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SIZING FACEWIDTHS IN CYLINDRICAL WORM DRIVES

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ABSTRACT - *A parametric study is performed in sizing the facewidths for cylindrical wormsets which is delineated into effective and allowance (non-effective) facewidths. The nominal facewidth of the worm is the sum of the effective and allowance facewidths. The tangent geometry method is commonly used in sizing wormgear facewidth. It is extended in this study for sizing the worm facewidth with a slight modification for both wormgear and worm facewidths. Three methods for sizing the worm facewidth are explored; namely the tangent geometry, axial pitch, and empirical design methods. The methods are applied to five design applications using four defined design method consistency factors. The tangent geometry method is found to be the most consistent when compared with the other methods. Based on the tangent geometry method, the wormgear nominal facewidth should be about 15% larger than the effective facewidth, the worm nominal facewidth should be about 25% larger than the effective facewidth, and the wormgear throat radial allowance should be about 75% of the normal module value. The study shows potential wormgear size reduction in three of the five examples presented when the tangent geometry is used for sizing the gear facewidth. The tangent geometry method provides adequate effective and consistent sizes for the gear facewidth, worm facewidth, and throat radial allowance. It reduces the arbitrariness of the choice of the effective and nominal facewidths for cylindrical worms drives. Therefore, the tangent geometry method is recommended as a preferred design method for sizing the facewidths of cylindrical wormsets.*

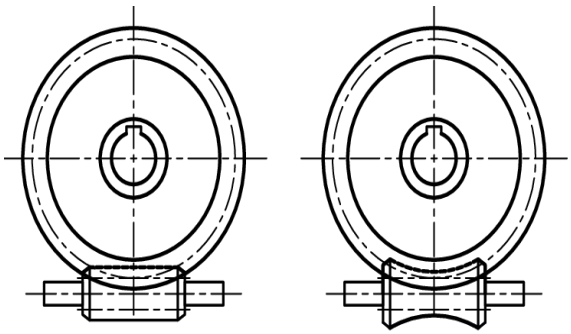
Keywords: geometry, facewidth, module, planes, sizing, tangent, wormgear, worm

1.0 INTRODUCTION

A wormset consists of a wormgear and a worm which is used when a large speed reduction or increase and non-intersecting shaft axes are required. The wormgear is a special type of a helical gear, while the worm is practically a power screw. The worm is made to drive the wormgear in a speed reduction gearbox, which is the most common configuration. The operation of wormsets is smooth and quiet as the worm threads continuously slide on the gear teeth [1]. Wormsets offer simplified assembly with compact profiles but are typically less efficient than other types of gear drives [2].

The types of wormsets are plain, spiroid, cylindrical, and globoidal, wormsets [3, 4]. The plain wormset consists of a straight worm and a regular helical gear. The spiroid wormset consists of a tapered worm and a throated wormgear. Plain and spiroid wormsets are not common and will not be discussed further.

The common types of wormsets are the cylindrical and globoidal wormsets, which are schematically shown in Fig. 1a and Fig. 1b, respectively. The cylindrical wormset has a throated wormgear and a non-throated or straight worm. It is also called a single-enveloping or singly-throated wormset. The wormgear is throated in its axial plane which corresponds to transverse (diametral) plane of the worm. Throating allows the gear to partially wrap around the worm, thereby enhancing greater contact between the pairs. It modifies what would be a point contact theoretically, into largely a line contact, allowing higher load capacity. In the globoidal wormset of Fig. 1b, both the worm and wormgear are throated. It is also called a double-enveloping wormset. The globoidal wormset has a higher load capacity [5] than the cylindrical wormset due partly to the greater contact area facilitated by the double throating and also, due to the higher number of gear teeth in mesh simultaneously in operation compared to cylindrical wormsets. But globoidal wormsets are more difficult to manufacture and assemble [2]. This study focuses on the cylindrical wormset which is the most popular of all wormsets, but the design methods presented may be applied to globoidal wormsets.



a) Cylindrical b) Globoidal

Fig. 1: Common wormsets

The length of the tooth of a gear along the shaft axis is called the facewidth. Other design parameters of a wormset are the normal module, pitch diameter, gear teeth number, and worm thread number. Other gear design parameters are usually obtained based on standard proportions prescribed by national and or international gear design organizations when the normal module and gear teeth numbers are known. The facewidth may be related proportionally to the normal module or the pitch diameter of the gear, especially in cylindrical and bevel gears. But this is not commonly done in the design of wormsets. The normal module of the gear may be determined from some suitable serviceability criterion such as pitting, scoring, or beam strength [6, 7]. Pitting resistance is proportional to the square root of gear facewidth [2, 7, 8, 9, 10]. The flash temperature number in scoring resistance is related to the ¾ power of gear facewidth [11]. The beam strength capacity of gears is directly proportional to gear facewidth [2, 7, 8, 9, 12]. According to Collins et al [13], the design of wormset is largely empirical due to the complexities in profile geometry, poorly understood cyclic contact stresses, and interactions between failure modes such as adhesive, abrasive, pitting, scoring wear. Consequently, wormset designers use different schemes for the estimation of the facewidths of worm and wormgear [11]. In fact, empirical methods are commonly used for determining facewidths of worm and wormgears, especially the worm facewidth. But the final sizes of the facewidths must be geometrically consistent with the assembly of the wormset.

The American Gear Manufacturing Association (AGMA) [14] recommends that the wormgear facewidth should be limited to 67% of the worm pitch diameter, while some authors use 73% of the worm pitch diameter [15, 16]. But other proportions such as 57.7% of the worm pitch diameter [11] or 50% of the outside diameter of the worm [17] are used also. Hamrock et al [2] provided two expressions for estimating the gear facewidth for cylindrical wormsets; one for axial pitch less than 4.06 mm, and the other for axial pitch greater or equal to 4.06 mm. Petrov et al [16] and Dobrovolsky et al [18] estimated the wormgear facewidth as 75% of the addendum diameter of the worm for worms with one or two threads, and 67% for those with more than 2 threads. Khurmi and Gupta, [19] provided an empirical formula for the wormgear facewidth as a linear function of the axial pitch and Mott [9] says an initial size of two axial pitches may be used. As per DIN recommendation, the gear facewidth should be about 80% of the worm pitch diameter [20].

It appears that a tangent geometric rule for estimating the gear facewidth has gained acceptance [3, 4, 8, 9, 11, 12, 16, 20, 21]. The rule states that the wormgear facewidth may be taken as the length of a tangent line drawn to the pitch circle diameter of the worm in the axial plane of the gear to intercept the worm addendum circle diameter on both sides of the worm midplane. This rule is called the tangent geometry method in this paper and it is currently based on the transverse module in the transverse plane of the worm. The method is considered to provide a safe wormgear facewidth for cylindrical wormsets [11].

The sizing of the worm facewidth is commonly done empirically with some designers using one expression for any number of threads on a worm [15, 19]. Others use one expression for worms with one or two threads, and another expression for worms with more than two threads [16, 18, 22]. These expressions are directly proportional to the worm axial pitch and wormgear pitch diameter. Generally, the estimates of the worm facewidth size from these empirical methods can vary from different designers who also give different interpretations to the size. Some consider it as the effective worm facewidth, while others treat it as the nominal facewidth size.

The manufacturing of wormgears and worms necessitate some precautions so as to ensure functionality and minimize cost. For example, it is recommended that the worm lead angle may be limited to 6^{o} per thread [11, 23] so as to minimize the difficulty of designing production tools and producing accurate curvature on worm threads and gear teeth [11]. Higher efficiency in wormsets depends on the hardness and surface roughness of the worm threads because frictional resistance is lower for smoother surfaces. Therefore, after worm threads have been cut and heat-treated, they are usually ground and sometimes polished [16]. Generally, some manufacturing allowances in facewidth sizes are necessary for worms and wormgears to avoid tool damage or enhance ease of manufacturing [16, 22].

2.0 MOTIVATION FOR STUDY

From the discussions above, it is obvious that the sizing of facewidths for cylindrical wormsets is not quite systematic. But, it is often desired that analytical solutions are available and used for engineering design problems in a systematic way. Though the empirical approaches for estimating cylindrical wormset facewidth sizes produce functional designs in practice, they appear arbitrary, and result in considerable variations in the facewidth sizes from different designers. Also, the worm facewidth size is interpreted differently by designers; some treating it as the effective size, while others treat it as the nominal size. In such a scenario, there appears to be a need for a more systematic and analytical model for the worm facewidth size.

The primary goal of this study is to investigate the possibility of using the tangent geometry method in sizing the facewidth of cylindrical worms. If the possibility is established, the method will provide a unified and systematic approach in the facewidth sizing of cylindrical wormsets. Because the wormgear is essential a helical gear, the analysis in the study is focused on the normal plane of the wormset, not on the transverse or diametral plane. The implication is that some equations for the wormgear and worm on the two planes, though similar, are not identical and can produce slight differences in addendum and dedendum diameters of the wormset.

To encourage and promote a systematic design and consistency of facewidth size interpretation, the facewidth of cylindrical wormsets is delineated into effective and allowance (non-effective) facewidths, the sum of which defines the nominal facewidth [6,7, 10]. The effective facewidth defines the length of the facewidth in the contact zone where meshing occurs between the thread(s) of a worm and some teeth on the wormgear. The allowance facewidth is any additional length to the effective facewidth. In the case of a worm, the allowance facewidth is the extension of the worm threaded length beyond the effective facewidth on both the leading and trailing sides. The gear teeth enter the contact zone by moving over the trailing part of the allowance facewidth and exit by moving over the leading part of the allowance facewidth. The wormgear teeth in the effective facewidth portion are active in transmitting the applied load on the wormset. If the effective facewidth of a cylindrical worm is not differentiated from the nominal facewidth, the load capacity of cylindrical wormset cannot be ascertain with clarity. This is due to the fact that it is only the effective facewidth of the worm, a portion of the nominal facewidth that determines the load capacity of the wormset. The differentiation is then very essential in understanding how cylindrical wormsets work and evaluating their load capability. Hence, this study appears to be necessary and relevant in wormset design and applications.

3.0 WORMGEAR TERMINOLOGY

A wormgear is a special helical gear with a nominal helix angle equal to the lead angle of the worm and the two have the same hand which may be a right or left hand. Fig. 2 shows the terms commonly associated with a helical gear tooth. The facewidth of the gear is measured parallel to the axis of the shaft on which the gear is mounted. The nominal helix angle is the inclination angle of the gear tooth to the shaft axis. The normal circular pitch is measured on a plane perpendicular to the nominal helix angle, while the transverse circular pitch is measured on a plane perpendicular to the shaft axis. The proportions of a wormgear tooth are usually taken as those of a regular helical gear [20] which are determined by national and or international standards.

Helical gears have teeth cut at the nominal helix angle to their cylindrical pitch diameter or shaft axis. The plane perpendicular to the nominal helix angle is called the normal plane. The plane perpendicular to the axis of the shaft carrying the gear is the transverse or diametral plane, and is the plane of rotation. Tooth proportions and functions of helical gears can be described in terms of the normal module measured in the normal plane or the transverse module measured in the transverse plane. Many of the proportions of helical gears are modified by the nominal helix angle for the same standard normal module value.

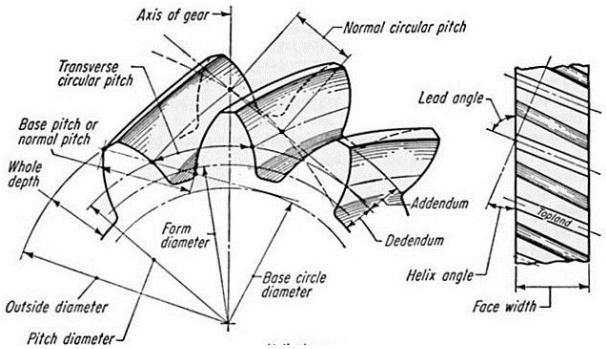


Fig. 2: Helical gear tooth terminology [24]

Several advantages may be associated with the nominal helix angle. Most notably, it enables gradual tooth engagement, reducing noise and increasing load capacity compared to spur gears with teeth parallel to the shaft axis. The nominal helix angle can reduce internal dynamic load in gearing to about 75% of the value in spur gears [20]. It causes the formation of inclined contact lines on the gear tooth faces and flanks that reduce the bending stress at the root of a helical gear tooth [16]. The nominal helix angle additionally has the effect of slightly reducing the transmission error in helical gearsets, making them slightly less sensitive to assembly and manufacturing errors [7, 17]. For the same normal module and normal pressure angle in the normal plane, standard spur gear hobs can be used to cut different helix angles, allowing spur gear hubs to be used often in generating helical gears [11]. In the transverse plane, a different hob is required for different helix angles even for the same transverse module and transverse pressure angle [11, 25]. Helical gears of standard pressure angle and module may be produced by shaping operations, but standard spur gear shaper cutters cannot be used to cut helical gears [11]. The gradual teeth engagement in helical

gears permits higher pitch velocity and greater power transmission capacity.

A major disadvantage of helical gears is the generation of a thrust force component on supporting shafts and bearings. The magnitude of the thrust increases with increasing nominal helix angle [8, 9, 16, 26]. Consequently, wormgears are not immuned from this disadvantage, but the influence is not significant in wormsets with nominal helix angles of low values. In non-parallel axis helical gearing, the mesh relative sliding velocity is influenced by the nominal helix angle [20] and sliding friction is significant in determining scoring capacity. Due to the influence of the nominal helix angle on the radial proportions of a helical gears, it is more difficult to obtain exact or integer values of pitch diameter and centre distance in helical gearsets. If a specific pitch diameter and or centre distance is required, gear profile shifting or a compromise helix angle would have to be used to achieve them [26].

4.0 WORM TERMINOLOGY

Fig. 3 shows the common terms associated with a worm which may be considered as a helical rack, but it is practically a special type of power screw. The nominal facewidth is the total threaded length of the worm parallel to the axis of the worm shaft. The worm can be made integral with the shaft or as a shell mounted and secured to the shaft, but most worms are made integral with the shaft [7, 10]. The thread profile of a worm may be trapezoidal, involute or some other profile [16]. The common worm thread profiles for cylindrical worm drives are designated as ZA, ZN, ZK, ZI, and ZC [10, 20]. In common practice, straight cylindrical worms with trapezoidal profile of 40° included angle and gear of involute profile are used in wormsets [16, 20, 27].

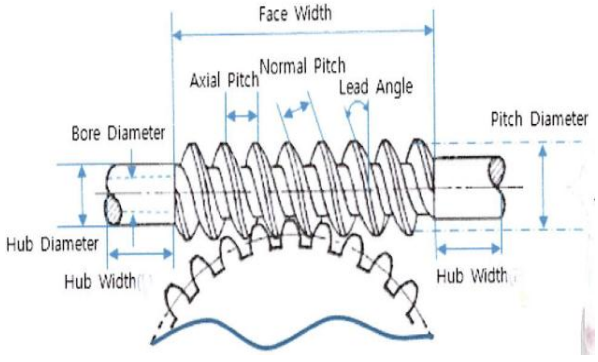


Fig. 3: Worm terminology [28]

The most popular worm type is the Archimedean worm that has straight-sided threads in the axial plane of the worm. Worms may have a single thread or multiple threads and the common numbers are 1, 2, 4, or 6 [3] for power gearing, but the number may be from 1 to 10 [8, 29] and more threads may be used in large units.

A functional wormset must have a worm and wormgear with the same normal pressure angle and normal module. Additionally, the nominal helix angle of the wormgear must be equal to the lead angle of the worm, and the hand of the lead angle must be the same as that of the nominal helix angle of the gear [30]. Eq. (1) provides the relationships between the basic kinematic parameters which characterize a wormset.

$$\psi = \gamma \qquad m_a = m_t = \frac{m_n}{\cos\psi} \qquad (1)$$

Note that Eq.(1) has two expressions which should be interpreted as Eq. (1a) and Eq. (1b) from left to right. All equations with more than one expression should be similarly interpreted. Eq. (1a) gives the relationship between the nominal helix angle of the wormgear and lead angle of the worm. Eq. (1b) gives the relationship between the normal and axial modules or between the normal and transverse modules. The axial and transverse modules are equal.

The circular pitch of a wormgear must match the axial pitch of a worm for them to mate properly. This is ensured by making the worm and wormgear to have the same normal module. The gear tooth size is determined by the normal module and other gear parameters depend on a gear tooth system, normal pressure angle and established tooth proportions based on the normal module. As mentioned earlier, the proportions of a wormgear tooth will be taken as those of a regular helical gear. The recommendations of the International Standardization Organizations (ISO) [31] for helical gear tooth proportions are chosen for this study.

$$p_n = \pi m_n \qquad p_t = p_a = \pi m_t \qquad (2)$$

Eq. (2a) gives normal pitch and Eq. (2b) gives the relationship for the transverse pitch, axial pitch, and transverse module of the gear. The axial plane of the worm corresponds to the transverse plane of the wormgear. Eq. (2a) refers to the normal plane which is common to both the worm and wormgear. Eq. (1a), Eq. (1b) and Eq. (2b) are required for kinematic functionality.

When the wormgear is treated as a helical gear, and based on ISO [31] gear standard proportions, the pitch diameters of the wormgear and worm are calculated from Eq. (3).

$$d_1 = z_w m_t \qquad d_2 = z_2 m_t \qquad (3)$$

The addendum and dedendum (root) diameters of the wormgear and worm are calculated from Eq. (4a) and Eq. (4b), respectively.

$$d_a = d\ + 2m_n \qquad d_f = d\ - 2.4m_n \qquad (4)$$

Eq. (5) gives different expressions for the lead angle of the worm.

$$\sin\gamma = \frac{m_n z_1}{d_1} \qquad \tan\gamma = \frac{m_t z_1}{d_1} \qquad \tan\gamma = \frac{z_1}{z_w} \qquad (5)$$

5.0 WORMGEAR EFFECTIVE FACEWIDTH

The effective facewidth of a wormgear is the portion of its total facewidth that is useful in transmitting the required load. This portion of the wormgear facewidth provides the contact surfaces for load transmission. Hence, the load capacity of a cylindrical wormset depends on the effective facewidth of the wormgear. Wormgear effective facewidth varies with the pitch diameter of the worm and the depth of thread [21]. Several empirical schemes may be used for the initial estimate of the wormgear facewidth size in worm drives. However, they do not provide consistent results and appear too arbitrary.

Some designers have estimated the effective portion of a wormgear facewidth from the length of a tangent line in the axial plane of the wormgear that is drawn through the worm pitch circle diameter to intersect the tip or addendum circle diameter of the worm [8, 11, 21]. This approach is called the tangent geometry method in this study. Fig. 4 shows a mesh of a cylindrical wormset which depicts the geometric layout for the tangent geometry method. The line O/D or O/D/ in Fig. 4 represents the addendum radius of the worm and the line O/B/ represents its pitch radius. From Eq. (4a), the addendum radius is one normal module more than the pitch radius of a gear in a helical gear. The line DD/ in Fig. 4 is drawn tangent to the pitch diameter circle of the worm. Therefore, the length of the line DD/, represents the useful or effective facewidth of the wormgear during operation according to this tangent line rule.

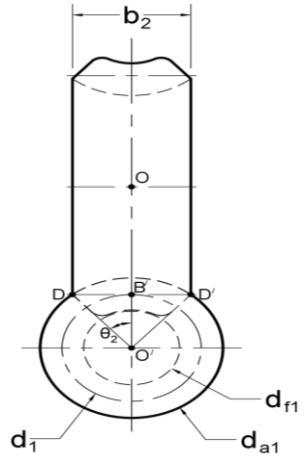


Fig. 4: Cylindrical wormgear facewidth

Referring to Fig. 4:

$$b_2 = DD^{/} = 2DB^{/} = 2\sqrt{O^{/}D^2 - O^{/}B^2} \qquad (6a)$$

When the worm parameters are substituted into Eq. (6a), the result is Eq. (6b).

$$b_2 = 2\sqrt{(0.5d_1 + m_n)^2 - (0.5d_1)^2} \qquad (6b)$$

Eq. (6b) simplifies to Eq. (7).

$$b_2 = 2\sqrt{m_n(d_1 + m_n)} \qquad (7)$$

Eq. (7) is similar to the expression for the gear facewidth in several presentations [3, 16, 18, 20, 22, 25]. However, the transverse module is used instead of the normal module in those expressions, essentially treating the wormgear as a spur gear [3, 8, 9, 20, 25, 32]. The transverse plane version of Eq. (7) is recommended by some authors [2, 8, 9, 11, 21]. According to Dudley [11], the size estimate from the transverse plane version of Eq. (7) is considered to be always safe [11] and may be made just slightly larger. Shigley and Mischke [21], considered such estimate as the maximum effective facewidth of the wormgear, and any additional facewidth is of very little value and a waste of material These opinions differ in interpretations of the size but essentially accept the estimate as being adequate for the wormgear.

6.0 WORM FACEWIDTH SIZING METHODS

The worm effective facewidth is the portion of its total threaded length that transmits the required load. Like the wormgear, this is the useful portion that provides the contact surfaces for load transmission. The nominal facewidth of a worm is the total threaded length.

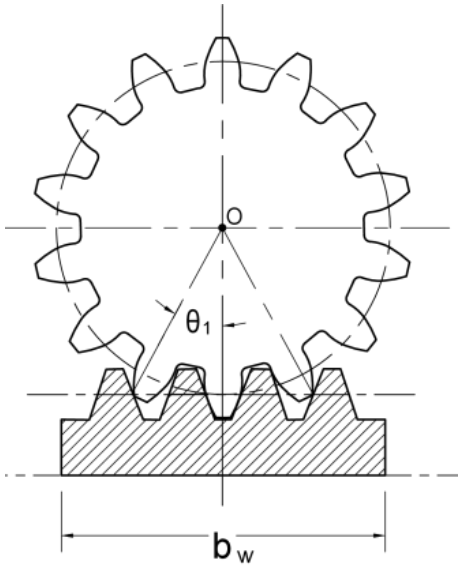


Fig. 5: Wormset mesh

Fig. 5 shows a schematic diagram of the engagement of wormgear teeth with a worm thread. The portion of the worm face enclosed by total worm face-angle ($2\theta_1$), is called the contact zone which defines the effective facewidth. This is observed to include two axial thicknesses. It is within this zone that gear teeth come into contact with the thread(s) of a worm and allows the transmission of load between the worm and worm gear. Any portion of the worm thread(s) outside the effective facewidth is termed the allowance zone and its length is the allowance length. This section discusses three approaches of axial pitch, tangent geometry, and empirical methods, that may be used to estimate the effective facewidth of cylindrical worms.

6.1 Axial Pitch Method

The axial pitch method seeks to assign some number of axial pitches as the effective facewidth of the worm. The nature of gear teeth engagement in a wormset is that of a gradual transfer of load from one tooth to the other. Therefore, at least two gear teeth should be in mesh with a worm thread at different depths in the thread groove for the smooth transmission of load. This is why it is generally required that the contact ratio of gear meshes should be greater than unity [2, 8, 9, 20]. For one gear tooth to carry the full load in wormsets, there will be one trailing and one leading tooth, resulting in three gear teeth in the contact zone. Each gear tooth is enclosed by one axial pitch, so three pitches would form the effective facewidth as shown in Fig. 5. If two gear teeth share the load in the contact zone, there will also be one trailing and one leading gear tooth associated with the two active gear teeth. Hence four axial pitches would form the effective facewidth. This should ensure that adequate number of gear teeth are within the contact zone. While three axial pitches may be about the minimum functional facewidth for a cylindrical worm, four axial pitches should ensure better gear teeth engagement with the worm threads and provide a more reasonable service life.

The number of enclosed gear teeth (n_g) in the contact zone is obtained as the worm effective facewidth divided by the axial pitch of the worm. Hence for a reasonable service life, Eq. (8a) is suggested and Eq. (8c) may be used to define the effective length of a cylindrical worm.

$$n_g = \frac{b_1}{p_a} \geq 4 \qquad b_1 \geq 4p_a \qquad b_1 \geq 4\pi m_t \qquad (8)$$

The major limitation inherent in the axial pitch approach estimate (Eq. (8c)) of the worm effective facewidth is that it does not explicitly consider the size of the wormgear. In a wormset assembly, the worm is placed most often directly below or above the wormgear. Sometimes it is placed vertically on the left or right side of the wormgear [33]. In all cases, the worm must fit into the space available in the gearbox which is determined primarily by the wormgear diameter. Therefore, the size of the worm facewidth should be somehow related to the wormgear size.

The above functional explanation of wormgear teeth and worm thread(s) meshing indicates that the entrance and exit of wormgear teeth into the groove of the worm thread in the contact zone requires non-contacting leading and trailing gear teeth. Also, some extra threaded length is required for ease of manufacture of the worm, when grinding of the worm threads is done [16, 22]. Therefore, some extra portion of threaded length is necessary for functionality, as well as for the manufacturing ease of the worm. According to Shigley and Mischke [21], the contact zone of the worm should extend beyond the point where contact between its thread and the teeth of the gear begins. Hence, the need to separate the worm total facewidth size into effective and allowance sizes is obvious.

6.2 Tangent Geometry Method

An attractive, and perhaps more, realistic approach for sizing the effective facewidth of worms is using an expression having the wormgear pitch diameter as a variable. As explained in the previous section, the wormset gearbox size is primarily determined by the wormgear diameter. Earlier, the useful portion of the wormgear facewidth was estimated in Eq. (7) using the tangent geometry method and has the worm pitch diameter as a variable. Such an approach, could well be applied for sizing the worm effective facewidth. That is, the effective facewidth of a worm may be defined as the distance between the intercepts of the wormgear addendum circle diameter and a line drawn as a tangent through the wormgear pitch circle diameter at its mid plane.

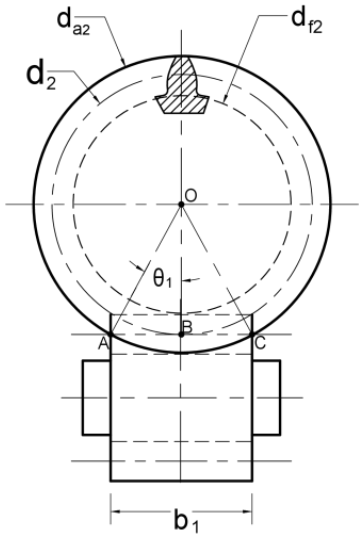


Fig. 6: Worm effective facewidth

The tangent rule condition is depicted in Fig. 6 where the line AC is a tangent line drawn to the pitch circle diameter of the wormgear at point B. The distance OA or OC is the addendum radius of the wormgear, and the distance OB is its pitch radius. In helical gears, the addendum radius is one normal module more than the pitch radius. The distance above point B to the addendum radius of the worm is also one normal module in helical gears. This allows a clearance between the gear root radius and the worm addendum radius. Based on the tangent geometry method, the effective worm facewidth is then defined by the length of the line AC.

Referring to Fig. 6:

$$b_1 = AC = 2AB \;\; = 2\sqrt{OA^2 - OB^2} \qquad (9a)$$

When the gear parameters are substituted into Eq. (9a), the result is Eq. (9b).

$$b_1 = 2\sqrt{(0.5d_2 + m_n)^2 - (0.5d_2)^2} \qquad (9b)$$

Eq. (9b) simplifies to Eq. (10), giving the expression for the effective facewidth of the worm.

$$b_1 = 2\sqrt{m_n(d_2 + m_n)} \qquad (10)$$

Eq. (10) gives the worm effective facewidth of a cylindrical worm as a function of the wormgear pitch diameter and the normal module. It is derived solely on geometric consideration of a wormset like the gear effective facewidth above in section 5.0. As discussed in the previous section, extra facewidth beyond the effective value is needed for functionality and manufacturing ease. The extra length can be added to the estimate of Eq. (10).

6.3 Empirical Methods

An empirical method uses design formulas or equations derived from observation, experience, and experimental data rather than pure theoretical science and engineering [34]. They are commonly used to predict the behavior of complex systems and estimate sizes in complicated design situations. Empirical formulas are often valid within narrow ranges of applications and may not have consistent units on both sides of the equation [34]. According to Collins et al [13], the design of a wormset is largely empirical for several reasons. Hence, the sizing of worm facewidth has largely been done with empirical formulas, and several are available. For instance, Eq. (11) and Eq. (12) are commonly used by designers in estimating the facewidth of cylindrical worms.

$$b_w = \left[4.5 + \frac{z_2}{\chi_1}\right] p_a \qquad b_w = \left[12.5 + \beta_1 z_2\right] m_t \qquad 11$$

$$b_w = \left[3.5 + \frac{z_2}{\alpha_1}\right] p_a \qquad b_w = \left[11 + \eta_1 z_2\right] m_t \qquad 12$$

Several designers [15, 19, 25, 32, 35, 36] used Eq. (11) for power transmitting worms of any number of threads. The common values of χ_1 and β_1 are 50 and 0.063, respectively. For some designers, the values of χ_1 and β_1 are 35 and 0.09 for worm with threads more than 2 [16, 18, 22]. Some other designers [16, 18, 22] used Eq. (12) for estimating the worm facewidth for worms with one or two threads, and the

values of α_1 and η_1 used are 52 and 0.06, respectively. The size of a worm facewidth from Eq. (12) is smaller than that from Eq. (11) for worms with one or two threads, though the same power may be transmitted.

In Eq. (11a) and Eq. (12a), the constants inside the brackets are 3.5 and 4.5, with an average of 4. This shows that about four (4) axial pieces are needed for proper engagement of worm thread(s) with wormgear teeth in cylindrical wormsets as suggested in section 6.1. Also, the product $z_2 m_t$ (Eq. (3b)) is the pitch diameter of the wormgear, and is in the empirical formulas above. This seems to be a recognition that using axial pitch only for facewidth estimation is inadequate. Therefore, the empirical formulas show that the pitch diameter of the wormgear ought to be a parameter in estimating the effective or nominal facewidth of the worm. The conclusion is that Eq. (10) meets all the functional and geometric requirements for estimating the worm effective facewidth.

7.0 WORM MAXIMUM FACEWIDTH

Fig. 7 shows the condition for the maximum facewidth of a worm. This condition is obtained when a line is drawn tangent to the clearance diameter circle of the wormgear as indicated by line AC. The line AC is made tangent to the clearance diameter circle of the gear, not the root circle diameter because some clearance is required between the outside radius of the worm and the root radius of the wormgear for proper functioning of the wormset. At this condition, the allowed clearance between the root of the wormgear and the tip radius of the worm is preserved. Hence, there is no interference between the addendum

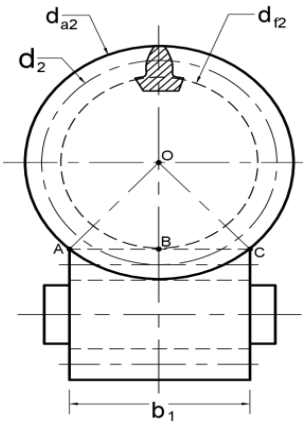


Fig. 7: Maximum worm facewidth

diameter size of the worm and the root diameter size of the wormgear.

In Fig. 7, the distance OA or OC is the addendum radius of the wormgear, and the distance OB is clearance circle radius of the gear. The length of line AC, therefore represents the maximum facewidth of the wormgear.

Referring to Fig. 7:

$$b_1' = AC = 2AB = 2\sqrt{OA^2 - OB^2} \qquad (13a)$$

When the gear parameters are substituted into Eq. (13a), the result is Eq. (13b).

$$b_1' = 2\sqrt{(0.5d_2 + m_n)^2 - (0.5d_2 - m_n)^2} \qquad (13b)$$

Eq. (13b) simplifies to Eq. (14) which gives the maximum facewidth for the worm.

$$b_1' = 2\sqrt{2m_n d_2} \qquad (14)$$

Eq. (14) gives the maximum effective facewidth of the worm that a wormset can geometrically accommodate and should yield consistent results for different designers. This worm facewidth size ensures maximum load capacity for the wormset [9]. Some allowance would have to be added to this size for manufacturing purposes. Eq. (14) has the normal module as a variable like Eq. (10) or Eq. (7). It is essentially the same as some models in literature which use the transverse module instead [2, 8, 9, 11, 14, 21]. AGMA [14] characterizes the size from Eq. (14) as the maximum facewidth for the worm. But, Shigley and Mischke [21], calls this the minimum worm facewidth, while Hamrock et al [2] says it is the maximum like AGMA [14]. However, no distinction is made between the effective and nominal facewidths by these designers with respect to the size estimate.

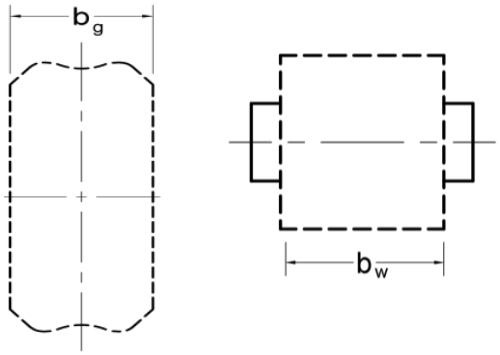
8.0 NOMINAL GEAR FACEWIDTH AND WORM FACE LENGTH

Fig. 8 depicts the gear and worm nominal facewidth sizes. Because, the effective gear facewidth size is set at a safe value [11] for functional and geometric reasons in Eq. (7), the nominal facewidth of the gear should be slightly more than the effective facewidth. It should be remembered that most wormgear blanks are often made by casting, and some manufacturing allowances are necessary for casting and machining. These may be at least 3 mm, though actual

allowances usually depend on the size of the wormgear, material, and casting method. The chamfer allowance should be kept in mind also.

Shigley and Mischke [21] consider the size estimate from Eq. (7) as maximum effective facewidth of the wormgear and that any additional facewidth is of very little value and a waste of material. Some designers made an allowance of 1.5 times the transverse module for the wormgear facewidth [25, 36]. The allowance on the wormgear facewidth should be minimized, especially when wormgears are made from bronze, that is an expensive material. The nominal gear facewidth may be estimated from Eq. (15a).

$$b_g \geq b_2 + m_n \qquad b_w \geq b_1 + p_a \qquad (15)$$



a) Gear b) Worm

Fig. 8: Nominal facewidth sizes in cylindrical wormset

Worm threads are usually finished ground and the grinding wheel should be free from damage when entering and leaving the worm threads. Therefore, to avoid the deformation of the working surfaces of the grinding wheel when it enters and leaves the worm threads, the worm facewidth may be extended by three transverse module (about 1.0 axial pitch) [16, 33], but Chernilevsky [22] suggested three to four transverse module (about 1.0 to 1.25 axial pitches). According to Hamrock et al [2], a worm facewidth of 5 axial pitches is reasonable and allows one axial pitch before and after the contact zone, which is in agreement with AGMA [14] recommendation. The allowance suggested by these authors varies between one and two axial pitches, the minimum value of one is taken in this study in order to minimize worm material and manufacturing cost. Therefore, Eq. 15b is suggested for the nominal facewidth of cylindrical wormsets. To prevent dynamic imbalance of the worm, the worm nominal facewidth should be made an even number of axial pitches [18]. However, this view is not commonly held by designers but seems reasonable, especially when the rotational speed of the worm is high.

The models of Eq. (6) to Eq. (15) differ from those in literature by using the normal module instead of the transverse module (Eq. 1b)). This is because, the wormgear is treated as a helical gear in the current study and not as a spur gear being implied in literature. The conceptual difference is in the plane of reference used for the gear system. The normal plane of the gear is the reference plane when the normal module is used in analysis. However, when the transverse module is used, the transverse plane becomes the reference plane. Note that the transverse or diametral of the gear corresponds to the axial plane of the worm or verse versa. But the worm and wormgear share a common normal plane and both the normal and transverse plane systems give the same pitch circle diameter for the worm or wormgear. The addendum diameter and dedendum diameter sizes are different for both planes because of the difference in values between the normal and transverse modules. This difference becomes significant at high lead or nominal helix angles. For instance, the difference in these sizes would be practically insignificant for wormsets with a lead angle less than 10^o because, at that value, the transverse module is only about 1.54% larger than the normal module. When the lead angle is above 10^o, some noticeable size difference may be observed, and probably significant difference when the lead angle in a wormset exceeds 15^o [20]. At a lead angle of 15^o, the transverse module is about 3.53% larger than the normal module. The tip diameters and facewidths of the worm and wormgear are affected by the reference plane system. A major advantage of the normal system is that a hob of a specific standard module and pressure angle can be used to cut a wormgear of any lead angle. In the transverse system, different hubs are required for different lead angles [11, 12].

9.0 WORMGEAR OUTSIDE DIAMETER

The wormgear outside diameter is also known as the tip diameter, or outer diameter [12]. Fig. 9a shows a straight tooth wormgear outside diameter and some other parameters. Fig. 9b shows a tapered tooth wormgear outside diameter and some other parameters. The main difference between the two types of gear tooth is that the throat face angle of Fig. 9a may be smaller than the gear face angle of Fig. 9b. Fig. 9a tooth style is often used for small gear sizes of 100 mm and below, while Fig. 9b style is commonly used for power transmission.

The outside diameter of a wormgear is influenced by the throat face allowance (a_t) as indicated in Fig. 9a and Fig. 9b. The throat face allowance is influenced by the face angle of

the gear. In Fig. 9a, the throat face angle is chosen arbitrarily with a minimum of about 45°, which gives a half-face angle of 22.5° [11]. A large gear face angle in Fig. 9a often leads to geometric undercutting during hobbing and manufacturing. Also gear alignment with the worm may become more difficult to achieve and load distribution on the gear face becomes uneven. The gear face angle in Fig. 9b is fixed by the gear facewidth and may not be chosen arbitrarily. Gear face angle between 60 to 100° are commonly used for the Fig. 9b style in industrial worm drives [11, 16, 18].

The gear or worm half-face angle can be determined from geometric consideration of the meshing of worm and wormgear. The total face angle for both gear and worm is twice the half-face angle. The gear half face-angle can be deduced from Fig. 4, and it is given by Eq. (16a). The worm half face-angle can be deduced from Fig. 6, and it is given by Eq. (16b).

$$\theta_2 = \sin^{-1}\left(\frac{b_2}{d_{a1}}\right) \qquad \theta_1 = \sin^{-1}\left(\frac{b_1}{d_{a2}}\right) \qquad (16)$$

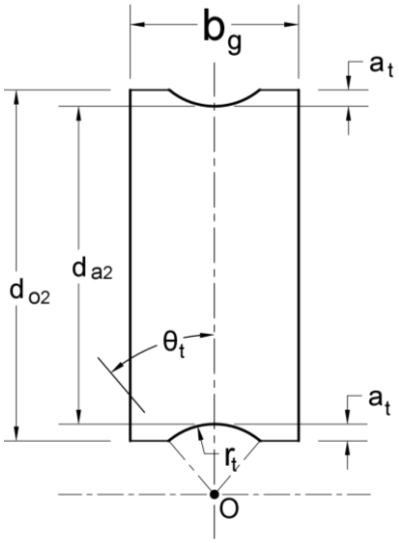


Fig. 9a: Straight face wormgear outside diameter

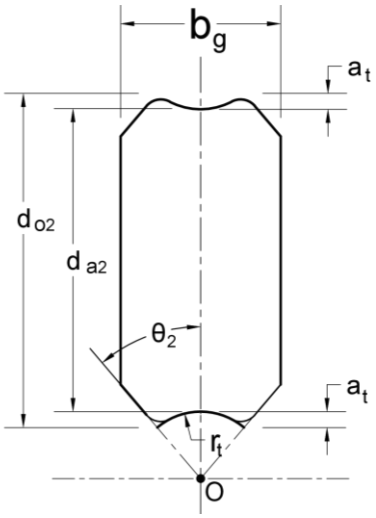


Fig. 9b: Tapered face wormgear outside diameter

In Fig. 9a or Fig. 9b, the throat radius is equal to the worm pitch radius less the addendum size, which is one normal module in helical gears as expressed in Eq. (17a). The throat radial allowance is given by Eq. (17b).

$$r_t = 0.5d_1 - m_n \qquad a_t = (1-\cos\theta_2)r_2 \qquad (17)$$

Note that the angle θ_2 is replaced with angle θ_t for a straight face wormgear of Fig. 9a in Eq. (17b).

The blank size of a wormgear used in manufacturing is derived from its outside diameter. It is larger than the addendum diameter of the wormgear due to throating and is often estimated by using empirical formulas [12, 16, 35]. Therefore, the outside diameter gear size is subject to variations for the same addendum diameter size and normal module. A size larger than necessary increases both material and manufacturing costs, and a size smaller than required, leads to more scraps during manufacturing. It is therefore, important that an adequate size is specified at the design stage. Geometrically, this size may be estimated by combining the gear half-face angle and the throat radius. The outside diameter of the wormgear may be estimated from Eq. (18).

A fillet is usually used to connect the throat radius with the face of the wormgear at the gear tip. The fillet radius at the tip of the gear is about 10% of the gear facewidth [11].

10.0 DESIGN METHOD CONSISTENCY

Design method consistency refers to the relative stability of solutions for a design method when applied to different design problems. This may be tested by comparing an important design parameter with one whose value is inherently stable. In this study, three methods are presented for the estimation of the worm facewidth. The first is the axial pitch method which was presented in section 6.1. The second is the tangent geometry method which is presented in section 6.2. The third is the empirical method which is presented in section 6.3 and was used in the cited references. The estimates from these methods may be compared with the maximum effective facewidth of the worm (Eq. (14)), a size of a wormset with inherent stability. This parameter is based only on the geometry of the wormset and has a definite value for each design problem. Therefore, the worm effective facewidth estimates from different design methods can be compared with the worm maximum effective facewidth.

A design method consistency factor for the worm facewidth may be defined as in Eq. (19a) and should yield a value above unity. When a design method is used for different applications, its value should be reasonably constant if the design method is inherently consistent.

$$\beta_m = \frac{b_1'}{b_1} \qquad \beta_w = \frac{b_w}{b_1} \qquad (19)$$

$$\beta_g = \frac{b_g}{b_2} \qquad \beta_t = \frac{a_t}{m_n} \qquad (20)$$

Other sizes that may be analyzed for consistency are the worm nominal facewidth, the wormgear nominal facewidth, and the throat radial allowance. Eq. (19b) may be used for judging the consistency of the worm nominal facewidth, Eq. (20a) may be used for judging the consistency of wormgear facewidth, and Eq. (20b) may be used for judging the consistency of the throat radial allowance. A fairly constant value of these factors for different design applications would imply that their design equations are consistent.

11.0 DESIGN APPLICATION EXAMPLES

The equations presented in the previous sections were coded in Microsoft Excel for computational efficiency. They were then used to analyze the gear facewidth and the worm facewidth, and other parameters discussed in the previous sections for five (5) previous design works. These are presented in Table 1 as EX1 [18, pp. 357 – 360]; EX2 [16, pp. 429 – 434]; EX3 [37]; EX4 [38]; and EX5 [33, pp. 541 – 542]. Examples 1, 2, and 5 are works believed to have been based on GOST (former Soviet Union) standards. Example 3 appears to be based in part on the European ISO standards, while Example 4 appears to be based on a combination of American and ISO standards. MITCalc works [37] is a software suite for mechanical engineering, industrial and technical calculations. KISSsoft [38] is a commercial software developed by a subsidiary of Gleason Corporations, a renowned gear manufacturer in the United States and developer of gear standards. Most of the wormset design examples did not provide the worm facewidth sizes. This may be due to the varied empirical methods used for the sizing of the facewidth.

Table 1 is a summary of the input data for the parametric study. Table 2 shows data about the gear facewidth and Table 3 shows data about the worm facewidth.

Table 1: Previous Design Data Summary for Examples

Example	z_1	γ (deg.)	m (mm)	m_n (mm)	d_1 (mm)	d_2 (mm)	b_g (mm)	b_w (mm)
EX1	3	15.26	7	6.753	77	343	68	132
EX2	2	14.04	10	9.701	80	320	75	130
EX3	3	12.10	6.443	6.300	90.20	483.22	77	98
EX4	2	12.53	4	3.905	36	160	31	64
EX5	2	9.12	6.3	6.220	78.75	201.6	67	100

Table 2: Gear Facewidth Data

Example	Previous	Tangent		β_g (#)+	Dev. (%)
	b_g (mm)	b_2 (mm)*	b_g (mm)**		

EX1	68	48	54	1.43	1.14	-20.12
EX2	75	59	69	1.27	1.16	-8.40
EX3	77	49	56	1.56	1.13	-27.77
EX4	31	25	29	1.24	1.16	-6.87
EX5	67	46	52	1.46	1.14	-22.09
*Based on Eq. (7)		**Based on Eq. (15a)		+Based on Eq. (20a)		

Table 3: Worm Facewidth Data

Example	Previous	Tangent		β_w (#)+		Dev. (%)
	b_w (mm)	b_1 (mm)*	b_w (mm)**			
EX1	132	97	119	1.36	1.23	-9.70
EX2	130	113	145	1.15	1.28	11.17
EX3	98	111	131	0.88	1.18	33.99
EX4	64	51	63	1.26	1.25	-1.31
EX5	100	72	92	1.39	1.28	-8.30
*Based on Eq. (10)		**Based on Eq. (15b)		+Based on Eq. (19b)		

Table 4: Worm Effective Facewidth Design Method Consistency

Example	Previous	Axial Pitch	Tangent
	β_m (#) (Eq. (19a))		
EX1	1.03	1.55	1.40
EX2	1.21	1.25	1.39
EX3	1.59	1.93	1.41
EX4	1.10	1.41	1.40
EX5	1.00	1.27	1.39

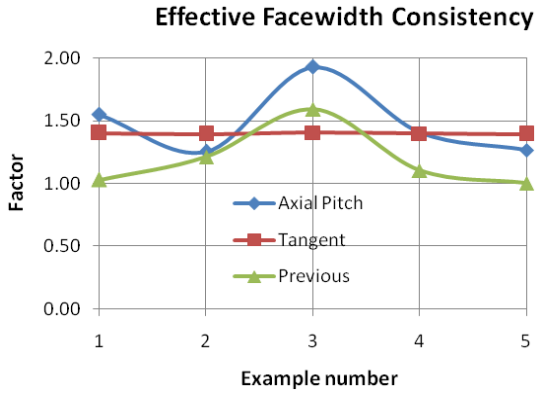


Fig. 10: Worm maximum facewidth consistency

12.0 DISCUSSIONS

Three design methods of axial pitch, tangent geometry, and empirical are discussed for estimating the worm facewidth and there is need to assess the consistency of each method. Table 4 shows the design consistency data generated from Eq. (19a) for the three design methods. Fig. 10 shows the design consistency plots of these methods with the design consistency data values of Table 4. The label "Previous" in the diagram refers to the empirical methods used to generate the solutions in the cited references. The Axial Pitch and Tangent refer to the Axial Pitch Method and Tangent Geometry Method, respectively. The method with the most variability is the axial pitch method. This is followed by the empirical method that was used in the previous solutions for the worm facewidth solutions. The tangent geometry method

shows very little variability, yielding a nearly constant factor value of 1.4. This indicates that the worm maximum facewidth is approximately 1.4 times the effective facewidth. Therefore, the tangent geometry method appears to be more reliable in predicting the size of the worm effective facewidth. Consequently, it is the method that needs further consideration in comparison with the empirical method used in the previous solutions for the design problems. The axial pitch method will no longer be discussed in this section.

Table 2 provides data for the wormgear facewidth. The analysis done for the gear facewidth are based on Eq. (7) and Eq. (15a) and the results are shown in column 3 and column 4 of Table 2. Column 5 of shows the wormgear nominal facewidth design consistency data based on Eq. (20a) for empirical method, while column 6 shows that for the Tangent geometry method. Column 7 of Table 2 shows the percentage deviations of the gear nominal facewidth from those in the cited references. The values indicate that the sizes in the cited references are larger than the current values, since the column data are all negative. This seems to suggest that the gear facewidth in empirical design practice tend to be on the conservative side. The variances of column 7 appear significant for Examples 1, 3, and 5; and somewhat low for Examples 2 and 3.

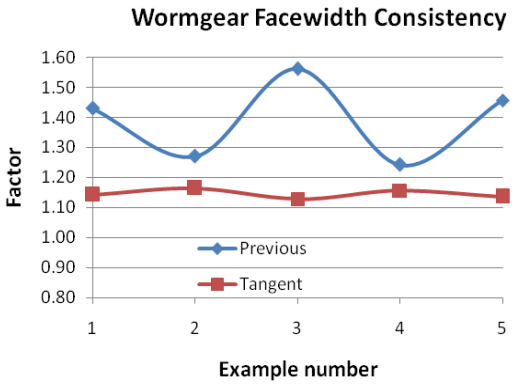


Fig. 11: Wormgear nominal facewidth consistency

Fig. 11 shows the consistency plots of the empirical and tangent geometry methods with design consistency data values of Table 2. The plots show that the empirical method is highly variable compared with the tangent geometry method which has a relatively flat curve. The average consistency factor for the tangent geometry method is about 1.15, indicating that the nominal facewidth is approximately 1.15 times the effective facewidth. This method is gaining acceptance in the gear design industry and appears really attractive for future use. The possible size reduction shown in column 6 of Table 2 means that using Eq. (7) for the gear effective facewidth could lead to possible savings in the costs of material and manufacturing for all the gears in the study. Large gears made of costly material like bronze will definitely benefit from such savings.

Table 3 provides data for the worm facewidth. The analysis done for the worm facewidth are based on Eq. (10), Eq. (15b), and Eq. (19b) and the results are shown in column 3, column 4, column 5, and column 6 of Table 3. Column 3 gives the worm effective facewidth data, while column 4 gives the worm nominal facewidth data. Column 5 gives the consistency data for the previous design method. From column 5, it is noted that the consistency factor values for examples 2 and 3 are close to unity and the worm facewidth from the previous solutions may be for the effective size. The values for examples 1, 4 and 5 are considerably above unity and the worm facewidth size from these previous solutions may be for the nominal size. The mixed interpretations of the results from the empirical method by the designers is thus confusing.

Table 3 column 6 gives the consistency data for the tangent geometry method and the values appear reasonably close. Column 7 of Table 3 shows the percentage deviations of the worm nominal facewidth from those in the cited references. The values in column 7 indicate that examples 1, 4, and 5 worm facewidth sizes in the cited references are slightly larger than those of the current size results. The examples 2 and 3 worm facewidth sizes for the previous solution are considerably smaller than the current size solutions. However, the extra facewidth sizes in these two example would allow more active gear teeth in the contact zone, and hence potentially give longer service lives to the gears which could minimize life cycle costs.

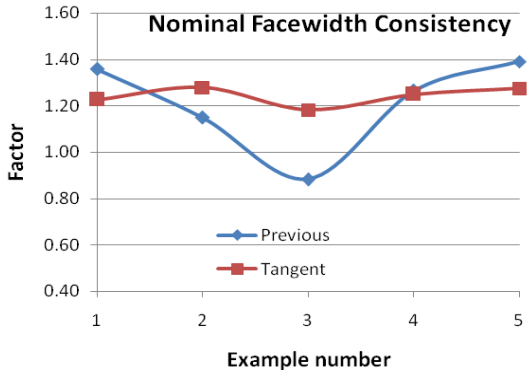


Fig. 12: Worm nominal facewidth consistency

Fig. 12 shows the plots of the worm nominal facewidth consistency factors based on Eq. (19b) using the data in

columns 5 and 6 of Table 3. The plot for the previous or empirical design method is observed to show more variability than the plot for the tangent geometry design method. The average value for the empirical method is 1.21, but it may lead to considerable differences in design if used. The average value for the tangent geometry design method is 1.24 with little variability. This suggests that the nominal facewidth for the tangent geometry method is about 1.24 times the effective facewidth. A value of 1.25 is suggested for design. Because the tangent geometry design method is more consistent than the empirical method, it should be the preferred design method.

Table 5 shows some values of pertinent design parameters for the previous solutions. Column 2 gives the number of gear teeth in the contact zone. Column 3 provides the gear half-face angle (Eq. (16a)). Column 4 shows the throat radial allowance (Eq. 17b)). Column 5 gives the outside diameter (not provided in all previous solutions) based on Eq. (18). Column 6 gives the throat radial allowance consistency data based on Eq. (20b).

Table 6 provides similar data as Table 5 for the current solutions up to column 6. The additional column 7 gives the wormgear tip diameter variances between the previous and current solutions.

Table 5: Some Intermediate Parameters: Previous Solutions

Example	n_g (#)	θ_2 (deg.)	a_t (mm)	d_{o2} (mm)	β_t (#)
EX1	6.00	49	9.27	375	1.37
EX2	4.14	49	9.56	359	0.99
EX3	4.84	49	10.87	518	1.73
EX4	5.09	45	3.40	175	0.87
EX5	5.05	47	8.78	232	1.41

Table 6: Some Intermediate Parameters: Current Solutions

Example	n_g (#)	θ_2 (deg.)	a_t (mm)	d_{o2} (mm)	β_t (#)	Dev. (%)
EX1	4.42	36	5.08	367	0.75	-2.23
EX2	3.60	42	7.08	354	0.73	-1.38
EX3	5.49	32	4.81	505	0.76	-2.34
EX4	4.03	40	2.65	173	0.68	-0.86
EX5	3.63	34	4.59	223	0.74	-3.61

The minimum number gear teeth in the contact zone in column 2 for Table 5 is 4.18 and that for Table 6 is 3.6. This lends credence to the possible use of 4 axial pitches for the worm facewidth effective size discussed in section 6.1, though it is not a consistent method. The gear half-face angle in column 3 of Table 5 is fairly consistent, being close to 50°. Values of the same parameter in column 3 of Table 6 show some variability but are lower than those in Table 5. Table 6, column 7 compares the previous and current outside diameters of the gears. The negative signs in these data indicate the current diameter values are smaller than the previous values. This means possible savings in material cost and potentially, manufacturing cost, also.

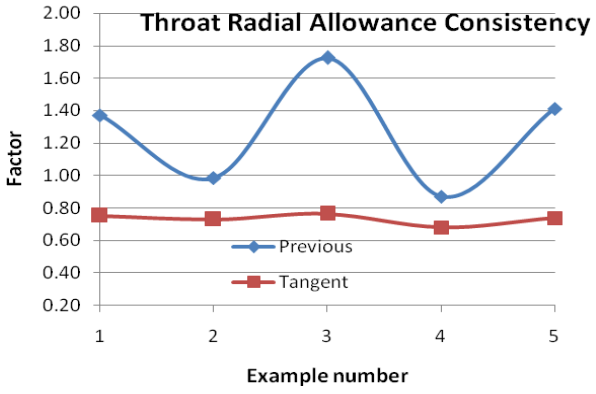


Fig. 13: Throat radial allowance consistency factor

Fig. 13 shows the plots of the throat radial allowance consistency factors based on Eq. (20b). The plot for the previous design method based on columns 1 and 6 of Table 5. It is observed to show high variability compared to the plot for the tangent geometry design method based on columns 1 and 6 of Table 6. The average value for the empirical method is 1.274, but the variability is too high for the average value to be recommended for design application. The average value for the tangent geometry method is 0.73 with low variability and a design value of 0.75 would be appropriate. Clearly, the tangent geometry design method is very consistent relative to the empirical method and should be the preferred design method.

The size deviations of column 7 in Table 2 are directly proportional to the facewidth, as well as the masses of the gears. In Table 6 column 7, the masses of the gears are proportional to the square of the gear diameters. Therefore size deviations in the gear diameters have a greater influence on the masses of wormgears than deviations in the facewidth. The combined size deviations in Table 2 and Table 6 could have substantial influence on material cost savings and perhaps manufacturing cost savings for the wormgears. In wormsets, the wormgear is usually the larger and more costly component. Usually, size reduction leads to lighter gears which have lower inertia. This means they would require less energy to start and stop, improving overall system responsiveness and efficiency [39]. The possibility of worm gear size reduction by the tangent geometry method should make it an attractive method in new designs of cylindrical wormsets.

From Table 5 column 3, the gear face angle is in the range of 90 ^{o} to 98^{o}. These values appear to be on the high side of wormset design practice with 100^{o} [16] being about the maximum. This suggests that precise alignment of wormgear axis with worm axis is necessary to ensure proper contact of mating surfaces and even distribution of contact load. The gear face angle from Table 6 column 3 is in the range of 64 ^{o} to 84^{o}. This range of gear face angle is more favorable for even load distribution on contact surfaces and may require less precise alignment of the axis of the wormgear with that of the worm.

According to AGMA [14], if the number of threads in contact with wormgear teeth is the usual 2 to 3, the worm facewidth can be taken to be five times axial pitch which provides for three contacting threads plus one extra thread at each end to provide support for the load carrying threads. This implies extra worm facewidth is necessary for proper functioning of a worm. The need to avoid damaging the grinding wheel during manufacturing adds a manufacturing requirement for some extra facewidth for worm. The extra facewidth requirements for functionality and manufacturing makes the nominal worm facewidth size different from the effective facewidth size. If this extra length allowance is not deliberately provided on a worm threaded length, it would mean the effective worm facewidth in actual operation of a cylindrical wormset is reduced to some extent. How much reduction occurs will probably depend on the quality of workmanship in the manufacturing process. But, any reduction of the effective facewidth of the worm means the number of gear teeth engaging the worm thread(s) is reduced, which leads to higher contact stress, and potentially, shorter service life. Consequently, providing adequate manufacturing allowance on the facewidth of the worm is very important. Therefore, the effective facewidth should be differentiated from the nominal facewidth for cylindrical worms.

13.0 CONCLUSIONS

The facewidth of cylindrical wormsets has been delineated into effective and allowance facewidth components. The wormgear effective facewidth has a history of being geometrically determined, but such an approach is not well established for the worm facewidth. This study has extended the geometric approach which is termed “the tangent geometry method” for sizing the wormgear facewidth to that of the worm with a slight modification. The nominal facewidth is the sum of the effective and allowance facewidths.

The effective facewidths are the portions of the wormgear and worm nominal facewidths that transmit load on the wormset. The gear allowance facewidth ensures proper gear teeth engagement with the worm thread(s). Similarly, the worm allowance facewidth is the extra portion of the threaded worm length that ensures proper engagement of worm thread(s) with the wormgear teeth and enhances manufacturing ease. The gear teeth enter the contact zone by moving over the trailing part of the worm allowance facewidth and exit it by moving over the leading part. The allowance facewidths are thus provided for both functional and manufacturing ease.

The application of the tangent geometry method in sizing the effective facewidths of the wormgear and worm is summarized in Eq. (21a) and Eq. (21b), respectively. These are reproduced from Eq. (10) and Eq. (7), respectively. Eq. (21) differs from currently available models because it uses the normal module of the wormgear instead of the transverse module.

$$b_1 = 2\sqrt{m_n(d_2 + m_n)} \qquad b_2 = 2\sqrt{m_n(d_1 + m_n)} \qquad (21)$$

The worm and gear nominal facewidths are given by Eq. (22a) and Eq. (22b), respectively, a reproduction of Eq. (15).

$$b_w \geq b_1 + p_a \qquad b_g \geq b_2 + m_n \qquad (22)$$

Eq. (21) and Eq. (22) provide systematic basis and consistent method for the size estimates of worm facewidth and gear facewidth that are functional and manufacturable. From the parametric analysis, the worm nominal facewidth is about 25% larger than the effective facewidth, and the gear nominal facewidth is about 15% larger than the effective facewidth. These results lead to the simplifications of Eq. (22a) into Eq. (23a), and Eq. (22b) into Eq. (23b), respectively.

$$b_w \geq 1.25 b_1 \qquad b_g \geq 1.15 b_2 \qquad (23)$$

Eq. (18b), reproduced as Eq. (24a) may be used to determine the outside diameter of the wormgear. The throat radial allowance is about 75% of the value of the normal module from the parametric analysis. This suggests that Eq. (24a) may be simplified to Eq. (24b).

$$d_{o2} \geq d_{a2} + 2a_t \qquad d_{o2} \geq d_{a2} + 1.5 m_n \qquad (24)$$

In the five examples investigated, the tangent geometry method provided adequate effective facewidth sizes for both the wormgear and worm. It was found that possible reduction of wormgear size may be achieved with the tangent geometry method. Generally, size reduction leads to lighter gears which have lower inertia, and thus require less energy to start and stop, improving overall system responsiveness and efficiency. Also, size reduction could result in material and manufacturing costs savings.

From both functionality and manufacturing ease perspectives, differentiating the worm effective facewidth from the nominal facewidth is necessary for the proper understanding of the operations and design of cylindrical wormsets. When the size of the worm facewidth is short of that required for proper engagement, fewer gear teeth are made to transmit the imposed load during the operation of the wormset. This can lead to an increase in the average contact stress on the gear teeth surfaces in the contact zone, resulting in higher rate of wear for the gear teeth and worm threads. The service life of the wormset may be shortened or more frequent maintenance may be required to adjust for the gear wear. Therefore, the functionality of the wormset is compromised without adequate effective size for the worm facewidth. The tangent geometry method provides adequate effective sizes for the gear facewidth, worm facewidth, and throat radial allowance. It provides a unified analytical and a systematic approach in sizing the facewidths of cylindrical wormsets. The tangent geometry method reduces the arbitrariness in the choice of the effective and nominal facewidths of the worm and wormgear and ensures consistent interpretation of the sizes. It offers a possibility of wormgear size reduction which potentially can lower the cost of wormsets. Therefore, the tangent geometry method is here recommended as a preferred design method for the facewidths of cylindrical wormsets.

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NOMENCLATURE

1 - subscript for worm
2 - subscript for gear
a_t – radial throat allowance (mm)
b – gear facewidth (mm)
b_1 – effective or active facewidth of worm (mm)
$b_1^{/}$ – maximum effective facewidth of worm (mm)
b_2 – effective or active facewidth of gear
b_g – nominal facewidth of gear
b_w – nominal facewidth of worm (mm)
d – gear pitch diameter (mm)
d_a – addendum diameter (mm)
d_f – dedendum diameter (mm)
d_{o2} – tip or outside diameter of wormgear (mm)
m_a – axial module (mm)
m_n – normal module (mm)
m_t – transverse module (mm)
n_g – number of gear teeth enclosed on effective facewidth of worm
p_a – axial pitch (mm)
p_t – transverse pitch (mm)
p_n – normal pitch (mm)
r_t – gear throat radius (mm)

z_1 – number of threads on worm

z_2 – number of gear teeth

z_w – worm pitch diameter or form number

α_1 – empirical worm facewidth axial pitch factor

β_1 – empirical worm facewidth module factor

β_m – worm maximum facewidth factor

β_w – worm nominal facewidth factor

β_g – wormgear nominal facewidth factor

β_t – radial throat allowance factor

χ_1 – empirical worm facewidth axial pitch factor

η_1 – empirical worm facewidth module factor

θ_1 – worm face half-angle (deg.)

θ_2 – gear face half-angle (deg.)

θ_t – throat face half-angle (deg.)

ϕ_n – normal pressure angle (deg.)

ϕ_t – transverse pressure angle (deg.)

γ – worm lead angle (deg.)

ψ – wormgear nominal helix angle (deg.)

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