state theory analysis, all points on the surface of the thin-walled cylinder are in a plane stress state [4].

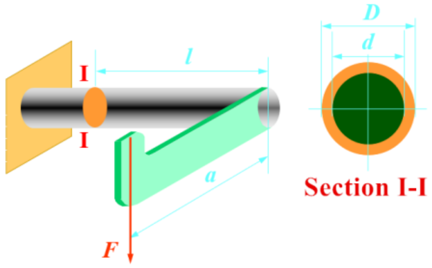


Fig.1: Experimental apparatus and cross-sectional schematic diagram for combined bending and twisting deformation

Take any point on the surface of the thin-walled circular tube for stress analysis, and the plane stress state of any point is shown in Figure 2. From Figure 2, it can be seen that the surface of the component is in a plane stress state. To obtain the magnitude and direction of the principal stress of the unit cell in the plane stress state, it is necessary to know the magnitude and direction of the stress in the two perpendicular directions of the unit cell, as well as the magnitude and direction of the shear stress [5].

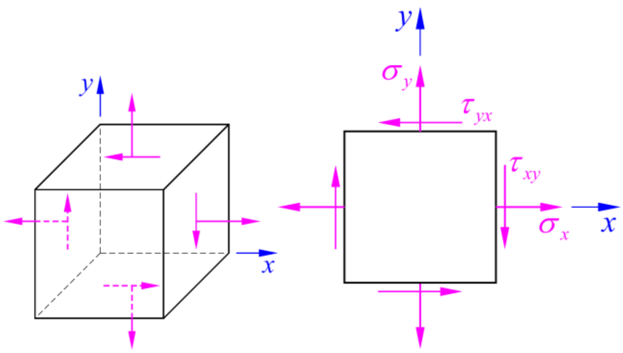


Fig.2: Stress analysis under plane stress state

According to Hooke's Law [6], it can be concluded that:

$$\begin{cases} \varepsilon_1 = \dfrac{1}{E}(\sigma_1 - \mu\sigma_2) \\ \varepsilon_2 = \dfrac{1}{E}(\sigma_2 - \mu\sigma_1) \end{cases} \quad (1)$$

Among them: σ_1 is the maximum principal stress, σ_2 is the minimum principal stress, ε_1 is the line strain in the direction of the maximum principal stress (σ_1), ε_2 is the line strain in the direction of the minimum principal stress (σ_2), E is the elastic modulus, and μ is Poisson's ratio.

The magnitude of the principal stress at any point can be obtained from equation (1) as follows:

$$\begin{cases} \sigma_1 = \dfrac{E}{1-\mu^2}(\varepsilon_1 + \mu\varepsilon_2) \\ \sigma_2 = \dfrac{E}{1-\mu^2}(\varepsilon_2 + \mu\varepsilon_1) \end{cases} \quad (2)$$

For the convenience of expressing strain in different directions, a coordinate system is set for the measuring point, and the strain components at the measuring point are defined as ε_x, ε_y, and γ_{xy}. The angle between the measuring point and the X-axis is defined as the main direction in the α direction, and the angle α is defined as positive when rotated counterclockwise. There are [7]:

$$\varepsilon_\alpha = \varepsilon_x \cos^2\alpha + \varepsilon_y \sin^2\alpha + \gamma_{xy} \sin\alpha\cos\alpha \quad (3)$$

After transforming the trigonometric relationship, we obtain:

$$\varepsilon_\alpha = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\alpha + \frac{\gamma_{xy}}{2}\sin 2\alpha \quad (4)$$

From equation (4), a strain circle can be obtained as shown in Figure 3, with ε on the x-axis and $-\gamma/2$ on the y-axis [8]. This strain circle can represent the variation of strain in different directions at a point under plane stress state.

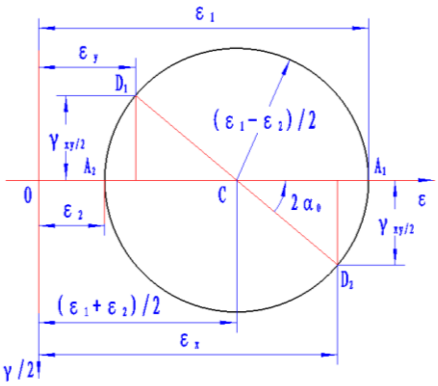


Fig.3: Strain Circle under Plane Stress State

According to the strain circle under the plane stress state shown in Figure 3, it can be concluded that:

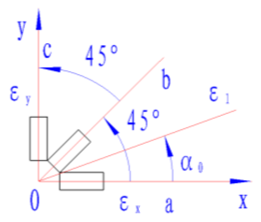
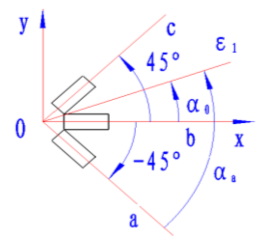
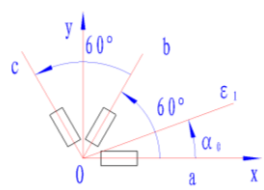
$$\begin{cases} \varepsilon_1 = \frac{1}{2}\left[(\varepsilon_x + \varepsilon_y) + \sqrt{(\varepsilon_x - \varepsilon_y)^2 + \gamma_{xy}^2}\right] \\ \varepsilon_2 = \frac{1}{2}\left[(\varepsilon_x + \varepsilon_y) - \sqrt{(\varepsilon_x - \varepsilon_y)^2 + \gamma_{xy}^2}\right] \\ 2\alpha_0 = \arctan\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} \end{cases} \quad (5)$$

3 TEST THE METHOD OF PASTING STRAING STRAIN GAUGES

In actual measurement, ε_x and ε_y can be directly measured, but γ_{xy} cannot be directly measured. The unique circle can be determined by three points, and as long as the line strain in any three directions is known, the unique strain circle can be determined. In actual measurement, when the direction of the principal stress at each point is unknown, it can be determined by arranging a right angle strain flower vertically pasted at the measured point, a right angle strain inclined 45 degree pasted method, or an equiangular strain flower pasted method [9].

According to equations (4) and (5), three strain flower pasting methods can be obtained, and the magnitude and direction of the principal strain are shown in Table 1.

Table 1: Three types of strain flower pasting methods and the magnitude and direction of the principal strain

Paste method	Schematic diagram of strain flower pasting method	Main strain magnitude and direction
Vertical pasting of right angle strain flower		$\varepsilon_1=\frac{1}{2}\left\{(\varepsilon_a+\varepsilon_c)+\sqrt{2\left[(\varepsilon_a-\varepsilon_b)^2+(\varepsilon_b-\varepsilon_c)^2\right]}\right\}$ $\varepsilon_2=\frac{1}{2}\left\{(\varepsilon_a+\varepsilon_c)-\sqrt{2\left[(\varepsilon_a-\varepsilon_b)^2+(\varepsilon_b-\varepsilon_c)^2\right]}\right\}$ $2\alpha_0=\arctan\frac{2\varepsilon_b-(\varepsilon_a+\varepsilon_c)}{\varepsilon_a-\varepsilon_c}$
Right angle strain inclined at 45° adhesive		$\varepsilon_1=\frac{1}{2}\left\{(\varepsilon_a+\varepsilon_c)+\sqrt{2\left[(\varepsilon_a-\varepsilon_b)^2+(\varepsilon_b-\varepsilon_c)^2\right]}\right\}$ $\varepsilon_2=\frac{1}{2}\left\{(\varepsilon_a+\varepsilon_c)-\sqrt{2\left[(\varepsilon_a-\varepsilon_b)^2+(\varepsilon_b-\varepsilon_c)^2\right]}\right\}$ $2\alpha_0=\arctan-\left[\frac{\varepsilon_a-\varepsilon_c}{2\varepsilon_b-(\varepsilon_a+\varepsilon_c)}\right]$
Adhesive of equiangular strain flower		$\varepsilon_1=\frac{\varepsilon_a+\varepsilon_b+\varepsilon_c}{3}+\frac{\sqrt{2}}{3}\sqrt{(\varepsilon_a-\varepsilon_b)^2+(\varepsilon_b-\varepsilon_c)^2+(\varepsilon_c-\varepsilon_a)^2}$ $\varepsilon_2=\frac{\varepsilon_a+\varepsilon_b+\varepsilon_c}{3}-\frac{\sqrt{2}}{3}\sqrt{(\varepsilon_a-\varepsilon_b)^2+(\varepsilon_b-\varepsilon_c)^2+(\varepsilon_c-\varepsilon_a)^2}$ $2\alpha_0=\arctan\frac{\sqrt{3}(\varepsilon_b-\varepsilon_c)}{2\varepsilon_a-\varepsilon_b-\varepsilon_c}$

By substituting the principal strain values of the three strain gauges in Table 1 into equation (2), the principal stress values corresponding to the three strain gauges can be obtained.

The magnitude of the principal stress for the vertical pasting method of the right angle strain flower and the 45 ° inclined pasting method of the right angle strain flower is:

$$\begin{cases}\sigma_1=\frac{E}{1-\mu^2}\left[\frac{1+\mu}{2}(\varepsilon_a+\varepsilon_c)+\frac{\sqrt{2}(1-\mu)}{2}\sqrt{(\varepsilon_a-\varepsilon_b)^2+(\varepsilon_b-\varepsilon_c)^2}\right]\\ \sigma_2=\frac{E}{1-\mu^2}\left[\frac{1+\mu}{2}(\varepsilon_a+\varepsilon_c)-\frac{\sqrt{2}(1-\mu)}{2}\sqrt{(\varepsilon_a-\varepsilon_b)^2+(\varepsilon_b-\varepsilon_c)^2}\right]\end{cases} \quad (6)$$

The principal stress magnitude of the adhesive method for equiangular strain flowers is:

$$\begin{cases}\sigma_1=\frac{E}{1-\mu^2}\left[\frac{1+\mu}{3}(\varepsilon_a+\varepsilon_b+\varepsilon_c)+\frac{\sqrt{2}(1-\mu)}{3}\sqrt{(\varepsilon_a-\varepsilon_b)^2+(\varepsilon_b-\varepsilon_c)^2+(\varepsilon_c-\varepsilon_a)^2}\right]\\ \sigma_2=\frac{E}{1-\mu^2}\left[\frac{1+\mu}{3}(\varepsilon_a+\varepsilon_b+\varepsilon_c)-\frac{\sqrt{2}(1-\mu)}{3}\sqrt{(\varepsilon_a-\varepsilon_b)^2+(\varepsilon_b-\varepsilon_c)^2+(\varepsilon_c-\varepsilon_a)^2}\right]\end{cases} \quad (7)$$

4 TESTING METHOD FOR COMBINED BENDING AND TWISTING DEFORMATION

Taking the method of pasting strain gauges at a 45 degree angle with a right angle strain inclination as an example, this study investigates the magnitude and direction of the principal stress at a specified point on the surface of a thin-walled cylinder, as well as the normal stress caused by the bending moment within the specified section, the shear stress caused by the torque, and the shear stress caused by the shear force.

Select four points, A、B、C、D, on the top, bottom, front, and back of the I-I section of the thin-walled cylinder shown in Figure 1. The positions of the four points are shown in Figure 4, and a set of strain flowers with a right angle strain inclination of 45 degrees are pasted at the points A、B、C、D, as shown in Figure 5.

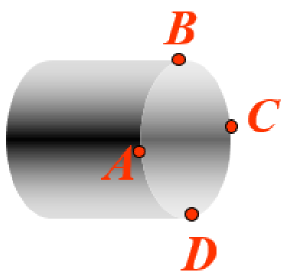


Fig.4: Surface test points on section I-I

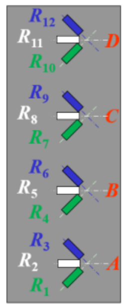


Fig.5: Schematic diagram of pasting strain gauges at test points

4.1 Testing method for magnitude and direction of principal stresses at points A、B、C、D

4.1.1 Theoretical analysis of the magnitude and direction of principal stresses at points A、B、C、D

According to the knowledge of material mechanics [10], there are three types of internal forces: bending moment M, torque T, and shear force Q on section I-I. Therefore, the stresses of the unit cells at points a/b/c/d on section I-Iare caused by these internal forces.

Points B、D: Point B experiences tensile stress, while point D experiences compressive stress. The shear stress caused by bending is zero, and the normal stress caused by bending and the shear stress caused by torsion constitute a biaxial stress state, as shown in Figure 6.

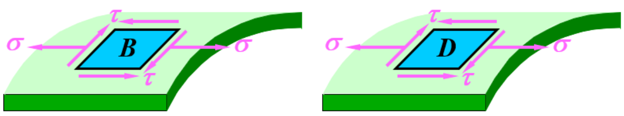


Fig.6: Stress state analysis of points B and D

According to the knowledge of material mechanics, the normal stress caused by bending moment M at points B and D and the shear stress caused by torsion T at points B and D are respectively:

$$\sigma_M = \frac{M}{W_z} = \frac{\Delta F \cdot L}{\frac{\pi D^3}{32}(1-\alpha^4)} \quad (8)$$

$$\tau_T = \frac{T}{W_t} = \frac{\Delta F \cdot a}{\frac{\pi D^3}{16}(1-\alpha^4)} \quad (9)$$

Among them, W_z is the bending section modulus of the circular tube; W_t is the torsional section modulus of the circular tube; $\alpha = d/D$ 。

According to Figure 6, the stress components at points B、D are as follows:

$$\begin{cases} \sigma_x = \sigma_M \\ \sigma_y = 0 \\ \tau_{xy} = \tau_T \end{cases} \quad (10)$$

The magnitude and principal direction of the normal stress at points B、D are:

$$\begin{cases} \sigma_{1,3} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{\sigma_M}{2} \pm \sqrt{\left(\frac{\sigma_M}{2}\right)^2 + \tau_T^2} \\ tg2\alpha = -\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -\frac{2\tau_T}{\sigma_M} \end{cases} \quad (11)$$

Among them, α is the angle between the main stress and the axis of the circular tube.

Two points A and C: Both points A and C are in a state of pure shear stress, and both points are on the neutral layer, so they will not cause principal stress. However, torque and shear force respectively cause shear stress, as shown in Figure 7.

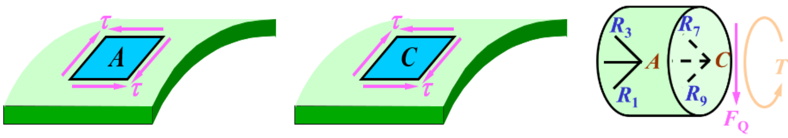


Fig.7: Stress state analysis of points A、C

According to the knowledge of material mechanics, the shear stress caused by the shear force Q at points B and D is:

$$\tau_Q = \frac{2Q}{A} = \frac{Q}{\pi R_0 t} \quad (12)$$

Among them, $Q = F$, $R_0 = (D+d)/2$, $t = (D-d)/2$.

According to Figure 7, the stress component at point A is:

$$\begin{cases} \sigma_x = \sigma_y = 0 \\ \tau_{xy} = \tau_T + \tau_Q \end{cases} \quad (13)$$

The stress component at point C is:

$$\begin{cases} \sigma_x = \sigma_y = 0 \\ \tau_{xy} = \tau_T - \tau_Q \end{cases} \quad (14)$$

The magnitude and direction of the principal stress at points A、C are:

$$\sigma_{1,3} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \pm\left(\tau_n \pm \tau_Q\right) \qquad (15)$$

$$\alpha = -45^\circ$$

4.1.2 Testing method for magnitude and direction of principal stresses at points A、B、C、D

Connect strain gauges $R_1 - R_{12}$ at four points A、B、C、D on section I-I using the half bridge common external compensation wiring method to the strain gauge, and measure the bridge as shown in Figure 8.

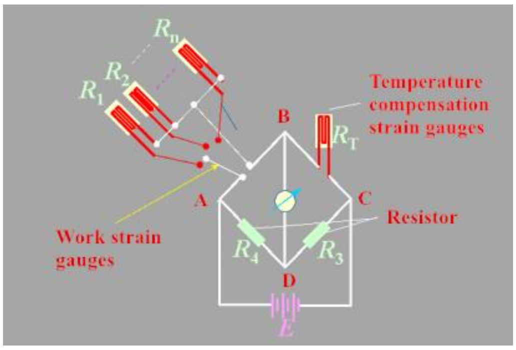


Fig.8: $R_1 - R_{12}$ Half Bridge Common Temperature Compensation Wiring Method

After loading the levels, the strains of points A、B、C、D can be measured separately. By substituting the strain ε_i of points A、B、C、D into the following formulas, the magnitude and direction of the principal stress at each point can be calculated.

$$\sigma_{1,2} = \frac{E\left(\varepsilon_{45^\circ} + \varepsilon_{-45^\circ}\right)}{2(1-\mu)} \pm \frac{\sqrt{2}}{2(1+\mu)}\sqrt{\left(\varepsilon_{0^\circ} - \varepsilon_{45^\circ}\right)^2 + \left(\varepsilon_{0^\circ} - \varepsilon_{-45^\circ}\right)^2}$$

$$tg\,2\alpha = \frac{\varepsilon_{45^\circ} - \varepsilon_{-45^\circ}}{2\varepsilon_{0^\circ} - \varepsilon_{45^\circ} - \varepsilon_{-45^\circ}} \qquad (16)$$

4.2 Testing method for normal stress caused by bending moment M

The theoretical value of the normal stress caused by bending M at points B and D is shown in equation (8).

Measure the normal stress caused by bending moment M by connecting strain gauges R_5 and R_{11} at 0° on the top and bottom (B、D) of the cylinder using the half bridge connection method, as shown in Figure 9.

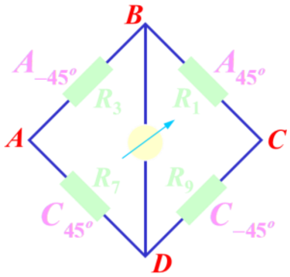


Fig.9: Wiring diagram for measuring normal stress caused by bending moment

According to the knowledge of electrical measurement method, the relationship between strain degree and positive strain caused by bending moment when wiring according to Figure 9 is:

$$\varepsilon_M = \frac{\varepsilon_{ds}}{2} \quad (17)$$

Among them: ε_{ds} is the measured strain reading; ε_M is the positive strain value caused by bending moment.

According to Hooke's Law, the normal stress caused by bending moment is:

$$\sigma_M = E\varepsilon_M = \frac{E\varepsilon_{ds}}{2} \quad (18)$$

Among them, σ_M is the normal stress caused by bending moment.

4.3 Testing method for shear stress caused by torque T

The theoretical value of the normal stress caused by torque T at points B and D is shown in equation (9).

Measure the shear stress caused by torque T by connecting four strain gauges R_1, R_3, R_7, and R_9 at two points -45 ° and 45 ° in front and behind the cylinder (A, C) to the entire bridge line, as shown in Figure 10.

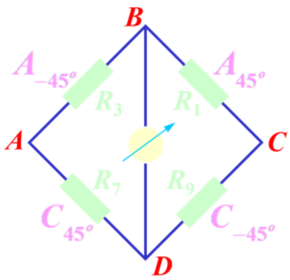


Fig.10: Wiring diagram of shear stress caused by torque measurement

According to the knowledge of electrical measurement method, the relationship between strain degree and strain caused by torque when wiring according to Figure 10 is:

$$\varepsilon_{ds} = 4\varepsilon_T \quad (19)$$

Among them, ε_T is the strain value caused by torque T.

The shear strain caused by torque is:

$$\gamma_T = 2\varepsilon_T = \frac{\varepsilon_{ds}}{2} \qquad (20)$$

Among them, γ_T is the shear strain caused by torque T.

According to Hooke's law of shear, the shear stress caused by torque is:

$$\tau_T = G\gamma_T = \frac{E\varepsilon_{ds}}{4(1+\mu)} \qquad (21)$$

Among them: τ_T is the shear stress caused by torque T; G is the shear elastic modulus of a thin-walled cylinder.

4.4 Testing method for shear stress caused by shear force Q

The theoretical value of shear stress caused by shear force Q at points B and D is shown in equation (12).

The shear stress caused by the shear force Q is measured by connecting four strain gauges R_1, R_3, R_7, and R_9 at -45 ° and 45 ° in front and behind the cylinder (A, C) to the entire bridge line, as shown in Figure 11.

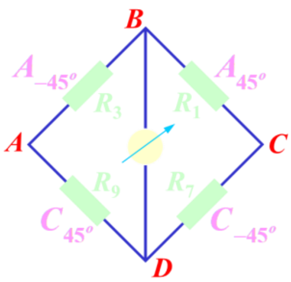


Fig.11: Wiring diagram of shear stress caused by shear force measurement

According to the knowledge of electrical measurement method, the relationship between the strain degree and the strain caused by shear force when wiring according to Figure 11 is:

$$\varepsilon_{ds} = 4\varepsilon_Q \qquad (22)$$

Among them, ε_Q is the strain value caused by shear force.

The shear strain caused by shear force is:

$$\gamma_Q = 2\varepsilon_Q = \frac{\varepsilon_{ds}}{2} \qquad (23)$$

Among them, γ_Q is the shear strain caused by shear force.

According to Hooke's law of shear, the shear stress caused by shear force is:

$$\tau_Q = G\gamma_Q = \frac{E\varepsilon_{ds}}{4(1+\mu)} \quad (24)$$

Among them, τ_Q is the shear stress caused by shear force.

5. CONCLUSIONS

This article first conducts stress analysis on the combined deformation of bending and twisting of thin-walled circular pipes to understand the stress state at any point on the surface of the thin-walled circular pipe; Then analyze the magnitude and direction of the principal strain at the designated point of the thin-walled circular tube under three different strain flower pasting methods; Finally, the electrical measurement method is used to study the magnitude and direction of the principal stress at a specified point on the surface of a thin-walled cylinder, as well as the normal stress caused by bending moment, shear stress caused by torque, and shear stress caused by shear force within a specified section when the thin-walled cylinder undergoes combined bending and twisting deformation. The above research not only provides a measurement method for separately measuring the strain component generated by a certain internal force in composite deformation, but also lays a solid practical foundation for students to engage in work related to engineering monitoring.

REFERENCES

[1] H Zheng, J Mo. Extended teaching of strain electrical measurement technology based on scientific research achievements. Mechanics and Practice, 2016, 38 (3): 324-327.
[2] F Zhu, P Bai, Lei D, et al. Discussion on the measurement scheme of bending shear stress in bending torsion combination experiment. Experimental Technology and Management, 2022, 39 (4): 138-144.
[3] H Li, Y Liu, M Wang, et al. Improvement of cylinder bending and twisting combination experiment. Experimental Technology and Management, 2017, 24 (11): 43-45.
[4] J Wu, M Yang, L Zhao, et al. Teaching of Bending Twist Combination Experiment in Mechanics Experiment. China Metallurgical Education, 2019 (6): 65-68.
[5] R Zhang, Y Lu. Experimental stress analysis. Mechanical Industry Press, 1983.
[6] X Sun, X Fang, L Guan. Mechanics of Materials (I). Higher Education Press, 2002.
[7] Y Kang, Y Cheng, P Xia, et al. Reform of Materials Mechanics Experimental Teaching under the Background of New Engineering. Education and Teaching Forum 2022 (18): 73-76.
[8] L Sun, J Wang, Mechanics Experiment. Nanjing: Hehai University Press, 2019.
[9] J Huang, R Tao, Z Jiang, et al. The influence of selecting different strain gauges on the results of principal stress testing. Laboratory research and exploration, 2016, 35 (7): 32-36.
[10] H Liu. Mechanics of Materials (I). Higher Education Press, 2004.