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A note on Trigonometric moments of Marshall – Olkin Stereographic Circular Logistic Distribution

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Abstract - *Trigonometric moments play a vital role in evaluating population characteristics. This note is aimed at obtaining trigonometric moments of Marshall – Olkin Stereographic Circular Logistic.*

Key Words: *Trigonometric moments, Marshall – Olkin, Stereographic projection, and Gradshteyn & Ryzhik formula*

1. INTRODUCTION

A random variable Φ_C on unit circle is said to have Marshall-Olkin Stereographic Circular Logistic distribution with location parameter μ, scale parameter $\sigma > 0$ and tilt parameter $\alpha > 0$ denoted by MOSCLG(μ, σ, α), if the probability density and cumulative distribution functions for $\sigma, \alpha > 0,\ -\pi \le \theta < \pi$ are respectively given by

$$g_{MO}(\theta) = \frac{\alpha}{2\sigma}\sec^2\left(\frac{\theta}{2}\right)\left[1+\alpha\exp\left(\frac{-\left(\tan\left(\frac{\theta}{2}\right)-\mu\right)}{\sigma}\right)\right]^{-2}\exp\left(-\left(\frac{\tan\left(\frac{\theta}{2}\right)-\mu}{\sigma}\right)\right) \quad (1.1)$$

$$G_{MO}(\theta) = \left[1+\alpha\exp\left(\frac{-\left(\tan\left(\frac{\theta}{2}\right)-\mu\right)}{\sigma}\right)\right]^{-1} \quad (1.2)$$

Graphs of probability density function and cdf of Marshall-Olkin Stereographic Circular Logistic Distribution for various values of σ, α and $\mu = 0$ are presented in Fig 1.1 through 1.3

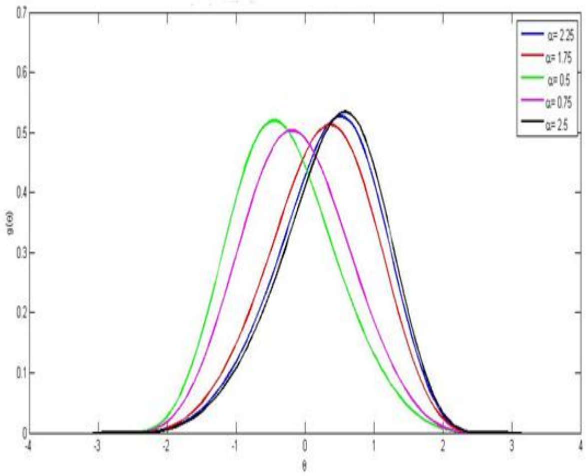

Fig 1.1: Graphs of pdf of Marshall-Olkin Stereographic Circular Logistic Distribution σ **=0.25**

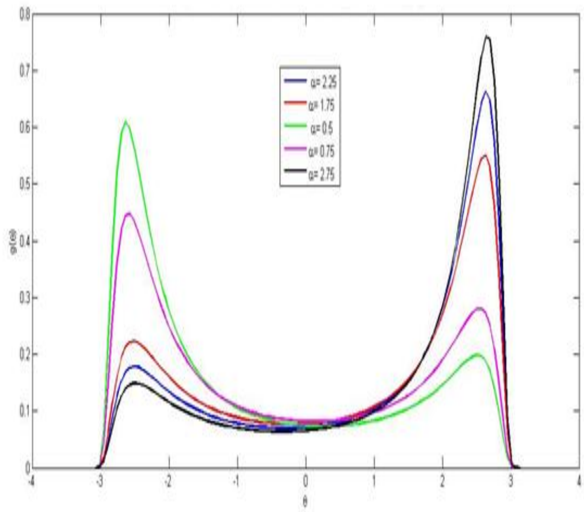

Fig 1.2 : Graphs of pdf of Marshall-Olkin Stereographic Circular Logistic Distribution σ = 1.5

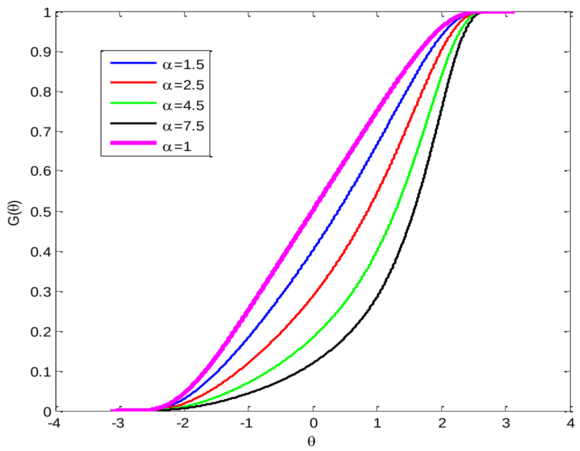

Fig 1. 3 : The graphs of the cdf of the Marshall-Olkin Stereographic Circular Logistic Distribution

Note: It can be observed from the graphs that the Marshall - Olkin Stereographic Circular Logistic distribution is

1. Asymmetric for $\alpha \neq 1$ and symmetric for $\alpha = 1$
2. **Unimodal** if $\sigma < 0.5$ and **bimodal** if $\sigma > 0.5$.

The characteristic function of the Marshall - Olkin Stereographic Circular Logistic model is given by

$$\varphi_{X_S}(p) = \int_{-\pi}^{\pi} e^{ip\theta} g_{MO}(\theta)\, d\theta$$

$$= \int_{-\infty}^{\infty} e^{ip\left(2\tan^{-1}(x)\right)} \frac{\alpha}{\sigma}\left[1 + \alpha e^{-\left(\frac{x}{\sigma}\right)}\right]^{-2} e^{-\left(\frac{x}{\sigma}\right)} dx \tag{1.3}$$

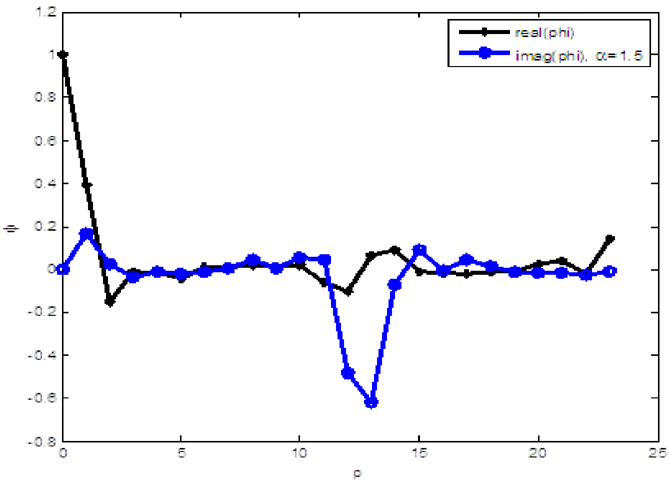

Fig 1.4: The Characteristic Function of the Marshall - Olkin Stereographic Circular Logistic model $\sigma = 0.5$

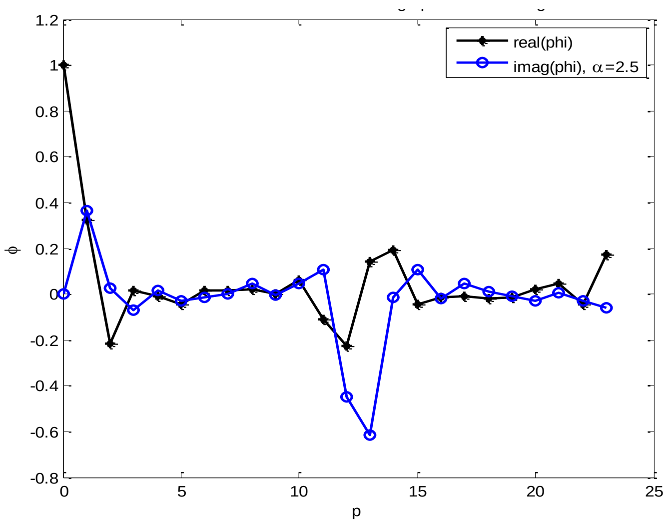

Fig 1.5 : The Characteristic Function of the Marshall - Olkin Stereographic Circular Logistic model $\sigma = 0.5$

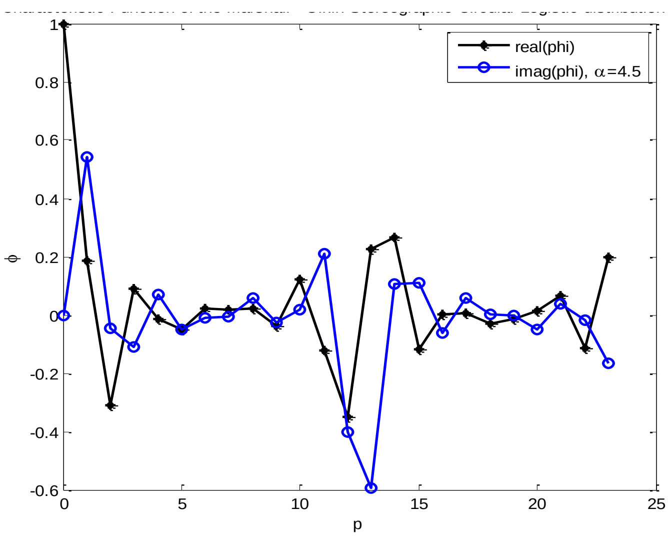

Fig 1.6 : The Characteristic Function of the Marshall - Olkin Stereographic Circular Logistic model $\sigma = 0.5$

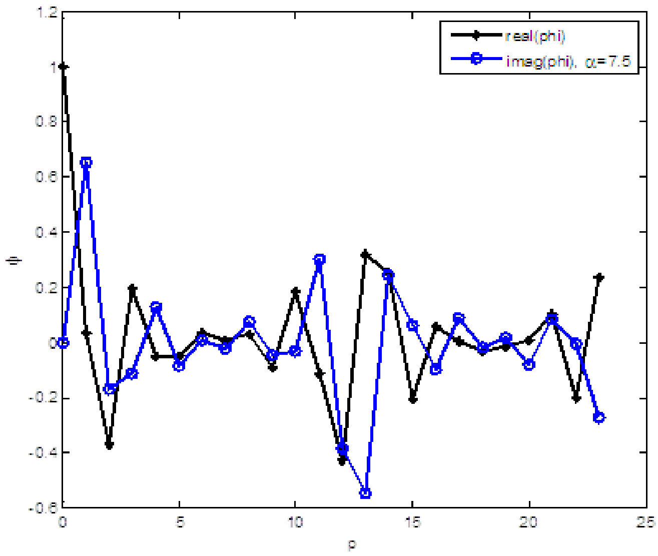

Fig 1.7 : The Characteristic Function of the Marshall - Olkin Stereographic Circular Logistic model $\sigma = 0.5$

2. TRIGONOMETRIC MOMENTS OF MARSHALL – OLKIN STEREOGRAPHIC CIRCULAR LOGISTIC DISTRIBUTION

The first two trigonometric moments are derived by us using formula from (Gradshteyn and Ryzhik, 2007) and are presented in the form of the following result.

Result: Under the pdf of the Marshall-Olkin Stereographic Circular Logistic Distribution with $\mu = 0$, the first two $\alpha_p = E(\cos p\theta)$ and $\beta_p = E(\sin p\theta)$, $p = 1, 2$. are given as follows:

$$\alpha_1 = 1 - \frac{1}{\sqrt{\pi}\sigma}\sum_{k=0}^{\infty}\left[\left(\frac{1}{\alpha}\right)^{k+1} + \alpha^{k+1}\right](-1)^k (k+1)\, G_{31}^{13}\left(\left(\frac{k+1}{2\sigma}\right)^2 \middle| \begin{matrix} \frac{-1}{2} \\ \frac{-1}{2}, 0, \frac{1}{2} \end{matrix}\right)$$

$$\beta_1 = \frac{1}{\sqrt{\pi}\sigma}\sum_{k=0}^{\infty}(-1)^k (k+1)\left(\alpha^{k+1} - \frac{1}{\alpha^{k+1}}\right) G_{31}^{13}\left(\left(\frac{k+1}{2\sigma}\right)^2 \middle| \begin{matrix} 0 \\ 0, 0, \frac{1}{2} \end{matrix}\right)$$

$$\alpha_2 = 1 + \frac{4}{\sqrt{\pi}\sigma}\sum_{k=0}^{\infty}(-1)^k (k+1)\left(\alpha^{k+1} + \left(\frac{1}{\alpha}\right)^{k+1}\right)$$
$$\left(G_{31}^{13}\left(\left(\frac{k+1}{2\sigma}\right)^2 \middle| \begin{matrix} -\frac{3}{2} \\ -\frac{1}{2}, 0, \frac{1}{2} \end{matrix}\right) + G_{31}^{13}\left(\left(\frac{k+1}{2\sigma}\right)^2 \middle| \begin{matrix} -\frac{1}{2} \\ -\frac{1}{2}, 0, \frac{1}{2} \end{matrix}\right)\right)$$

$$\beta_2 = \frac{2}{\sqrt{\pi}\sigma}\sum_{k=0}^{\infty}(-1)^k (k+1)\left(\alpha^{k+1} - \left(\frac{1}{\alpha}\right)^{k+1}\right)$$
$$\left[G_{31}^{13}\left(\left(\frac{k+1}{2\sigma}\right)^2 \middle| \begin{matrix} 0 \\ 0, 0, \frac{1}{2} \end{matrix}\right) - 2G_{31}^{13}\left(\left(\frac{k+1}{2\sigma}\right)^2 \middle| \begin{matrix} -1 \\ 0, 0, \frac{1}{2} \end{matrix}\right)\right]$$

where

$$\int_0^{\infty} x^{2\nu-1}\left(u^2+x^2\right)^{Q-1} e^{-\mu x}dx = \frac{u^{2\nu+2Q-2}}{2\sqrt{\pi}\,\Gamma(1-Q)} G_{13}^{31}\left(\frac{\mu^2 u^2}{4} \middle| \begin{matrix} 1-\nu \\ 1-Q-\nu, 0, \frac{1}{2} \end{matrix}\right)$$

for $|\arg u\pi| < \frac{\pi}{2}, \operatorname{Re}\mu > 0$ and $\operatorname{Re}\nu > 0$ and

$$G_{13}^{31}\left(\frac{\mu^2 u^2}{4} \middle| \begin{matrix} 1-\nu \\ 1-Q-\nu, 0, \frac{1}{2} \end{matrix}\right)$$ is called as

Meijer's **G**-function (Gradshteyn and Ryzhik, 2007, formula no. 3.389.2).

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