-Carlo simulation in EXCEL. The model has been made to run for 100 times in ETABS 2013 and analyzed. Moment and axial load is obtained in different position for 100 times in symmetrical and unsymmetrical loading of RC frame. The variables included in the study are dimensions, material properties, loads. The current study fits a normal distribution for all variables by using Chi-square Test for goodness-of-test in MATLAB. Data is grouped for frequency analysis, statistical parameters like mean, standard deviation, coefficient of variation are worked out. The randomness is modeled with probability distributions. The results are presented in the form of graph by using MATLAB.

a) Symmetrical loading
b) Unsymmetrical loading

Similarly, the histogram, probability distribution curve, cumulative density function and probability plot of the generated sample are obtained for all the moment and axial load for different location.

4. COMPUTATION OF PROBABILITY OF FAILURE

- For random variations in different grades of concrete, characteristic strength, dimensions and live load, corresponding moment and axial values are obtained from ETABS.
- Compute deterministic value of Pt.
- Find Pt\textsubscript{i} where i = 1, 2, 3, ..., 100 (ETABS values)
- If Pt < Pt\textsubscript{i} then it is failure.
- Compute probability of failure, P\textsubscript{f} = (Number of failure/Total number of observation)

4.1 Symmetrical Loading on RC frame

<table>
<thead>
<tr>
<th>Location</th>
<th>Mean ((\mu))</th>
<th>M2</th>
<th>P2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f_c) (N/mm(^2))</td>
<td>20</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>(f_y) (N/mm(^2))</td>
<td>415</td>
<td>415</td>
<td></td>
</tr>
<tr>
<td>Mean ((\mu))</td>
<td>108.96</td>
<td>114.71</td>
<td></td>
</tr>
<tr>
<td>Standard deviation ((\sigma))</td>
<td>34.97</td>
<td>34.49</td>
<td></td>
</tr>
<tr>
<td>Coefficient of variance (COV)</td>
<td>32.10</td>
<td>30.06</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Location</th>
<th>Mean ((\mu))</th>
<th>M2</th>
<th>P2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f_a) (N/mm(^2))</td>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(f_y) (N/mm(^2))</td>
<td>415</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probability of failure, P\textsubscript{f}</td>
<td>0.32</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reliability, R</td>
<td>0.68</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.2 Unsymmetrical loading on RC frame
**Table -6**: Mean, SD and COV of moment and axial load

<table>
<thead>
<tr>
<th>Location</th>
<th>M2</th>
<th>P2</th>
<th>M4</th>
<th>P4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{ck}$(N/mm$^2$)</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>$f_y$(N/mm$^2$)</td>
<td>415</td>
<td>415</td>
<td>415</td>
<td>415</td>
</tr>
<tr>
<td>Mean($\mu$)</td>
<td>34.88</td>
<td>33.28</td>
<td>50.03</td>
<td>50.34</td>
</tr>
<tr>
<td>Standard deviation ($\sigma$)</td>
<td>6.26</td>
<td>4.51</td>
<td>11.714</td>
<td>10.42</td>
</tr>
<tr>
<td>Coefficient of variance (COV)</td>
<td>17.94</td>
<td>13.54</td>
<td>23.413</td>
<td>20.70</td>
</tr>
</tbody>
</table>

**Table -7**: Probability of failure

<table>
<thead>
<tr>
<th>Location</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{ck}$ (N/mm$^2$)</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>$f_y$(N/mm$^2$)</td>
<td>415</td>
<td>415</td>
</tr>
<tr>
<td>Probability of failure, $P_f$ %</td>
<td>$10^{-6}$</td>
<td>$10^{-6}$</td>
</tr>
<tr>
<td>Reliability, R %</td>
<td>$\approx 1$</td>
<td>$\approx 1$</td>
</tr>
</tbody>
</table>

5. CONCLUSION

Attempt is made to quantify the safety level in terms of probability failure of a reinforced concrete frame. A typical RC frame is selected and designed using deterministic approach, as per the provision of IS 456-2000. Using ETABS software the frame is analysed and repeated for 100 times, every time using the generated values of design variables as input. The generated values of Action (S) in terms of bending moments at typical locations are statistically analysed and probabilistically modeled using MATLAB software. If the value of $P_f$ (probabilistic design) more than $P_t$ provided by the deterministic design, it is considered as failure. Such failures are counted during simulation. It is observed that rate of failure of the column section is about 32% or probability failure is 0.32. i.e reliability 68% in symmetrical loading on the RC frame where as in unsymmetrical loading on RC frame the probability failure is very low i.e $10^{-6}$ and reliability is almost 100% but in reality $P_f$ will not be equal to zero and reliability will not be equal to 100%. For important structure the $P_f$ has to keep as low as possible.

REFERENCES

6. IS 456:2000 "Indian Standard Code of Practice for Plain and Reinforced Concrete" (Fourth revision), Bureau of Indian Standards.

BIOGRAPHIES

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