Abstract - Interleaver is an integral component of many digital communication systems. The interleaver is situated between the two forward error correcting encoders to avoid the burst errors by splitting or by spreading out error-bursts. One problem with classical interleavers is that they are normally designed to give a specific interleaving depth. This is fair if burst of errors created not exceeds the designed interleaver depth, but the created burst of errors are smaller than the designed interleaver depth then it is wasteful if the interleaver is overdesigned and error bursts are typically much shorter than the interleaver depth. Of course this interleaver is not good for longer bursts of errors. Hence in this paper performance of turbo codes has been analyzed with golden relative prime interleaver and compared the results with block interleaver.

Key Words: WDM networks, optical switches, FEC (Forward error correction), Turbo code, Golden section, Block interleaver, Golden relative prime interleaver, BCJR decoding, RSC(Recursive Systematic convolutional encoder).

1. INTRODUCTION

In data networks errors are introduced by several noise sources. Interleavers are simple devices that permute sequences, they are widely used for improving error correction capabilities of coding schemes over bursty channels. The basic “Turbo” interleaver is based on the use of N×N matrix with a form relative prime indexing, but the design was more complicated. This interleaver was found that it works well, but this interleaver was not suited to variable block sizes. Another type of interleaver that designed is the “random” interleaver or spread interleaver. The random interleaver simply executes a random or pseudo-random permutation of the elements without bounds. This interleaver is found very useful, and most used in calculating error-rate bounds. The spread interleaver is nothing but a semi random interleaver. The spread interleaver is purely depends on the random generation of N integers from 0 to N-1, but with the constraint that is, once the integer is selected randomly that integer should compare with the S most recently selected integers. If the currently selected integer found within the most recently selected integers then it will reject the currently selected integer and it will selects a new integer, and this continues until the previous condition is satisfied. This process is repeated until all N integers are extracted. The search time increases with S, and there is no guarantee that the process will finish successfully. Problems associated with classical interleavers that they are normally designed to give a specific interleaving depth. This is fair if burst of errors created not exceeds the designed interleaver depth, but the created burst of errors are smaller than the designed interleaver depth then it is wasteful if the interleaver is overdesigned (too long) and error-bursts are typically much shorter than the interleaver depth. In this project another interleaving technique is introduced to recover the drawbacks of classical interleavers by reducing the searching time, memory requirement and to achieve less decoding complexity even if block length varies.

2. TURBO ENCODER

A general block diagram of turbo encoder is shown in Figure 1. It consists two similar systematic recursive convolutional encoders connected in parallel with an interleaver before the second recursive convolutional encoder. The two recursive convolutional encoders are called the powered parts of the Turbo encoder. The information bits are encoded by both RSC encoders. The data frame, length of size N, inserts directly into the first encoder and after interleaving of length N, it feeds the second encoder. Therefore, N systematic bits can generate 2N parity bits and it gives a code rate of 1/3. The input applied to the second encoder is an interleaved sequence of the systematic input X, thus the outputs of convolutional encoder 1 and convolutional encoder are the codes displaced by time and both the codes are generated through the same systematic input X. The input sequence X, the outputs of the two coders constitutively produce a rate R=1/3 code. This is shown in Figure 1. The design of interleaver plays a major role on code performance. If a encoder produce a low weight code then it can cause poor error performance, hence one or both of the coders should produce codes with good weight. If an encoder 1 produces a low weight code then the second encoder which is a interleaved code of input X should produce good weight code.
3. INTERLEAVER

The interleaver design is a key factor which determines the good performance of a turbo code. The output sequences of RSC encoder usually have high weight. However, some input sequences still cause the output sequence of RSC encoder to generate low weight code word. Therefore, the interleaver scrambles the input sequence and generates randomness to the input sequence as a result high code word can obtain.

3.1 Block interleaver

Block interleaver is easy to implement in practice. This simplest interleaver is a memory in which data is written row-wise and read column-wise. It is also known as the “row-column” interleaver.

Table 1: Writing data row wise in memory

<table>
<thead>
<tr>
<th>A1</th>
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<td>A17</td>
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3.2 Concept of Golden Section

Figure 2 shows the principle of the golden section, consider a line segment of length 1, here dividing the entire line segment into long segment g and shorter segment 1-g is the real problem, hence from that line segment we can say that the ratio of the longer segment g to the entire segment (that is 0 to 1) is the same as the ratio of the shorter segment 1-g to the longer segment. That is,

\[ \frac{g}{1} = \frac{1-g}{g} \] .................. (1)

By solving equation (1)

\[ g = \sqrt{5} - \frac{1}{2} = 0.618 \]

Now consider a starting point as 0 and points generated from starting point 0 and adding increments of g, using modulo-1 arithmetic. After incrementing the first time there are two points they are 0 and g and these points are separated by the distance of 1-g using modulo-1 arithmetic. To have the option of selecting a first point starting anywhere in the line segment modulo distances are used. From equation (2.1), we can observe that the distance of 1-g is also equals to \( g^2 \). After finishing the second increment the 1st and 3rd points gives the minimum distance and this is equal to \( g^3 \). After finishing the 3rd increment the 1st and 4th points gives the minimum distance and this equal to \( g^4 \). This process continues, and the minimum distance will not decrease more than a factor of g. We can also generate the same distances by using Complement increment that is (1-g) = \( 0.382 \), but the beginning minimum distances are decreased to the smaller increment value.

Figure 2: Illustration of the golden section principle.

3.3 Golden Relative Prime Interleaver

The index values in golden relative prime interleaver, are computed as

\[ \alpha (i)= (s + i.t) \mod L_{info}, \quad i=0...L_{info}-1 \] ....(3.5)

Where s=integer initial index,
\[ t \] =increment in integer index,
\[ L_{info} \] =length of the interleaver.

Here we should note that \( t \) and \( L_{info} \) must be relative primes to make sure that each element in the sequence is read one and only once.
The initialising index of ‘s’ is usually set to zero, but increment of index is chosen like it should have close value to one of the non integer value of the following:

\[ c = \frac{L_{\text{info}} (gm+j)}{r} \] 

(3.6)

Where \( g \) = Golden section value, \( m \) = any positive integer greater than zero, \( r \) = Index spacing between nearby elements, \( j \) = any integer modulo \( r \).

The favored values for \( m \) are normally 1 or 2. In a normal execution the neighboring elements should be highly spread to avoid the burst errors, \( j \) is put to 0 and \( r \) is put to 1. To obtain good spreading properties for the elements which are placed in \( r \) distance apart, we can set greater values for \( j \) and \( r \). For example, to spread the error events to the extent we can set \( r \) as the repetition period of the feedback polynomial in the RSC encoder. For previously defined values of \( L_{\text{info}}, m, j, r \) the simple way is to select the relative prime \( t \) closest to \( c \), the result is a golden relative prime interleaver with quantization error. Quantization error for short error burst lengths in large blocks is normally not considerable, but the quantization error may grow and can be considerable after adding many increments. To avoid quantization error we can conduct a search for the better relative prime increment ‘\( t \)’ in the neighborhood of ‘\( c \)’, by considering highest number of elements, the minimum distance property between interleaved indexes used. Possibly, the better increment in relative prime, \( t \), in the neighborhood of \( c \), is decided by the summing the minimum distances between interleaved indexes for all numbers from initial element to the highest number of elements considered.

4. TURBO DECODER

The two main types of decoder are BCJR or Maximum A Posteriori (MAP) and the Soft Output Viterbi Algorithm (SOVA). MAP looks for the most likely symbol received, SOVA looks for the most likely sequence. Both MAP and SOVA perform similarly at high Eb/No. At low Eb/No MAP has a distinct advantage, gained at the cost of added complexity. MAP looks for the most probable value for each received bit by calculating the conditional probability of the transition from the previous bit, given the probability of the received bit. The focus on transitions, or state changes within the trellis, makes LLR a very suitable probability measure for use in MAP.
It is simplest to view the decoding process as 2 stages, initializing the decoder and decoding the sequence. The demodulator output contains the soft values of the sequence $x'$ and the parity bits $p1'$ and $p2'$. These are used to initialize the decoder, as shown in Figure 4. The interleaved sequence is sent to decoder 2, while the sequence derived from $x'$ is sent to decoder 1 and presented to decoder 2 through an interleaver. This re-sequences bits from streams $x'$ and $p1'$ so that bits generated from the same bit in $x$ are presented simultaneously to decoder 2, whether from $x$, $p1'$ or $p2'$.

5. SIMULATION RESULTS

The figure 5 shows a plot for the parameters minimum distances versus the number of points considered, in golden relative prime interleaver, after finding the smaller section and larger section we will get the golden value and by using this golden value we will increment by adding some points in modulo 1 arithmetic. For every particular point the upper bound is also as shown in the figure 5. If we consider $n$ points, these $n$ points systematically placed by a minimum distance of $1/n$. We can notice that even when there is a drop in minimum distance, the preceding minimum distance will be maintain through their neighbours. The red colour line shows the upper bound and the blue line shows the golden section increment.

Figure 5: Minimum distance between points versus number of points with a golden increment.

The figure 6 shows the performance of golden relative prime interleaver with block interleaver for the block size of 1200 and the information length of $10^5$. As a block length increases the performance of block interleaver degrade as shown in figure 6.

Figure 6: Performance comparison of block interleaver with golden relative prime interleaver.

The figure 7 shows the performance of golden relative prime interleaver with block interleaver for the block size of 1200 and the information length of $10^6$. By increasing the message bits from $10^5$ to $10^6$ the performance of the golden relative prime interleaver also improves this can be observed by comparing figure 6 and figure 7. Along with the bit error rate even signal to noise ratio also improved in golden relative prime interleaver if we compared it with the block interleaver.

Figure 7: Performance comparison of block interleaver with golden relative prime interleaver for message length $10^6$. 
6. CONCLUSION
Golden section interleaver for turbo code is implemented and the performance is compared against the block interleaver for various block size and signal to noise ratio. The results in bit error rate are found to be better performing in case of golden section interleaver. And the performance betters with increasing SNR as well as increase in block size of the packets.

The golden relative prime interleaver, although highly structured with no random component, worked surprisingly well for high code rates. Further improvements should be possible by incorporating more specific knowledge about the punctured component RSC codes into the interleaver design. The various ‘golden’ interleavers have excellent spreading properties in general and are thus useful for many applications other than Turbo-codes. There are no restrictions on the block size, and a time-consuming search is not required. Thus, interleavers can be easily generated on and as needed basis for any block length.

REFERENCES

BIOGRAPHIES

Mr. Purushottama G B obtained B.E degree in Electronics and Communication Engineering from Visvesvaraya Technological University during 2008-11, pursuing MTech in Digital Electronics and Communication Systems at Malnad College of Engineering, Hassan. Presently, he undertakes his academic project on performance analysis of turbo codes with golden section interleaver at MCE, HASSAN.

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