International Research Journal of Engineering and Technology (IRJET) e-ISSN: 2395-0056
Volume: 11 Issue: 08 | Aug 2024 www.irjet.net p-ISSN: 2395-0072

Introducing Hidden Order Derivatives: Sine and Cosine Enjoying their Oscillation

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Abstract- *Sine and cosines are well known to be oscillatory functions. They are oscillating between (-1) and 1 for all such values permissible into sines and cosines. These elementary plane trigonometric ratios are appropriate for each other. The oscillation of sine and cosine functions is a pivotal concept with broadened applications in various fields. This versatility makes them essential tools for observing and analyzing periodic phenomena. Here, we introduce our proposal for new conventions related to the hidden order of the derived function. In this article, we have developed very elementary level fundamentals to expand this field in the future. This study explores the relationship between higher order derivatives of sines and cosines. At the beginning, it seems fairly obvious, but it has underlying mysteries. Our methodology addresses new notations, per the convenience of derivations. Through our outcomes, we seek to unfold other properties of periodic functions, 'hidden' beyond sines and cosines. Later, we conclude our work with an attempt to explore the properties of functions other than sines and cosines. Our work on such periodic entities represents a new avenue to the field of modern mathematics.*

Key Words: Oscillatory function, Derived function, Plane trigonometric ratios.

1. Introduction

Trigonometry focuses on the interesting story of triangles and their corresponding arms and angles. The history of 'numerical relations of triangles' found in 'Ancient Egypt' and 'Babylonia' paved the way for 'early trigonometry'. However, the trigonometry we use today is found in ancient Greece. In addition to Egypt, Babylonia and Greece made contributions to trigonometry, Indian mathematicians such as Aryabhata-I also performed many works on this topic [3]. Initially, this inspired many applications, such as real-life approaches to problems corresponding 'Height & Distance', and problems related to triangles. Sine, Cosine, Tangent, Cosecant, Secant and Cotangent are six ratios that fall under the plane trigonometric function [5]. Currently, the foundation of trigonometry is very strong, and mathematicians are making great innovations in this field. Trigonometric functions are easy to use as sine and cosine functions are bounded functions in their domain, and their use can be helpful in solving complex problems. The well-known Fourier series depends upon trigonometric representations such as sines and cosines. The basic objective of such a representation is to disintegrate periodic functions into well-known sine and cosine components [2]. This is very easy because of the use of elementary trigonometric ratios. In addition, in this study, we adopted a new convention for further exploration. We have explored, 'relation within the derivatives' of sine and cosine functions in some particular order. In later sections, we present some results. We expect our methodology to penetrate curious minds, for a bright avenue in mathematics.

2. Intuitive preliminaries

For the general readers, we have listed some of the basic preliminaries in this section. Here, our objective is to demonstrate a very crystal-clear introduction of the periodicity of function.

2.1 Functions

Let A and Bbe two nonvoid sets then a $relation$ R from set A to set B is a subset of $A \times B$. Thus, a relation R from A to B if the ordered pair $(a, b) \in R \Rightarrow aRb$ in that case, it is abbreviated as 'a is related to b by $relation$ R'. In accordance with these requirements, there are distinct types of relations, such as universal, void, identity, reflexive, symmetric, anti-symmetric and transitive. The foundations and allied consequences of function are tremendously important in mathematics and other disciplines as well. For two nonvoid sets A and B a $relation$ f $from$ A to B (i.e., a subset of $A \times B$ is called a function or mapping or a map) from A to B if the following is maintained:

1. For each a$\in A$, b$\in B$ such that$(a, b) \in f$
2. If $(a, b) \in f$ and$(a, c) \in f$, then $a = c$

Hence, we write $f: A \rightarrow B$, where A is called the domain and where B is the codomain of the function f. When the values of B are associated with real numbers, the corresponding function is said to be a real-valued function. Thus, we can infer that the functions are special kinds of 'relations' with the abovementioned inherent property. Depending upon the requirements, functions are classified into one-one, many-one, onto and into functions. The trigonometric functions given in this article are many-one functions in their corresponding domain.

2.2 Periodic functions

A real-valued function $f: X \to Y$ is said to be a periodic function with a period T, if there exists a least positive value T, for which $f(x+T) = f(x)\ \forall x \in X$. The integral multiples of $T$are also then periods of the preceding function f, but the lowest such value for which the above relation holds is said to be the "principal period". Geometrically speaking, every periodic function can be demonstrated over an interval of one period within the domain, as the same pattern will oscillate over the domain under consideration. The trigonometric functions are well-known periodic functions in the world of mathematics, e.g., sine and cosines have periods2π in their domain of existence.

Now, one can question what the consequent functions are if we apply little bit calculus in the periodic function: "is it remains periodic?". Here, we explore the possible extensions, and the basic relationship lies in the differentiation of the periodic functions.

2.3 Plane trigonometric ratios

As we are already familiar with them, very few notations and important relations are given as per the formalities. There are many results and formulae in the theory of trigonometric functions, and we provide some identities and frequently used terminologies in the following subsection below.

2.3.1 Important identities

Some standard trigonometric identities are given as follows:

1. $\sin^2\theta + \cos^2\theta = 1$
2. $\sec^2\theta - \tan^2\theta = 1$
3. $\csc^2\theta - \cot^2\theta = 1$
4. $(\cos^2\theta - \sin^2\theta) = (2\cos^2\theta - 1) = \{1 - 2\sin^2\theta\} = \cos(2\theta)$
5. $2\sin\theta\cos\theta = \sin(2\theta)$
6. $\sin(A+B) = (\sin A\cos B + \cos A\sin B)$
7. $\sin(A-B) = (\sin A\cos B - \cos A\sin B)$
8. $\cos(A+B) = (\cos A\cos B - \sin A\sin B)$
9. $\cos(A+B) = (\cos A\cos B + \sin A\sin B)$

2.4 On differentiation of trigonometric ratios

We have started our discussion within the differentiation of periodic functions. Then, it is natural to determine the flow of the trigonometric ratios. Our article is mainly interested in exploring the related consequences for the differentiation of sine and cosine functions up to a higher order in their domain.

From the basic notion of the differentiation of real-valued functions, we know that

$f: X \to \mathbb{R}$ then the differentiation of $y = f(x)$ with respect to x as the limit exists; then,
$f'(x) = \lim_{h\to 0}\left\{\frac{f(x+h)-f(x)}{h}\right\} = \frac{dy}{dx}$
Thus,$\frac{d(\sin x)}{dx} = \cos x\,;\ \frac{d(\cos x)}{dx} = (-\sin x)$

Taking those factors into account, we further extend their relations of higher-order derivatives. Halt! For a second, do not confuse our objective with the 'Leibnitz's rule for nth derivatives'. This rule only investigates the general term obtained after the nth derivative for a real-valued function. This work explores the relationships between the higher-order derivative of sines and cosine functions. As far as we have investigated, there appears to be some camouflage between the sine and cosine derivatives, which are very easy to observe but need to be explored further. This is why we named it the 'hidden order derivative'.

3. Research Objectives

With the newly adopted convention, we have derived few results in later sections. Our objective is as follows:

1. In addition to their simple identities, the relationships between sines and cosines can be extended.
2. When we adopt notations for differentiating sines and cosines, we explore the consequences.
3. A new branch of enhanced trigonometric functions is obtained.

4. Methodology

Functions are special kinds of relations between two nonvoid sets. Trigonometric functions are real-valued functions. We define the sine and cosine in their domain as the real line, and the corresponding range is $[-1,1]$. For convenience, we assume the notations below.

$F_S(x) = \sin x\,, F_C(x) = \cos x\,;$ for the derivatives $\frac{d^n}{dx^n}\{F_S(x)\} = \frac{d^n}{dx^n}\{\sin x\} = F_S{}^{(n)}(x)$ similarly, for the other $\frac{d^n}{dx^n}\{F_C(x)\} = \frac{d^n}{dx^n}\{\cos x\} = F_C{}^{(n)}(x)$.

When $F_S{}^{(4)}(x) = F_S{}^{(8)}(x) = F_S{}^{(12)}(x) = F_S{}^{(16)}(x) = F_S{}^{(20)}(x) \ldots = F_S{}^{(4n)}(x)$

and, ${F_C}^{(4)}(x) = {F_C}^{(8)}(x) = {F_C}^{(12)}(x) = {F_C}^{(16)}(x) = {F_C}^{(20)}(x) \ldots = {F_C}^{(4n)}(x)$. For example, ${F_S}^{(20)}(x)$ and ${F_S}^{(40)}(x)$ are the same, and ${F_C}^{(8)}(x)$ and ${F_C}^{(56)}(x)$ are also indistinguishable. For an arbitrary case, ${F_X}^{(aN)}(x) = f\ (say)$, then f is also the same as ${F_X}^{(N)}(x)$;, where $X(x)$ is any function (called the component function) and ${F_X}^{(aN)}(x) = {F_X}^{(N)}(x)$ for some 'a' and 'N'. Henceforth, the order is found to be hidden such that the equality holds with the derived function after a certain period. Thus,'N'corresponds to the hidden order. The$F_S(x), F_C(x)$ associated hidden order is 4. Our methodology relies upon observing the 'simplicity of differentiation' of sines and cosines with a periodic nature.

5. Discussion and Results

The consequents results are given in this section as follows:

$${F_S}^{(4)}(x) = {F_S}^{(8)}(x) = {F_S}^{(12)}(x) = \cdots {F_S}^{(4n)}(x)$$
$${F_C}^{(4)}(x) = {F_C}^{(8)}(x) = {F_C}^{(12)}(x) = \cdots {F_C}^{(4n)}(x) \quad (5.1)$$

[using those elementary understandings of the patterns, we are going to settle possible relationships among them].

On settling, $F_S(x) = X_0$ denotes the sine function only; $F_C(x) = Y_0$ denotes the cosine function only. Now, by successive differentiation, the results obtained are given below. Where ${F_X}^{(N)}(x)$represents the N^{th} derivative of $F_X(x)$ (arbitrary function of x using the component function, X).

$F_S(x) = X_0$ on successive differentiation,
$${F_S}^{(1)}(x) = X_1 = F_C(x),$$
$${F_S}^{(2)}(x) = X_2 = -F_S(x)$$
(5.2)

Hence, it is easy to verify that
$$F_S(x) + {F_S}^{(2i)}(x) = 0; i = (2k-1)$$
(5.3)

The results are the same for the cosine,
$$\{F_C(x) + {F_C}^{(2i)}(x)\} = 0; i = (2k-1) \quad (5.4)$$
$$\frac{F_S(x)}{F_C(x)} - \frac{{F_S}^{(2i)}(x)}{{F_C}^{(2i)}(x)} = 0$$
$$\Longrightarrow \frac{F_S(x)}{F_C(x)} = \frac{{F_S}^{(2i)}(x)}{{F_C}^{(2i)}(x)}; i = (2k-1)$$
(5.5)

Using(5.1), we obtain
$$\frac{{F_S}^{(4i)}(x)}{{F_C}^{(4i)}(x)} = \frac{{F_S}^{(2i)}(x)}{{F_C}^{(2i)}(x)}; i = (2k-1)$$
(5.6)

$F_C(x) = Y_0$ on successive differentiation,
$${F_C}^{(1)}(x) = Y_1 = -F_S(x)$$
$${F_C}^{(2)}(x) = Y_2 = -F_C(x)$$
${F_C}^{(3)}(x) = Y_3 = F_S(x)$, on the other hand,
$${F_S}^{(1)}(x) = {F_S}^{(5)}(x) = \cdots = F_C(x)$$
$$\Rightarrow {F_S}^{(4k-3)}(x) = F_C(x) = {F_C}^{(4i)}(x)$$
(5.7)
$${F_C}^{(3)}(x) = {F_C}^{(7)}(x) = \cdots = F_S(x)$$
$$\Rightarrow {F_C}^{(4k-1)}(x) = F_S(x) = {F_S}^{(4i)}(x)$$
(5.8)

Finally, we obtained the following:
$$\frac{{F_C}^{(4k-1)}(x)}{{F_S}^{(4k-3)}(x)} = \frac{{F_S}^{(2i)}(x)}{{F_C}^{(2i)}(x)}; i = (2k-1)$$
(5.9)

We often denote, $\left|\frac{d^H F_S(x)}{dx^H}\right| = \left|\frac{d^H F_C(x)}{dx^H}\right| = (4n)$ (which does not mean $H = 4n$, is natural number); 'H' denotes the hidden order of preceding functions.

6. Conclusion

By adopting the used notation used in the previous section, we can derive many results.

$(5.1) - (5.9)$addresses various results related to the differentiation of higher-order $F_S(x), F_C(x)$. We expect that the methodology used will help us explore the relationships among the different periodic functions. In the case of the exponential, algebraic functions, we do not encounter any kind of sufficient result to enlist. We can assure that this methodology is also useful for such functions. The sine and cosine functions have hidden order: $(4n)$, and the exponential function has hidden order: (n).

7. Future scope of the current research topic

1. Extending the methodology we have adopted here can be useful for settling possible relationships among higher-order derivatives of different periodic functions.

2. Finding the consequent geometric backbone of the obtained results.

8. References

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