OBSERVATIONS ON THE BIQUADRATIC EQUATION WITH FIVE UNKNOWNS

\[ 2(x - y)(x^3 + y^3) = (1 + 3k^2)(x^2 - y^2)w^2 \]

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Abstract: We obtain infinitely many non-zero integer quintuples \( x, y, X, Y, w \) satisfying the biquadratic equation with five unknowns

\[ 2(x - y)(x^3 + y^3) = (1 + 3k^2)(x^2 - y^2)w^2 \]

and various interesting relations between the solutions and special numbers, octahedral numbers, centered polygonal & pyramidal numbers are exhibited.

Key Words: Bi-Quadratic equation with five unknowns, Integral solutions, polygonal number, Pyramidal numbers, Centered polygonal.

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NOTATIONS USED:

1. Polygonal number of rank ‘n’ with sides \( m \)

\[ t_{m,n} = n \left( 1 + \frac{(n-1)(m-2)}{2} \right) \]

2. Stella octangular number of rank ‘n’

\[ SO_n = n(2n^2 - 1) \]

3. Pyramidal number of rank ‘n’ sides \( m \)

\[ P_m^a = \frac{n(n+1)}{6}[(m-2)n + (5-m)] \]

4. Pronic number of rank ‘n’

\[ Pr_n = n(n+1) \]

5. Octahedral number of rank ‘n’

\[ OH_n = \frac{1}{3}[n(2n^2 + 1)] \]

1. INTRODUCTION:

Bi-quadratic Diophantine Equations, homogeneous and non- homogeneous, have aroused the interest of numerous
Mathematicians since ambiguity as can be seen from [1,2,17-19]. In the context one may refer [3-16] for varieties of problems on the Diophantine equations with two, three and four variables. This communication concerns with the problems of determining non-zero integral solutions of bi-quadratic equation in six unknowns represented by

\[ 2(x - y)(x^3 + y^3) = (1 + 3k^2)(X^2 - Y^2)w^2 \]

A few interesting relations between the solutions and special polygonal numbers are presented.

2. METHOD OF ANALYSIS:

The Diophantine equation representing the bi-quadratic equation with five unknowns under consideration is

\[ 2(x - y)(x^3 + y^3) = (1 + 3k^2)(X^2 - Y^2)w^2 \]  \hspace{1cm} (1)

Introducing the linear transformations

\[ x = u + v \]
\[ y = u - v \]
\[ X = 2u + v \]  \hspace{1cm} (2)
\[ Y = 2u - v \]

in (1), it simplifies to

\[ u^2 + 3v^2 = (1 + 3k^2)w^2 \]  \hspace{1cm} (3)

The above equation (3) is solved through different methods and thus, one obtains distinct sets of integer solutions to (1)

2.1 set.1

Let \[ w = a^2 + 3b^2 \]  \hspace{1cm} (4)

Substituting (4) in (3) and using the methods of factorization, define

\[ (u + i\sqrt{3}v) = (1 + i\sqrt{3}k)(a + i\sqrt{3}b)^2 \]  \hspace{1cm} (5)

Equating real and imaginary parts, we have

\[ u = a^2 - 3b^2 - 6abk \]  \hspace{1cm} (6)
\[ v = ka^2 - 3b^2k + 2ab \]

Substituting the values of u & v in (2), the non-zero distinct integral solutions of (1) are given by

\[ x(a,b) = a^2 + ka^2 - 3b^2 - 3b^2k + 2ab - 6abk \]
\[ y(a,b) = a^2 - ka^2 - 3b^2 + 3b^2k - 2ab - 6abk \]
\[ X(a,b) = 2a^2 + ka^2 - 6b^2 - 3b^2k + 2ab - 12abk \]
\[ Y(a,b) = 2a^2 - ka^2 - 6b^2 + 3b^2k - 2ab - 12abk \]
\[ w(a,b) = a^2 + 3b^2 \]  \hspace{1cm} (7)
2.2 Properties:

A few interesting properties obtained as follows:

(i) \( x(a,a+1) + y(a,a+1) + 4(t_{4a} - 6kt_{t_{4a}}) = 6(\mod 2) \)

(ii) \( x(a,a+1) - y(a,a+1) + 2k(2t_{4a} + 6a + 3) - 4p_k = 0 \)

(iii) \( x(2a^2 + 1) + y(a,2a^2 + 1) + 36OH_a + 2(1t_{4a} + 12t_{t_{4a}} + 3) = 0 \)

(iv) \( x(a,2a^2 - 1) + w(a,2a^2 - 1) + 2(2a_{4a} + SO_a) + k(-13t_{4a} + 12t_{t_{4a}} + 6SO_a + 3) = 0 \)

(v) \( x(a,2a - 1) - w(a,2a - 1) + 2(2a_{4a} - t_{4a}) + k(1t_{4a} + 6t_{t_{4a}} - 12a + 3) = -6(\mod 24) \)

(vi) \( X(a,3a - 1) + Y(a,3a - 1) + 104t_{4a} + 48kt_{t_{4a}} = -12(\mod 72) \)

(vii) \( X(a,4a - 3) - Y(a,4a - 3) - 2k(-47t_{4a} + 72a - 27) - 4t_{t_{4a}} = 0 \)

2.3 Note:

In (5) replace \((1 + i\sqrt{3}k)\) by \((-1 + i\sqrt{3}k)\)

\( \therefore (u + i\sqrt{3}v) = (-1 + i\sqrt{3}k)(a + i\sqrt{3}b)^2 \)

(8)

Following the procedure presented in set. 1 a different solution is given by

\( x(a,b) = -a^2 + ka^2 + 3b^2 - 3b^2k - 2ab - 6abk \)

\( y(a,b) = -a^2 - ka^2 + 3b^2 + 3b^2k + 2ab - 6abk \)

\( X(a,b) = -2a^2 + ka^2 + 6b^2 - 3b^2k - 2ab - 12abk \)

\( Y(a,b) = -2a^2 - ka^2 + 6b^2 + 3b^2k + 2ab - 12abk \)

\( w(a,b) = a^2 + 3b^2 \)

(9)

3.1 set.2

(3) can be written as

\( u^2 + 3v^2 = (1 + 3k^2)w^2 \)

(10)

Write 1 as \( 1 = \frac{(1 + i\sqrt{3})(1 - i\sqrt{3})}{4} \)

(11)

Using (11) in (10) and employing the method of factorization, define

\( (u + i\sqrt{3}v) = (1 + i\sqrt{3k})(a + i\sqrt{3b})^2 \cdot \frac{(1 + i\sqrt{3})}{2} \)

(12)

Equating real & imaginary parts & replacing \( a \) by \( 2a \) & \( b \) by \( 2b \), we have

\( u = 2a^2 - 6ka^2 - 6b^2 + 18b^2k \)

\( -12ab - 12abk \)

(13)

\( v = 2a^2 + 2ka^2 - 6b^2 - 6b^2k \)

\( + 4ab - 12abk \)

Using (13) & (2) we get the integral solutions of (1) to be
\[ x(a, b) = 4a^2 - 4ka^2 - 12b^2 + 12b^2k - 8ab - 24abk \]
\[ y(a, b) = -8ka^2 + 24b^2k - 16ab \]
\[ X(a, b) = 6a^2 - 10ka^2 - 18b^2 + 30b^2k - 20ab - 36abk \]
\[ Y(a, b) = 2a^2 - 14ka^2 - 6b^2 + 42b^2k - 28ab - 12abk \]
\[ w(a, b) = 4a^2 + 12b^2 \]

\[ (14) \]

3.2 Properties:

A few interesting properties obtained as follows:

(i). \[ x(a, 2a^2 - 1) + y(a, 2a^2 - 1) + 4(-13t_{4,a} + 12t_{4,a}^2 + 6SO_a + 3) + k(15t_{4,a} - 144t_{4,a}^2 + 24SO_a - 36) = 0 \]

(ii). \[ x(a, a + 1) - y(a, a + 1) + 8(t_{4,a} - P_a) + 4k(2t_{4,a} + 6P_a + 6a + 3) \equiv -12 \pmod{24} \]

(iii). \[ x(a, 2a^2 + 1) + w(a, 2a^2 + 1) + 8(-t_{4,a} - 3OH_a) - 2k(22t_{4,a} + 24t_{4,a}^2 - 36OH_a + 6) = 0 \]

(iv). \[ x(a, a + 1) - w(a, a + 1) + 8(3t_{4,a} + 2t_{3,a}) + 4k(2t_{4,a} - 12t_{3,a} + 6a + 3) \equiv -24 \pmod{48} \]

(v). \[ y(a^2, a + 1) + w(a^2, a + 1) + 8k(t_{4,a}^2 - 3t_{4,a} - 6a - 3) + 4(8P_a^2 - t_{4,a}^2 - 3t_{4,a}) \equiv 12 \pmod{24} \]

(vi). \[ y(a, 2a - 1) - w(a, 2a - 1) + 4(13t_{4,a} + 4t_{6,a}) - 4k(22t_{4,a} - 24a + 6) \equiv -12 \pmod{48} \]

(vii). \[ X(a, 3a - 2) + Y(a, 3a - 2) - 8(-26t_{4,a} - 6t_{8,a}) + 8k(78t_{4,a} - 6t_{8,a} - 72) \equiv -96 \pmod{288} \]

(viii). \[ x(a, 7a - 5) = y(a, 7a - 5) - 8(-73t_{4,a} + 2t_{9,a}) - 4k(-146t_{4,a} - 12t_{9,a} + 210a - 75) \equiv -300 \pmod{840} \]

3.3 Note:

\[ \ln(12) \text{ replace } \frac{(1 + i\sqrt{3})}{2} \text{ by } \frac{(-1 + i\sqrt{3})}{2} \]

\[ \therefore (u + i\sqrt{3}) = (1 + i\sqrt{3})(a + i\sqrt{3}b)^2 \cdot \frac{(-1 + i\sqrt{3})}{2} \]

(15)

Following the procedure presented in set.2 a different solution is given by

\[ x(a, b) = -8ka^2 + 24b^2k - 16ab \]
\[ y(a, b) = -4a^2 - 4ka^2 + 12b^2 + 12b^2k - 8ab + 24abk \]
\[ X(a, b) = -2a^2 - 14ka^2 + 6b^2 + 42b^2k - 28ab + 12abk \]
\[ Y(a, b) = -6a^2 - 10ka^2 + 18b^2 + 30b^2k - 20ab + 36abk \]
\[ w(a, b) = 4a^2 + 12b^2 \]

(16)

Case 1:

\[ u^2 + 3v^2 = (1 + 3k^2)w^2 \cdot 1 \]

Instead of (11), write 1 as

\[ 1 = \frac{(1 + i4\sqrt{5})(1 - i4\sqrt{3})}{49} \]

(17)

Using (17) in (10) and employing the method of factorization, define
(u + i\sqrt{3}v) = (1 + i\sqrt{5})(a + i\sqrt{3}b)^2 * (1 + i4\sqrt{3}) \over 7
(18)

Equating real & imaginary parts & replacing a by
7a & b by 7b
u = 7a^2 - 84ka^2 - 21b^2 + 252b^2k - 168ab - 42abk
v = 28a^2 + 7ka^2 - 84b^2 - 21b^2k + 14ab - 168abk
(19)

Using (19) & (2) we get the integral solutions of
(1) to be

\begin{align*}
x(a, b) &= 35a^2 - 77ka^2 - 105b^2 + 23b^2k \\
&- 154ab - 210abk \\
y(a, b) &= -21a^2 - 91ka^2 + 63b^2 + \\
&273b^2k - 182ab + 126abk \\
x(a, b) &= 42a^2 - 161ka^2 - 126b^2 \\
&+ 48b^2k - 322ab - 252abk \\
Y(a, b) &= -14a^2 - 175ka^2 + 42b^2 \\
&+ 525b^2k - 350ab + 84abk \\
w(a, b) &= 49a^2 + 147b^2
\end{align*}

Case 2:

\begin{align*}
u^2 + 3v^2 &= (1 + 3k^2)w^2 * 1 \\
1 &= (11 + i5\sqrt{3})(11 - i5\sqrt{3}) \\
&\over 196
(23)

Using (23) in (10) and employing the method of factorization, define

\begin{align*}
(u + i\sqrt{3}v) &= (1 + i\sqrt{3})(a + i\sqrt{3}b)^2 * (11 + i5\sqrt{3}) \\
&\over 14
(24)

Equating real & imaginary parts & replacing a by
14a & b by 14b

\begin{align*}
u &= 154a^2 - 210ka^2 - 462b^2 + \\
&630b^2k - 420ab - 924abk \\
v &= 70a^2 + 154ka^2 - 210b^2 \\
&- 462b^2k + 308ab - 420abk
(25)
\end{align*}
Using (25) & (2) we get the integral solutions of (1) to be

\[
x(a, b) = 224a^2 - 56ka^2 - 672b^2 + 168b^2k - 112ab - 1344abk
\]
\[
y(a, b) = 84a^2 - 364ka^2 - 252b^2 + 1092b^2k - 728ab - 504abk
\]
\[
X(a, b) = 378a^2 - 266ka^2 - 1134b^2 + 798b^2k - 532ab - 2268abk
\]
\[
Y(a, b) = 238a^2 - 574ka^2 - 714b^2 + 172b^2k - 1148ab - 1428abk
\]
\[
w(a, b) = 196a^2 + 588b^2
\]

3.5 Note

In (24) replace

\[
\frac{(11 + i5\sqrt{3})}{14} \text{ by } \frac{(-11 + i5\sqrt{3})}{14}
\]

\[
\therefore (u + i\sqrt{3}v) = (1 + i\sqrt{3}k)(a + i\sqrt{3}b)^2 \cdot \frac{(-11 + i5\sqrt{3})}{14}
\]

(27)

Following the similar procedure as in case 2, the corresponding integer solutions of (1) are obtained as

\[
x(a, b) = -84a^2 - 364ka^2 + 252b^2 + 1092b^2k - 728ab - 504abk
\]
\[
y(a, b) = -224a^2 - 56ka^2 + 672b^2 + 168b^2k - 112ab + 1344abk
\]
\[
X(a, b) = -238a^2 - 574ka^2 + 714b^2 + 172b^2k - 1148ab + 1428abk
\]
\[
Y(a, b) = -378a^2 - 266ka^2 + 1134b^2 + 798b^2k - 532ab + 2268abk
\]
\[
w(a, b) = 196a^2 + 588b^2
\]

Case 3:

\[u^2 + 3v^2 = (1 + 3k^2)w^2 \times 1\]

Instead of (23), write 1 as

\[
1 = \frac{(13 + i3\sqrt{3})(13 - i3\sqrt{3})}{196}
\]

(29)

Following the similar procedure as in case 2, the corresponding integer solutions of (1) are obtained as

\[
x(a, b) = 224a^2 + 56ka^2 - 672b^2 - 168b^2k + 112ab - 1344abk
\]
\[
y(a, b) = 140a^2 - 308ka^2 - 420b^2 + 924b^2k - 616ab - 840abk
\]
\[
X(a, b) = 406a^2 - 70ka^2 - 1218b^2 + 210b^2k - 140ab - 2436abk
\]
\[
Y(a, b) = 322a^2 - 434ka^2 - 966b^2 + 1302b^2k - 868ab - 1932abk
\]
\[
w(a, b) = 196a^2 + 588b^2
\]

3.6 Note
In (29) replace \( \frac{(13 + i3\sqrt{3})}{14} \) by \( \frac{(-13 + i3\sqrt{3})}{14} \)

\[ \therefore (u + i\sqrt{3}v) = (1 + i\sqrt{3})(a + i\sqrt{3}b)^2 \times \frac{(11 + i4\sqrt{3})}{13} \]  
(34)

Equating real & imaginary parts & replacing a by 13a & b by 13b, we have

\[ u = 143a^2 - 156ka^2 - 429b^2 
+ 468b^2k - 32ab - 858abk \]  
(35)

Using (35) & (2) we get the integral solutions of (1) to be

\[ x(a,b) = 195a^2 - 13ka^2 - 585b^2 
- 39b^2k - 26ab - 1170abk \]
\[ y(a,b) = 91a^2 - 299ka^2 - 273b^2 
+ 897b^2k - 598ab - 546abk \]
\[ X(a,b) = 338a^2 - 169ka^2 - 1014b^2 
+ 507b^2k - 338ab - 2028abk \]
\[ Y(a,b) = 234a^2 - 455ka^2 - 702b^2 
+ 1365b^2k - 910ab - 1404abk \]
\[ w(a,b) = 169a^2 + 507b^2 \]  
(36)

**Case 4:**

\[ u^2 + 3v^2 = (1 + 3k^2)w^2 \times 1 \]

Instead of (29), write 1 as

\[ 1 = \frac{(11 + i4\sqrt{3})(11 - i4\sqrt{3})}{169} \]  
(33)

Using (33) in (10) and employing the method of factorization, define

\[ \therefore (u + i\sqrt{3}v) = (1 + i\sqrt{3})(a + i\sqrt{3}b)^2 \times \frac{(-11 + i4\sqrt{3})}{13} \]  
(37)
Following the similar procedure as in case 3, the corresponding integer solutions of (1) are found to be

\[ x(a, b) = -91a^2 - 299ka^2 + 273b^2 + 897b^2k - 598ab + 546abk \]
\[ y(a, b) = -195a^2 - 13ka^2 + 585b^2 + 39b^2k - 26ab + 1170abk \]
\[ X(a, b) = -234a^2 - 455ka^2 + 702b^2 + 1365b^2k - 910ab + 1404abk \]
\[ Y(a, b) = -338a^2 - 169ka^2 + 1014b^2 + 507b^2k - 338ab + 2028abk \]
\[ w(a, b) = 169a^2 + 507b^2 \] (38)

4.1 set.III

Rewrite (3), we get

\[ u^2 - w^2 = 3(k^2w^2 - v^2) \] (39)

The above equation can be written in the form of ratio as

\[ \frac{u + w}{kw + v} = \frac{3(kw - v)}{u - w} = \frac{\alpha}{\beta}, \beta \neq 0 \] (40)

(40) is equivalent to the system of double equations

\[ \frac{u + w}{kw + v} = \frac{\alpha}{\beta}, \frac{3(kw - v)}{u - w} = \frac{\alpha}{\beta} \]

\[ \Rightarrow \beta u - \alpha v + (\beta - c\alpha)w = 0 \]
\[ \Rightarrow \alpha u + 3\beta v + (-3\beta k - \alpha)w = 0 \] (41)

Solving the above equations by applying the method of cross multiplication, we have

\[ u = \alpha^2 - 3\beta^2 + 6\alpha\beta k \] (42)
\[ v = -k\alpha^2 + 3\beta^2 k + 2\alpha\beta \]

Using the value of \( u \) & \( v \) in (2), we get the corresponding non-zero integer solutions to (1) to be

\[ x(a, b) = \alpha^2 - k\alpha^2 - 3\beta^2 + 3\beta^2 k + 2\alpha\beta \]
\[ y(a, b) = \alpha^2 + k\alpha^2 - 3\beta^2 - 3\beta^2 k + 2\alpha\beta + 6\alpha\beta k \]
\[ X(a, b) = 2\alpha^2 - k\alpha^2 - 6\beta^2 + 3\beta^2 k - 2\alpha\beta + 12\alpha\beta k \] (43)
\[ Y(a, b) = 2\alpha^2 + k\alpha^2 - 6\beta^2 - 3\beta^2 k - 2\alpha\beta + 12\alpha\beta k \]
\[ w(a, b) = \alpha^2 + 3\beta^2 \]

4.2 Properties:

A few interesting properties obtained as follows:

\[ x(a,2a^2 + 1) + y(a,2a^2 + 1) - 2(13k\alpha_{a,4} - 12\alpha_{a,1} + 18kOH_{a} - 3) = 0 \]
\[ x(a,2a^2 - 1) - y(a,2a^2 - 1) - 2k(-13\alpha_{a,4} + 12\alpha_{a,1} + 3) - 4SO_{a} = 0 \]
\[ x(a, a + 1) + w(a, a + 1) - 2(t_{4,a} + P_a) \\
- k(2t_{4,a} + 6P_a + 6a + 3) = 0 \]
\[ x(a^2, a + 1) - w(a^2, a + 1) \\
- k(-t_{4,a^2} + 3t_{3,a} + 12P_a^4 + 6a + 3) \\
- 2(-3t_{4,a} + 2P_a^5) \equiv -6 \text{(mod 2)} \]
\[ y(a, 4a - 3) + w(a, 4a - 3) - 2(t_{4,a} - t_{10,a}) \\
- k(-47t_{4,a} + 6t_{10,a} + 72a - 27) = 0 \]
\[ y(a, 5a - 3) - w(a, 5a - 3) \\
- k(-74t_{4,a} + 12t_{10,a} + 90a - 27) \\
+ 2(75t_{4,a} + 2t_{10,a}) \equiv -54 \text{(mod 80)} \]
\[ X(a, 3a - 2) + Y(a, 3a - 2) \\
- 8(-13t_{4,a} + 3kt_{8,a}) \equiv -48 \text{(mod 44)} \]
\[ X(a, a + 1) - Y(a, a + 1) \\
- 2k(2t_{4,a} + 6a + 3) - 8t_{3,a} = 0 \]

In addition to (40), (39) may also be expressed in the form of ratios in three different cases that are presented below.

**Case 1:**

\[
\frac{u + w}{3(kw + v)} = \frac{kw - v}{u - w} = \frac{\alpha}{\beta}, \beta \neq 0 \quad (44)
\]

**Case 2:**

\[
\frac{u + w}{3(kw - v)} = \frac{kw + v}{u - w} = \frac{\alpha}{\beta}, \beta \neq 0 \quad (45)
\]

**Case 3:**

\[
\frac{u + w}{kw - v} = \frac{3(kw + v)}{u - w} = \frac{\alpha}{\beta}, \beta \neq 0 \quad (46)
\]

Solving each of the above system of equation by the following procedure as presented in set 3, the corresponding integer solution to (1) are found to be as given below.

**4.3 Solution for case 1:**

\[
x(a, b) = 3\alpha^2 - 3k\alpha^2 - 2\beta^2 \\
+ \beta^2k + 2\alpha\beta + 6\alpha\beta k \\
y(a, b) = 3\alpha^2 + 3k\alpha^2 - \beta^2 \\
- \beta^2k - 2\alpha\beta + 6\alpha\beta k \\
X(a, b) = 6\alpha^2 - 3k\alpha^2 - 2\beta^2 \\
+ \beta^2k + 2\alpha\beta + 12\alpha\beta k \\
Y(a, b) = 6\alpha^2 + 3k\alpha^2 - 2\beta^2 \\
- \beta^2k - 2\alpha\beta + 12\alpha\beta k \\
w(a, b) = 3\alpha^2 + \beta^2
\]

**Solution for case 2:**

\[
x(a, b) = -3\alpha^2 - 3k\alpha^2 + \\
\beta^2 + 3\beta^2k + 2\alpha\beta - 6\alpha\beta k \\
y(a, b) = -3\alpha^2 + 3k\alpha^2 + \\
\beta^2 - 3\beta^2k - 2\alpha\beta - 6\alpha\beta k \\
X(a, b) = -6\alpha^2 - 3k\alpha^2 + \\
2\beta^2 + 3\beta^2k + 2\alpha\beta - 12\alpha\beta k \\
Y(a, b) = -6\alpha^2 + 3k\alpha^2 + \\
2\beta^2 - 3\beta^2k - 2\alpha\beta - 12\alpha\beta k \\
w(a, b) = -3\alpha^2 - \beta^2
\]
4.4 Solution for case

\[ x(a,b) = -\alpha^2 - k\alpha^2 + 3\beta^2 + 3\beta^2 k + 2\alpha\beta - 6\alpha\beta k \]
\[ y(a,b) = -\alpha^2 + k\alpha^2 + 3\beta^2 - 3\beta^2 k - 2\alpha\beta - 6\alpha\beta k \]

\[ X(a,b) = -2\alpha^2 - k\alpha^2 + 6\beta^2 + 3\beta^2 k + 2\alpha\beta - 12\alpha\beta k \]
\[ Y(a,b) = -2\alpha^2 + k\alpha^2 + 6\beta^2 - 3\beta^2 k - 2\alpha\beta - 12\alpha\beta k \]
\[ w(a,b) = -\alpha^2 - 3\beta^2 \]

5. Conclusion

In this paper, we have presented different choices of integer solutions to homogenous the biquadratic equation with five unknowns,

\[ 2(x - y)(x^3 + y^3) = (1 + 3k^2)(X^2 - Y^2)w^2 \]

It is worth mentioning here that in (2), the linear transformation for \( X \) & \( Y \) may also be considered as (1)\( X=uv+2, Y=uv-2 \) and (2) employing the above two forms of transformations for \( X, Y \) are obtained. To conclude, as bi-quadratic equations are rich in variety, one may consider other forms of bi-quadratic equations & search for corresponding properties.

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7. REFERENCES


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x^3 + y^3 + z^3 = 3xyz + 2(x + y)w^3, Antarctica

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