STATISTICAL HYPOTHESIS TESTING THROUGH TRAPEZOIDAL FUZZY INTERVAL DATA

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Abstract - ***Trapezoidal fuzzy numbers have many advantages over triangular fuzzy numbers as they have more generalized form. In this paper, we have approached a new method where trapezoidal fuzzy numbers are defined in terms of*** α - level ***of trapezoidal interval data based on this approach, the test of hypothesis is performed.***

Key Words: *Fuzzy set,* α - level *set, Fuzzy Numbers, Trapezoidal fuzzy number (TFN), Trapezoidal Interval Data, Test of Hypothesis, Confidence Limits, t-Test.*

"Statistics is a method of decision making in the face of uncertainty on the basis of numerical data and calculated risks"
– Prof. Ya-Lun-Chou

Introduction

Trapezoidal fuzzy numbers of the forms (a, b, c, d) have many advantages over linear and non-linear membership functions [17]. Firstly, trapezoidal fuzzy numbers form the most generic class of fuzzy numbers with linear membership function. This class of fuzzy numbers spreads entirely the widely discussed class of triangular fuzzy numbers which implies its generic property. And therefore, the trapezoidal fuzzy numbers have numerous applications in modeling linear uncertainty in scientific and applied engineering problems including fully fuzzy linear systems, fuzzy transportation problems, ranking problems etc.

An interesting problem is to approximate general fuzzy intervals by means of trapezoidal interval data, so as to overcome the complications in the proposed calculations.

This article is divided into five parts namely 1. Some footprints of previous research about fuzzy environments 2. Preliminaries and definitions 3. One – sample t - test 4. Test of hypothesis for interval data 5. Test of hypothesis for fuzzy data using TFN 6. Conclusion and References.

1. Some footprints in the field of fuzzy environments

The following are the footprints in the field of fuzzy environments

a. Arnold [4] discussed the fuzzy hypotheses testing with crisp data.
b. Casals and Gil [8] and Son et al. analysed the Neyman-Pearson type of testing hypotheses [16].
c. Saade [14, 15] analysed the binary hypotheses testing and discussed the likelihood functions in the process of decision making.
d. Akbari and Rezaei [2] analysed a notable method for inference about the variance based on fuzzy data.
e. Grzegorzewski [10], Watanabe and Imaizumi [20] analysed the fuzzy tests for hypotheses testing with vague and ambiguous data.
f. Wu [21] discussed and analysed the statistical hypotheses testing for fuzzy data by using the notion of degrees of optimism and pessimism.
g. Viertl [18, 19] found some methods to construct confidence intervals and statistical test for fuzzy valued data.
h. Wu [22] approached a new method to construct fuzzy confidence intervals for the unknown fuzzy parameter.
i. Arefi and Taheri [3] found a new approach to test the fuzzy hypotheses upon fuzzy test statistic for imprecise and vague data.
j. Chachi et al. [9] found a new method for the problem of testing statistical hypotheses for fuzzy data using the relationship between confidence intervals and hypotheses testing.
k. B. Asady [5] introduce a method to obtain the nearest trapezoidal approximation of fuzzy numbers.

l. Abhinav Bansal [1] explored some arithmetic properties of arbitrary trapezoidal fuzzy numbers of the form (a, b, c, d).
m. Zadeh [23] analysed some notions and criterions about fuzzy probabilities.

In this paper, we provide the decision rules that are used to accept or reject the null and alternative hypothesis. In the proposed test, we split the observed interval data into two different sets of crisp data such as upper level data and lower level data, then the appropriate test statistic for the two sets of crisp data is found then, we take a decision about the given sample and the given population in the light of decision rules. In this testing method, we do not use the degrees of optimism, pessimism and h-level set.

Moreover, two numerical examples are demonstrated and in example-2 we have used the testing procedure through trapezoidal interval data which are already analysed by Wu [21], Chachi et al. [9] through triangular fuzzy numbers. But we have extended this idea to trapezoidal interval data with some modifications.

2. Preliminaries and definitions

Definition-3.1 Membership function

A characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each of the members in X. This function can be generalized to a function $\mu_{\widetilde{A}}$ such that the value assigned to the element of the universal set X fall within the specified range. That is, $\mu_{\widetilde{A}} : X \to [0, 1]$. The assigned value indicates the membership grade of the element in the set A. The function $\mu_{\widetilde{A}}$ is called the 'membership function'.

Definition-3.2 Fuzzy set

A fuzzy set $\widetilde{A}$ of a universal set X is defined by its membership function $\mu_{\widetilde{A}} : X \to [0, 1]$ and we write $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)) : x \in X\}$.

Definition-3.3 α - level set of a fuzzy set $\widetilde{A}$

The α - cut or α - level set of a fuzzy set $\widetilde{A}$ is defined by $\widetilde{A} = \{x : \mu_{\widetilde{A}}(x) \geq \alpha\}$ where $x \in X$. And $\widetilde{A}_0$ is the **closure** of the set $\{x : \mu_{\widetilde{A}}(x) \neq 0\}$.

Definition-3.4 Normal fuzzy set

A fuzzy set $\widetilde{A}$ is called normal fuzzy set if there exists an element (member) 'x' such that $\mu_{\widetilde{A}}(x) = 1$.

Definition-3.5 Convex fuzzy set

A fuzzy set $\widetilde{A}$ is called convex fuzzy set if $\mu_{\widetilde{A}}(\alpha x_1 + (1 - \alpha) x_2) \geq \min\{\mu_{\widetilde{A}}(x_1), \mu_{\widetilde{A}}(x_2)\}$ where $x_1, x_2 \in X$ and $\alpha \in [0, 1]$.

Definition-3.6 Fuzzy Number

A fuzzy set $\widetilde{A}$, defined on the universal set of real number R, is said to be 'fuzzy number' if its membership function has the following characteristics:

i. $\widetilde{A}$ is convex,
ii. $\widetilde{A}$ is normal,
iii. $\mu_{\widetilde{A}}$ is piecewise continuous.

Definition-3.7 Non-negative fuzzy number

A fuzzy number $\widetilde{A}$ is said to be non-negative fuzzy number if and only if $\mu_{\widetilde{A}}(x) = 0, \ \forall x < 0$.

Definition-3.8 Trapezoidal fuzzy number

A fuzzy number $\widetilde{A} = (a, b, c, d)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0 & ; \ x < a \\ \dfrac{x - a}{b - a} & ; \ a < x \leq b \\ 1 & ; \ b < x < c \\ \dfrac{d - x}{d - c} & ; \ c \leq x < d \\ 0 & ; \ x > d \end{cases}$$

where $a \leq b \leq c \leq d$. A trapezoidal fuzzy number is a triangular fuzzy number if $b = c$.

Definition-3.9 Core of a fuzzy set

The core of a fuzzy set is the area for which the elements have maximum degree of membership to the fuzzy set $\widetilde{A}$. That is, $c_{\widetilde{A}} = \{x : \mu_{\widetilde{A}}(x) = 1\}$.

Definition-3.10 Height of a fuzzy set

This indicates the maximum value of the membership function of a fuzzy set $\widetilde{A}$.
That is, $h_{\widetilde{A}} = \max\{\mu_{\widetilde{A}}(x)\}$.

Definition-3.11 Support of a fuzzy set

The support of a fuzzy set $\widetilde{A}$ is the area where the membership function is greater than zero. That is, $s_{\widetilde{A}} = \{x: \mu_{\widetilde{A}}(x) > 0\}$.

Definition-3.12 Non-negative trapezoidal fuzzy number

A trapezoidal fuzzy number $\widetilde{A} = (a, b, c, d)$ is said to be non-negative (non-positive) trapezoidal fuzzy number

that is, $\widetilde{A} \geq 0$ $(\widetilde{A} \leq 0)$ if and only if $a \geq 0$ $(c \leq 0)$.

A trapezoidal fuzzy number is said to be positive (negative) trapezoidal fuzzy number

that is, $\widetilde{A} > 0$ $(\widetilde{A} < 0)$ if and only if $a > 0$ $(c < 0)$.

Definition-3.13 Equality of trapezoidal fuzzy numbers

Two trapezoidal fuzzy numbers $\widetilde{A}_1 = (a, b, c, d)$ and $\widetilde{A}_2 = (e, f, g, h)$ are said to be equal,

that is, $\widetilde{A}_1 = \widetilde{A}_2$ if and only if $a = e, b = f, c = g, d = h$.

Definition-3.14 Zero trapezoidal fuzzy number

A zero trapezoidal fuzzy number is denoted by $\tilde{0} = (0, 0, 0, 0)$

Definition-3.15

Let $\widetilde{A}_1 = (a, b, c, d)$ and $\widetilde{A}_2 = (e, f, g, h)$ be two non - negative trapezoidal fuzzy numbers then,

i. $\widetilde{A}_1 \oplus \widetilde{A}_2 = (a, b, c, d) \oplus (e, f, g, h) = (a + e, b + f, c + g, d + h)$

ii. $\widetilde{A}_1 \Theta \widetilde{A}_2 = (a, b, c, d) \Theta (e, f, g, h) = (a - h, b - g, c - f, d - e)$

iii. $-\widetilde{A}_1 = -(a, b, c, d) = (-d, -c, -b, -a)$

iv. $\widetilde{A}_1 \otimes \widetilde{A}_2 = (a, b, c, d) \otimes (e, f, g, h) = (ae, bf, cg, dh)$

v. $1/\widetilde{A} \cong (1/d, 1/c, 1/b, 1/a)$

Definition-3.16

Let $A = [a, b]$ and $B = [c, d] \in D$. Then,

(i) $A \leq B$ if $a \leq c$ and $b \leq d$

(ii) $A \geq B$ if $a \geq c$ and $b \geq d$

(ii) $A = B$ if $a = c$ and $b = d$.

3. One – sample t-test for single mean

In case if we want to test (i) if a random (small) sample $x_i (i = 1, 2, ..., n)$ of size n < 30 has been drawn from a normal population with a specified mean, say μ_0 or (ii) if the sample mean differs significantly from the hypothetical value μ_0 of the population mean, then under the null hypothesis H_0:

(a) The sample has been drawn from the population with mean μ_0 or

(b) There is no significant difference between the sample mean $\bar{x}$ and the population mean μ_0, in this case, **the test statistic** is given by

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \quad \text{where } \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\text{and } s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

follows Student's t-distribution with $(n - 1)$ degrees of freedom.

We now compare the calculated value of 't' with the tabulated value at certain level of significance. Let t_α be the tabulated value at α level of significance and 't' be the calculated value and we set the null hypothesis as $H_0: \mu = \mu_0$ and alternative hypothesis as below:

Alternative Hypothesis	Rejection Region
$H_A: \mu > \mu_0$	$t \geq t_{\alpha, n-1}$ [Upper tailed test]
$H_A: \mu < \mu_0$	$t \leq -t_{\alpha, n-1}$ [Lower tailed test]
$H_A: \mu \neq \mu_0$	$\lvert t \rvert \geq t_{\alpha/2, n-1}$ [Two tailed test]

That is, if $|t| > t_\alpha \Rightarrow$ The null hypothesis H_0 is rejected (one tailed test) and if $|t| < t_\alpha \Rightarrow$ the null hypothesis H_0 may be accepted (one tailed test) at the level of significance adopted. If $|t| < t_{\alpha/2} \Rightarrow$ the null hypothesis H_0 is accepted (two tailed test).

Now, the $100(1-\alpha)\%$ confidence limits for the population mean μ corresponding to the given sample are given below:

$$\bar{x} - t_{\alpha/2, n-1}\left(\frac{s}{\sqrt{n}}\right) < \mu < \bar{x} + t_{\alpha/2, n-1}\left(\frac{s}{\sqrt{n}}\right)$$

4. Test of Hypothesis for Interval Data

Let $\{[x_i, y_i];\ i = 1, 2, ..., n\}$ be a random sample of size $n(<30)$ such that $\{x_i;\ i = 1, 2, ..., n\}$ and $\{y_i;\ i = 1, 2, ..., n\}$ are the random samples from two distinct normal population with the population means η and μ and the sample means η_0 and μ_0 of the sample respectively.

Then the null hypothesis $H_0:[\eta, \mu] = [\eta_0, \mu_0]$, that is, $\eta = \eta_0$ and $\mu = \mu_0$ and the alternative hypotheses

$$H_A:[\eta, \mu] < [\eta_0, \mu_0], \text{ that is } \eta < \eta_0 \text{ and } \mu < \mu_0$$

$$H_A:[\eta, \mu] > [\eta_0, \mu_0], \text{ that is } \eta > \eta_0 \text{ and } \mu > \mu_0$$

$$H_A:[\eta, \mu] \neq [\eta_0, \mu_0], \text{ that is } \eta \neq \eta_0 \text{ or } \mu \neq \mu_0$$

We now consider the random sample of lower value and upper value of the given interval data be $S_L = \{x_i;\ i = 1, 2, ..., n\}$ and $S_U = \{y_i;\ i = 1, 2, ..., n\}$ respectively. The sample mean of S_L and S_U are s_x and s_y respectively.

The proposed test statistic is given by $t^L = \frac{(\bar{x} - \eta)}{s_x/\sqrt{n}}$ and $t^U = \frac{(\bar{y} - \mu)}{s_y/\sqrt{n}}$

The rejection region of the alternative hypothesis for α level of significance is given by

Alternative Hypothesis H_A	Rejection Region for α Level Test
$H_A:[\eta, \mu] > [\eta_0, \mu_0]$	$t^L \geq t_{\alpha, n-1}$ and $t^U \geq t_{\alpha, n-1}$ (Upper tailed test)
$H_A:[\eta, \mu] < [\eta_0, \mu_0]$	$t^L \leq -t_{\alpha, n-1}$ and $t^U \leq -t_{\alpha, n-1}$ (Lower tailed test)
$H_A:[\eta, \mu] \neq [\eta_0, \mu_0]$	$\|t^L\| \geq t_{\alpha/2, n-1}$ or $\|t^U\| \geq t_{\alpha/2, n-1}$ (Two tailed test)

If $|t^L| < t_{\alpha, n-1}$ (One tailed test) and
$|t^U| < t_{\alpha, n-1}$ (One tailed test)

$\Rightarrow$ The null hypothesis H_0 is accepted.

$\Rightarrow$ The difference between $[\eta, \mu]$ and $[\eta_0, \mu_0]$ is not significant at α level. Otherwise the alternative hypothesis H_A is accepted

If $|t^L| < t_{\alpha/2, n-1}$ (Two tailed test) and
$|t^U| < t_{\alpha/2, n-1}$ (Two tailed test)

$\Rightarrow$ The null hypothesis H_0 is accepted

$\Rightarrow$ The difference between $[\eta, \mu]$ and $[\eta_0, \mu_0]$ is not significant at α level. Otherwise the alternative hypothesis H_A is accepted.

Also, the $100(1-\alpha)\%$ confidence limits for the population mean $[\eta, \mu]$ corresponding to the given sample are given below:

$$\left[\bar{x} - t_{\alpha/2,n-1}\left(\frac{s_x}{\sqrt{n}}\right),\ \bar{y} - t_{\alpha/2,n-1}\left(\frac{s_y}{\sqrt{n}}\right)\right] < [\eta,\ \mu] < \left[\bar{x} + t_{\alpha/2,n-1}\left(\frac{s_x}{\sqrt{n}}\right),\ \bar{y} + t_{\alpha/2,n-1}\left(\frac{s_y}{\sqrt{n}}\right)\right]$$

Example-1

The department of career and placement of an inspection committee of a University claims that the placement percentage of the students from its affiliated institution is between 50 and 65. Only 12 colleges in that zone are selected at random. The colleges are observed and the placement percentages of each of the colleges are recorded. Due to some limited resources, the minimum and maximum of the placement percentage of each of the colleges can only be observed. Therefore, the placement percentages of the colleges are taken to be 'intervals' as follows [11]:

[44, 53], [40, 38], [61, 69], [52, 57], [32, 46], [44, 39], [70, 73], [41, 48], [67, 73], [72, 74] [53, 60], [72, 78].

Solution

We now consider the test hypotheses be $H_0{:}\mu = \mu_0$ and $H_A{:}\mu \neq \mu_0$ (Two tailed test)

Here, the size of the sample n = 12 and the population mean is [50, 65] with unknown sample S.D. We use 5% level of significance. Now, the sample mean value of the lower and upper interval values are

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \Rightarrow \bar{x}^L = 54 \text{ and } \bar{x}^U = 59$$

And the sample S.D. of the lower and upper interval values are

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$$

$$\Rightarrow S^L = 14.06 \text{ and } S^U = 14.30.$$

Now, the tabulated value of t at $(n-1) = 11$ degrees of freedom at 5% level of significance is $T_\alpha = 2.20$. And the lower value of μ_0 is $\mu_0^L = 50$ and the upper value of μ_0 is $\mu_0^U = 65$.

The test statistic:

$$t^L = \frac{\left(\bar{x}^L - \mu_0^L\right)}{s^L/\sqrt{n}} = 0.9855 \text{ and}$$

$$t^U = \frac{\left(\bar{x}^U - \mu_0^U\right)}{s^U/\sqrt{n}} = -1.4535$$

Since $\left|t^L\right| < T_\alpha$ and $\left|t^U\right| < T_\alpha$, the null hypothesis " the population mean $\mu = \mu_0$" is accepted and the 95% confidence limits for the population mean μ is

$$\left[\bar{x}^L - t_{\alpha/2,n-1}\left(\frac{s^L}{\sqrt{n}}\right),\ \bar{x}^U - t_{\alpha/2,n-1}\left(\frac{s^U}{\sqrt{n}}\right)\right] < [\eta,\ \mu] < \left[\bar{x}^L + t_{\alpha/2,n-1}\left(\frac{s^L}{\sqrt{n}}\right),\ \bar{x}^U + t_{\alpha/2,n-1}\left(\frac{s^U}{\sqrt{n}}\right)\right]$$

$$\Rightarrow [45.0707,\ 49.9183] < [\eta,\ \mu] < [62.9293,\ 68.0817]$$

5. Testing Hypothesis for Fuzzy Data using TFN

Trapezoidal Fuzzy Number to Interval

Let a trapezoidal fuzzy number be $\widetilde{A} = (a, b, c, d)$, then the fuzzy interval [17] in terms of α - cut interval is defined as follows:

$$\widetilde{A}_\alpha = \left[a + (b-a)\alpha,\ d - (d-c)\alpha\right]; \quad 0 \le \alpha \le 1 \text{ -- [R1]}$$

Suppose that the given sample is a fuzzy data that are trapezoidal fuzzy numbers and if we want to test the hypothesis about the population mean, then we can transfer the given fuzzy data into interval data by using the relation [R1].

Example-2

The marketing department for a tire and rubber company wants to claim that the average life of a tire, the company recently developed exceeds the well-known average tire life of a competitive brand, which is known to be 32000 miles. Only 24 new tires were tested because the tests are tedious and take considerable time to complete. Six cars of a particular model and brand were used to test the tires. Car model and brand were fixed so that other car-related aspects did not come into play. The situation was that the life of the tires were not known exactly. The obtained life of the tire was around a number. Therefore the tire life numbers were taken to be trapezoidal fuzzy numbers as follows [12]:

$\tilde{x}_1 = (33266,\ 33671,\ 34177,\ 34889)$

$\tilde{x}_2 = (32093,\ 32613,\ 33149,\ 33255)$

$\tilde{x}_3 = (32585,\ 32885,\ 33215,\ 33787)$

$\tilde{x}_4 = (31720,\ 32143,\ 32819,\ 33497)$

$\tilde{x}_5 = (32806,\ 33026,\ 33346,\ 33908)$

$\tilde{x}_6 = (31977,\ 32237,\ 32817,\ 33034)$

$\tilde{x}_7 = (33065,\ 33345,\ 33993,\ 34131)$

$\tilde{x}_8 = (31943,\ 32303,\ 33053,\ 33212)$

$\tilde{x}_9 = (30743,\ 31325,\ 31994,\ 32460)$

$\tilde{x}_{10} = (32169,\ 32955,\ 33148,\ 33968)$

$\tilde{x}_{11} = (32415,\ 33101,\ 33955,\ 34072)$

$\tilde{x}_{12} = (32900,\ 33452,\ 33873,\ 34335)$

$\tilde{x}_{13} = (32687,\ 33195,\ 33431,\ 33908)$

$\tilde{x}_{14} = (30327,\ 30975,\ 31295,\ 31445)$

$\tilde{x}_{15} = (32185,\ 32723,\ 32960,\ 33186)$

$\tilde{x}_{16} = (31187,\ 31436,\ 31999,\ 32237)$

$\tilde{x}_{17} = (33423,\ 34017,\ 34258,\ 34771)$

$\tilde{x}_{18} = (33208,\ 33860,\ 34280,\ 34876)$

$\tilde{x}_{19} = (31639,\ 32481,\ 33026,\ 33542)$

$\tilde{x}_{20} = (30945,\ 31634,\ 32325,\ 32739)$

$\tilde{x}_{21} = (31511,\ 32038,\ 32727,\ 33064)$

$\tilde{x}_{22} = (30826,\ 31610,\ 32406,\ 32913)$

$\tilde{x}_{23} = (33063,\ 33762,\ 34181,\ 34449)$

$\tilde{x}_{24} = (33464,\ 34062,\ 34388,\ 34974)$

Let us consider the testing hypotheses

$\tilde{H}_0 :\ \tilde{\mu} \cong \widetilde{32000}$ and

$\tilde{H}_A :\ \tilde{\mu} > \widetilde{32000}$

where $\widetilde{32000}$ means "around 32000", which regarded as a "linguistic data".

Therefore,

$\tilde{H}_0$: The average life of the tire is around 32000.

$\tilde{H}_A$: The average life of the tire is greater than 32000.

We may assume the membership function of $\widetilde{32000}$ as $\widetilde{32000} = (30000,\ 32000,\ 33000,\ 34000)$. The size of the sample $n = 24$ and the population mean is $\tilde{\mu}_0 = (30000,\ 32000,\ 33000,\ 34000)$ with unknown sample standard deviation.

Using relation [R1], we convert the fuzzy data into interval data and are listed below:

$\tilde{x}_1 = [33266 + 405\alpha,\ 34889 - 712\alpha]$

$\tilde{x}_2 = [32093 + 520\alpha,\ 33255 - 106\alpha]$

$\tilde{x}_3 = [32585 + 300\alpha,\ 33787 - 572\alpha]$

$\tilde{x}_4 = [31720 + 423\alpha,\ 33497 - 678\alpha]$

$\tilde{x}_5 = [32806 + 220\alpha,\ 33908 - 562\alpha]$

$\tilde{x}_6 = [31977 + 260\alpha,\ 33034 - 217\alpha]$

$\tilde{x}_7 = [33065 + 280\alpha,\ 34131 - 138\alpha]$

$\tilde{x}_8 = [31943 + 360\alpha,\ 33212 - 159\alpha]$

$\tilde{x}_9 = [30743 + 582\alpha,\ 32460 - 466\alpha]$

$\tilde{x}_{10} = [32169 + 786\alpha,\ 33968 - 820\alpha]$

$$\tilde{x}_{11} = [32415 + 686\alpha, 34072 - 117\alpha]$$

$$\tilde{x}_{12} = [32900 + 552\alpha, 34335 - 462\alpha]$$

$$\tilde{x}_{13} = [32687 + 508\alpha, 33908 - 477\alpha]$$

$$\tilde{x}_{14} = [30327 + 648\alpha, 31445 - 150\alpha]$$

$$\tilde{x}_{15} = [32185 + 538\alpha, 33186 - 226\alpha]$$

$$\tilde{x}_{16} = [31187 + 249\alpha, 32237 - 238\alpha]$$

$$\tilde{x}_{17} = [33423 + 594\alpha, 34771 - 513\alpha]$$

$$\tilde{x}_{18} = [33208 + 652\alpha, 34876 - 596\alpha]$$

$$\tilde{x}_{19} = [31639 + 842\alpha, 33542 - 516\alpha]$$

$$\tilde{x}_{20} = [30945 + 689\alpha, 32739 - 414\alpha]$$

$$\tilde{x}_{21} = [31511 + 527\alpha, 33064 - 307\alpha]$$

$$\tilde{x}_{22} = [30826 + 784\alpha, 32913 - 507\alpha]$$

$$\tilde{x}_{23} = [33063 + 699\alpha, 34449 - 268\alpha]$$

$$\tilde{x}_{24} = [33464 + 598\alpha, 34974 - 586\alpha]$$

Now, we obtain the mean value of the lower and upper interval values and are as given below:

$$\bar{x}^L = 32168.63 + 529.25\alpha \text{ and}$$
$$\bar{x}^U = 33610.50 + 408.63\alpha$$

And the sample S. D. of the lower and upper interval values are:

$$S^L = \sqrt{\frac{1}{n-1}\left[\sum_{i=1}^{n}\left(x_i^L - \bar{x}^L\right)^2\right]}$$

$$\Rightarrow \quad S^L = \sqrt{33316.02\alpha^2 - 80004.54\alpha + 838201.99}$$
$$\text{and } S^U = \sqrt{740506.58\alpha^2 - 169304.62\alpha + 815219.02}$$

Now, we set the null hypothesis: $H_0: \tilde{\mu} = \tilde{\mu}_0$ and the alternative hypothesis: $H_A: \tilde{\mu} > \tilde{\mu}_0$ (Upper tailed test) Choosing 5% level of significance and the table value of 't' for 23 degrees of freedom is $T_\alpha = 1.714$

And the interval representation of $\tilde{\mu}_0$ is given by

$$\tilde{\mu}_0 = \left[\mu_0^L,\ \mu_0^U\right]$$

where $\mu_0^L = 30000 + 2000\alpha$ and

$\mu_0^U = 34000 - 1000\alpha$; $0 \le \alpha \le 1$.

And therefore,

$$t^L = \frac{\bar{x}^L - \mu_0^L}{s^L/\sqrt{n}} = \begin{cases} 11.6042\ ; & \alpha = 0 \\ 3.8429\ ; & \alpha = 1 \end{cases}$$

$$\Rightarrow t^L > T_\alpha = 1.714 \text{ for all } \alpha,\ 0 \le \alpha \le 1.$$

And,

$$t^U = \frac{\bar{x}^U - \mu_0^U}{s^U/\sqrt{n}} = \begin{cases} -2.1134\ ; & \alpha = 0 \\ 1.8105\ ; & \alpha = 0.53 \\ 1.8757\ ; & \alpha = 0.54 \\ 4.2402\ ; & \alpha = 1 \end{cases}$$

$$\Rightarrow t^U > T_\alpha \text{ for } 0.53 \le \alpha \le 1$$

6. Conclusion

Since for $0.53 \le \alpha \le 1$, $t^L > T_\alpha$ and $t^U > T_\alpha$, the alternative hypothesis is accepted. Therefore, $\tilde{H}_A$: The average tire life is approximately greater than 32000 is accepted based on the given fuzzy data with condition $0.53 \le \alpha \le 1$.

Remark

The results obtained from Example-2 differ by 0.01 level of lower limit of α $(0.53 \le \alpha \le 1)$ when compared with the results in Wu [22], Chachi et al. [9] and D. Kalpanapriya et al. [12] which is $(0.54 \le \alpha \le 1)$.

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